

# Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.5-u-a+b-arctan-c+d-x^p

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## Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Listing of CAS systems tested . . . . .	3
1.2	Results . . . . .	3
1.3	Performance . . . . .	5
1.4	list of integrals that has no closed form antiderivative . . . . .	6
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	6
1.6	list of integrals solved by CAS but failed verification . . . . .	6
1.7	Timing . . . . .	7
1.8	Verification . . . . .	7
1.9	Important notes about some of the results . . . . .	7
1.10	Design of the test system . . . . .	8
<b>2</b>	<b>detailed summary tables of results</b>	<b>11</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	11
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	12
2.3	Detailed conclusion table specific for Rubi results . . . . .	22
<b>3</b>	<b>Listing of integrals</b>	<b>25</b>
3.1	$\int (ce + dex)^3 (a + b \tan^{-1}(c + dx)) dx$ . . . . .	25
3.2	$\int (ce + dex)^2 (a + b \tan^{-1}(c + dx)) dx$ . . . . .	29
3.3	$\int (ce + dex) (a + b \tan^{-1}(c + dx)) dx$ . . . . .	32
3.4	$\int \frac{a+b \tan^{-1}(c+dx)}{ce+dex} dx$ . . . . .	35
3.5	$\int \frac{a+b \tan^{-1}(c+dx)}{(ce+dex)^2} dx$ . . . . .	38
3.6	$\int \frac{a+b \tan^{-1}(c+dx)}{(ce+dex)^3} dx$ . . . . .	42
3.7	$\int (ce + dex)^3 (a + b \tan^{-1}(c + dx))^2 dx$ . . . . .	45
3.8	$\int (ce + dex)^2 (a + b \tan^{-1}(c + dx))^2 dx$ . . . . .	50
3.9	$\int (ce + dex) (a + b \tan^{-1}(c + dx))^2 dx$ . . . . .	55
3.10	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{ce+dex} dx$ . . . . .	59
3.11	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^2} dx$ . . . . .	63
3.12	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^3} dx$ . . . . .	67
3.13	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^4} dx$ . . . . .	71

3.14	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^5} dx$	76
3.15	$\int (ce+dex)^2 (a+b \tan^{-1}(c+dx))^3 dx$	81
3.16	$\int (ce+dex) (a+b \tan^{-1}(c+dx))^3 dx$	88
3.17	$\int \frac{(a+b \tan^{-1}(c+dx))^3}{ce+dex} dx$	94
3.18	$\int \frac{(a+b \tan^{-1}(c+dx))^3}{(ce+dex)^2} dx$	99
3.19	$\int \frac{(a+b \tan^{-1}(c+dx))^3}{(ce+dex)^3} dx$	104
3.20	$\int \frac{(a+b \tan^{-1}(c+dx))^3}{(ce+dex)^4} dx$	109
3.21	$\int \frac{\tan^{-1}(1+x)}{2+2x} dx$	114
3.22	$\int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$	117
3.23	$\int (a+bx)^2 \sqrt{\tan^{-1}(a+bx)} dx$	120
3.24	$\int (e+fx)^3 (a+b \tan^{-1}(c+dx)) dx$	122
3.25	$\int (e+fx)^2 (a+b \tan^{-1}(c+dx)) dx$	126
3.26	$\int (e+fx) (a+b \tan^{-1}(c+dx)) dx$	130
3.27	$\int (a+b \tan^{-1}(c+dx)) dx$	134
3.28	$\int \frac{a+b \tan^{-1}(c+dx)}{e+fx} dx$	137
3.29	$\int \frac{a+b \tan^{-1}(c+dx)}{(e+fx)^2} dx$	140
3.30	$\int \frac{a+b \tan^{-1}(c+dx)}{(e+fx)^3} dx$	144
3.31	$\int (e+fx)^2 (a+b \tan^{-1}(c+dx))^2 dx$	149
3.32	$\int (e+fx) (a+b \tan^{-1}(c+dx))^2 dx$	155
3.33	$\int (a+b \tan^{-1}(c+dx))^2 dx$	160
3.34	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{e+fx} dx$	164
3.35	$\int \frac{(a+b \tan^{-1}(c+dx))^2}{(e+fx)^2} dx$	168
3.36	$\int (e+fx)^2 (a+b \tan^{-1}(c+dx))^3 dx$	176
3.37	$\int (e+fx) (a+b \tan^{-1}(c+dx))^3 dx$	183
3.38	$\int (a+b \tan^{-1}(c+dx))^3 dx$	188
3.39	$\int \frac{(a+b \tan^{-1}(c+dx))^3}{e+fx} dx$	192
3.40	$\int \frac{(a+b \tan^{-1}(c+dx))^3}{(e+fx)^2} dx$	196
3.41	$\int (e+fx)^m (a+b \tan^{-1}(c+dx)) dx$	204
3.42	$\int (e+fx)^m (a+b \tan^{-1}(c+dx))^2 dx$	207
3.43	$\int (e+fx)^m (a+b \tan^{-1}(c+dx))^3 dx$	209
3.44	$\int x^3 \tan^{-1}(a+bx) dx$	211
3.45	$\int x^2 \tan^{-1}(a+bx) dx$	215
3.46	$\int x \tan^{-1}(a+bx) dx$	218
3.47	$\int \tan^{-1}(a+bx) dx$	221
3.48	$\int \frac{\tan^{-1}(a+bx)}{x} dx$	224
3.49	$\int \frac{\tan^{-1}(a+bx)}{x^2} dx$	227
3.50	$\int \frac{\tan^{-1}(a+bx)}{x^3} dx$	231
3.51	$\int \frac{\tan^{-1}(a+bx)}{x^4} dx$	235
3.52	$\int \frac{\tan^{-1}(a+bx)}{c+dx^3} dx$	239

3.53	$\int \frac{\tan^{-1}(a+bx)}{c+dx^2} dx$ . . . . .	244
3.54	$\int \frac{\tan^{-1}(a+bx)}{c+dx} dx$ . . . . .	248
3.55	$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}} dx$ . . . . .	251
3.56	$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}} dx$ . . . . .	255
3.57	$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}} dx$ . . . . .	260
3.58	$\int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}} dx$ . . . . .	265
3.59	$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx$ . . . . .	270
3.60	$\int \frac{\tan^{-1}(a+bx)}{1+x^2} dx$ . . . . .	276
3.61	$\int \frac{\tan^{-1}(d+ex)}{a+bx^2} dx$ . . . . .	280
3.62	$\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2} dx$ . . . . .	284
3.63	$\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} dx$ . . . . .	290
3.64	$\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} dx$ . . . . .	293
3.65	$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$ . . . . .	296
3.66	$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$ . . . . .	299
3.67	$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} dx$ . . . . .	302
3.68	$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} dx$ . . . . .	305
3.69	$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$ . . . . .	309
3.70	$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$ . . . . .	312

#### 4 Listing of Grading functions

315



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 70 ]. This is test number [ 151 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 70 )	% 0. ( 0 )
Mathematica	% 95.71 ( 67 )	% 4.29 ( 3 )
Maple	% 98.57 ( 69 )	% 1.43 ( 1 )
Maxima	% 45.71 ( 32 )	% 54.29 ( 38 )
Fricas	% 40. ( 28 )	% 60. ( 42 )
Sympy	% 32.86 ( 23 )	% 67.14 ( 47 )
Giac	% 40. ( 28 )	% 60. ( 42 )

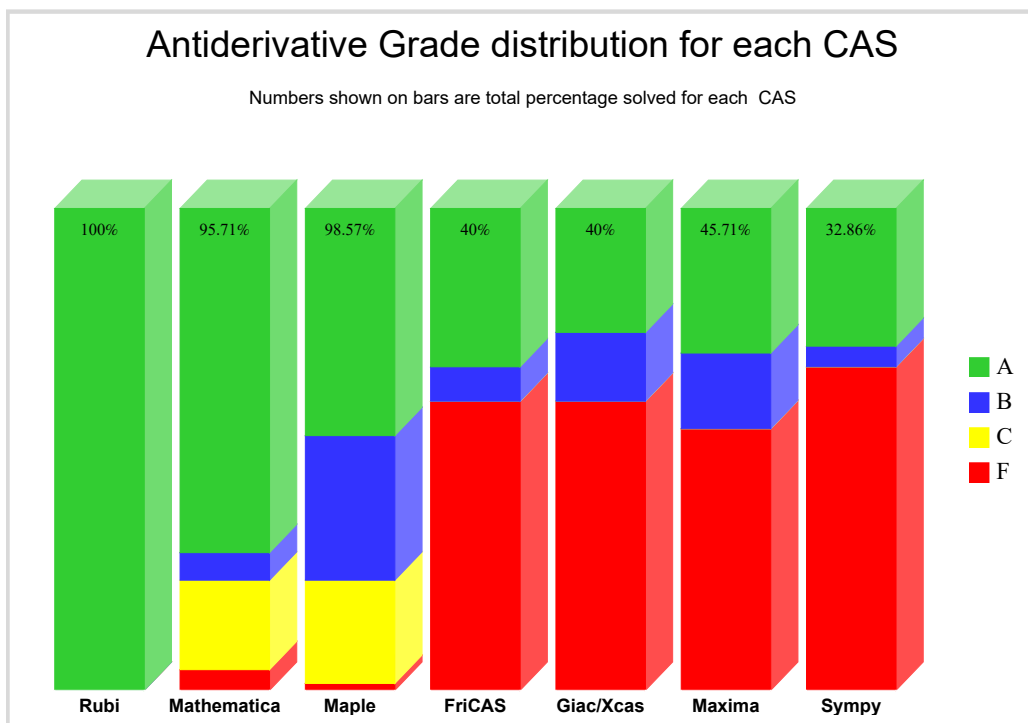
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

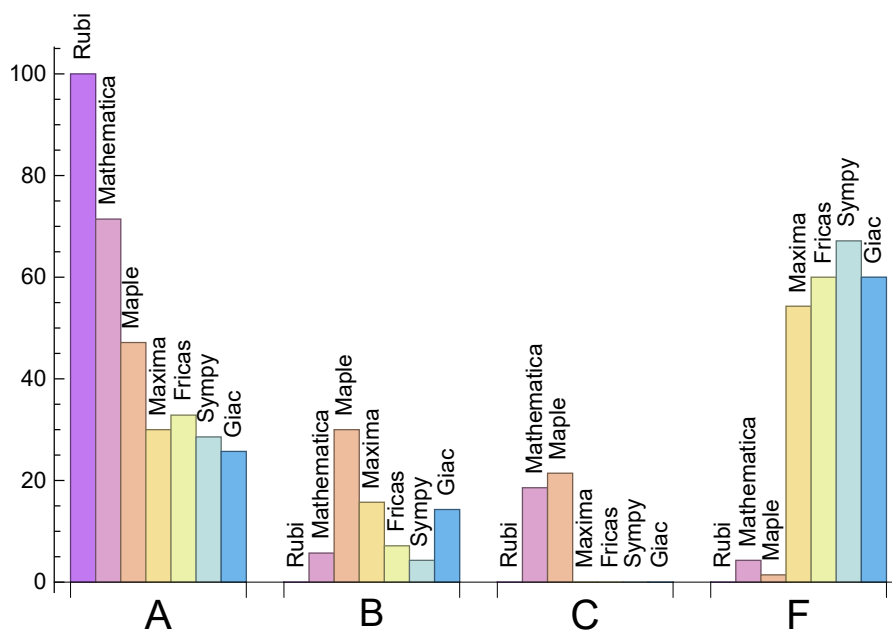
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	71.43	5.71	18.57	4.29
Maple	47.14	30.	21.43	1.43
Maxima	30.	15.71	0.	54.29
Fricas	32.86	7.14	0.	60.
Sympy	28.57	4.29	0.	67.14
Giac	25.71	14.29	0.	60.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	228.04	0.9	159.5	1.
Mathematica	1.23	266.03	1.45	163.	0.99
Maple	0.44	1882.19	4.99	225.	1.83
Maxima	1.59	240.47	2.18	164.	1.7
Fricas	1.74	289.43	2.54	193.5	2.68
Sympy	10.38	397.17	4.7	178.	2.3
Giac	1.16	311.54	2.37	126.	1.63

## 1.4 list of integrals that has no closed form antiderivative

{23, 42, 43, 65, 66, 69, 70}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {65, 66, 69, 70}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {8, 11, 13, 15, 16, 18, 19, 20, 28, 31, 32, 33, 35, 36, 37, 38, 48, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.



## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

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# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 32, 33, 35, 37, 38, 41, 42, 43, 47, 48, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 31, 36, 55, 56 }

C grade: { 6, 24, 25, 26, 29, 30, 44, 45, 46, 49, 50, 51, 57 }

F grade: { 34, 39, 40 }

#### 2.1.3 Maple

A grade: { 5, 6, 12, 14, 23, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 11, 13, 16, 19, 21, 22, 24, 31, 32, 38, 53, 60, 61, 62 }

C grade: { 10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 56, 57, 58, 59 }

F grade: { 41 }

#### 2.1.4 Maxima

A grade: { 5, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 48, 49, 50, 51, 55, 60, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 6, 7, 9, 12, 14, 21, 22, 54 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 56, 57, 58, 59, 61, 62, 63, 64, 67, 68 }

## 2.1.5 FriCAS

A grade: { 3, 5, 6, 9, 12, 24, 25, 26, 27, 29, 42, 43, 44, 45, 46, 47, 49, 50, 51, 65, 66, 69, 70 }

B grade: { 1, 2, 7, 14, 30 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 5, 6, 7, 9, 12, 23, 24, 25, 26, 27, 44, 45, 46, 47, 65, 66, 69 }

B grade: { 49, 50, 51 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 70 }

## 2.1.7 Giac

A grade: { 5, 6, 23, 27, 29, 42, 43, 44, 45, 46, 47, 49, 50, 51, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 7, 9, 14, 24, 25, 26, 30 }

C grade: { }

F grade: { 4, 8, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	225	500	315	231	317
normalized size	1	1.	0.78	3.12	6.94	4.38	3.21	4.4
time (sec)	N/A	0.251	0.06	0.035	1.518	1.577	4.44	1.151

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	161	321	278	178	216
normalized size	1	1.	0.81	2.4	4.79	4.15	2.66	3.22
time (sec)	N/A	0.055	0.017	0.037	1.514	1.699	2.473	1.169

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	92	162	139	95	178
normalized size	1	1.	0.83	1.92	3.38	2.9	1.98	3.71
time (sec)	N/A	0.03	0.012	0.036	1.499	1.548	1.567	1.139

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	132	0	0	0	0
normalized size	1	1.	0.83	2.1	0.	0.	0.	0.
time (sec)	N/A	0.058	0.022	0.046	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	73	124	184	2814	100
normalized size	1	1.	0.82	1.2	2.03	3.02	46.13	1.64
time (sec)	N/A	0.047	0.018	0.041	0.988	1.679	125.41	1.087

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	71	162	162	314	113
normalized size	1	1.	0.81	1.13	2.57	2.57	4.98	1.79
time (sec)	N/A	0.045	0.013	0.041	1.483	1.688	5.35	1.126

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	216	543	806	703	575	784
normalized size	1	1.	1.38	3.46	5.13	4.48	3.66	4.99
time (sec)	N/A	0.221	0.098	0.046	5.514	1.797	11.764	1.375

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	163	593	0	0	0	0
normalized size	1	1.	0.89	3.24	0.	0.	0.	0.
time (sec)	N/A	0.221	0.362	0.123	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	220	294	335	240	294
normalized size	1	1.	1.13	2.32	3.09	3.53	2.53	3.09
time (sec)	N/A	0.119	0.058	0.046	5.202	1.723	3.045	1.238

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	170	1433	0	0	0	0
normalized size	1	1.	0.93	7.83	0.	0.	0.	0.
time (sec)	N/A	0.339	0.063	0.588	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	135	471	0	0	0	0
normalized size	1	1.	1.13	3.96	0.	0.	0.	0.
time (sec)	N/A	0.187	0.191	0.125	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	194	182	362	470	986	0
normalized size	1	1.	1.66	1.56	3.09	4.02	8.43	0.
time (sec)	N/A	0.153	0.108	0.053	1.621	1.879	6.516	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	163	547	0	0	0	0
normalized size	1	1.	0.84	2.82	0.	0.	0.	0.
time (sec)	N/A	0.257	0.606	0.125	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	245	242	721	937	0	1902
normalized size	1	1.	1.44	1.42	4.24	5.51	0.	11.19
time (sec)	N/A	0.227	0.271	0.054	1.879	2.094	0.	1.494

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	349	3242	0	0	0	0
normalized size	1	1.	1.29	11.96	0.	0.	0.	0.
time (sec)	N/A	0.439	0.53	0.947	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	196	567	0	0	0	0
normalized size	1	1.	1.2	3.46	0.	0.	0.	0.
time (sec)	N/A	0.243	0.267	0.132	0.	0.	0.	0.



Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	252	2894	0	0	0	0
normalized size	1	1.	0.9	10.37	0.	0.	0.	0.
time (sec)	N/A	0.458	0.092	0.377	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	263	2696	0	0	0	0
normalized size	1	1.	1.61	16.54	0.	0.	0.	0.
time (sec)	N/A	0.3	0.449	0.4	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	225	631	0	0	0	0
normalized size	1	1.	1.25	3.51	0.	0.	0.	0.
time (sec)	N/A	0.319	0.226	0.138	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	360	7083	0	0	0	0
normalized size	1	1.	1.25	24.68	0.	0.	0.	0.
time (sec)	N/A	0.5	0.824	0.864	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	68	59	0	0	0
normalized size	1	1.	1.	2.19	1.9	0.	0.	0.
time (sec)	N/A	0.038	0.003	0.046	1.622	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	98	166	0	0	0
normalized size	1	1.	0.83	2.39	4.05	0.	0.	0.
time (sec)	N/A	0.045	0.007	0.046	1.682	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	5.408	0.472	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	157	494	467	667	627	961
normalized size	1	1.	0.67	2.12	2.	2.86	2.69	4.12
time (sec)	N/A	0.381	0.253	0.042	1.543	1.624	10.232	3.444

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	118	283	297	432	357	562
normalized size	1	1.	0.76	1.83	1.92	2.79	2.3	3.63
time (sec)	N/A	0.19	0.139	0.041	1.532	1.466	4.998	1.516

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	163	146	157	232	177	269
normalized size	1	1.	1.68	1.51	1.62	2.39	1.82	2.77
time (sec)	N/A	0.112	0.055	0.039	1.483	1.422	2.468	1.205

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	49	42	49	119	51	49
normalized size	1	1.	1.29	1.11	1.29	3.13	1.34	1.29
time (sec)	N/A	0.018	0.013	0.033	0.973	1.515	0.566	1.082

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	160	224	0	0	0	0
normalized size	1	1.	0.99	1.38	0.	0.	0.	0.
time (sec)	N/A	0.15	0.104	0.072	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	121	205	239	435	0	393
normalized size	1	1.	0.8	1.36	1.58	2.88	0.	2.6
time (sec)	N/A	0.121	0.157	0.045	1.533	3.038	0.	1.094

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	175	438	552	1415	0	1723
normalized size	1	1.	0.77	1.93	2.43	6.23	0.	7.59
time (sec)	N/A	0.303	0.555	0.046	1.562	11.938	0.	6.762

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	801	1622	0	0	0	0
normalized size	1	1.	2.1	4.25	0.	0.	0.	0.
time (sec)	N/A	0.574	3.655	0.133	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	264	748	0	0	0	0
normalized size	1	1.	1.19	3.37	0.	0.	0.	0.
time (sec)	N/A	0.373	0.379	0.119	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	109	180	0	0	0	0
normalized size	1	1.	1.07	1.76	0.	0.	0.	0.
time (sec)	N/A	0.107	0.073	0.105	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	0	2149	0	0	0	0
normalized size	1	1.	0.	8.23	0.	0.	0.	0.
time (sec)	N/A	0.165	5.445	1.339	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	419	1087	0	0	0	0
normalized size	1	1.	0.74	1.91	0.	0.	0.	0.
time (sec)	N/A	1.351	6.315	0.132	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	564	564	1844	6682	0	0	0	0
normalized size	1	1.	3.27	11.85	0.	0.	0.	0.
time (sec)	N/A	0.937	8.962	2.95	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	592	16362	0	0	0	0
normalized size	1	1.	1.76	48.55	0.	0.	0.	0.
time (sec)	N/A	0.632	0.679	1.261	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	212	359	0	0	0	0
normalized size	1	1.	1.48	2.51	0.	0.	0.	0.
time (sec)	N/A	0.214	0.115	0.155	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	372	0	4389	0	0	0	0
normalized size	1	1.	0.	11.8	0.	0.	0.	0.
time (sec)	N/A	0.203	71.529	0.765	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	4764	0	0	0	0
normalized size	1	1.	0.	3.86	0.	0.	0.	0.
time (sec)	N/A	2.311	70.268	1.013	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.288	1.284	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	4.445	1.105	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.405	1.097	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	95	132	140	203	155	139
normalized size	1	1.	0.9	1.25	1.32	1.92	1.46	1.31
time (sec)	N/A	0.111	0.07	0.039	1.504	1.685	2.368	1.108

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	114	95	115	161	117	111
normalized size	1	1.	1.44	1.2	1.46	2.04	1.48	1.41
time (sec)	N/A	0.092	0.048	0.036	1.503	1.749	1.432	1.093

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	90	66	92	123	78	81
normalized size	1	1.	1.5	1.1	1.53	2.05	1.3	1.35
time (sec)	N/A	0.055	0.031	0.035	1.514	1.693	0.965	1.1

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	39	36	42	97	46	42
normalized size	1	1.	1.18	1.09	1.27	2.94	1.39	1.27
time (sec)	N/A	0.011	0.014	0.035	1.008	1.748	0.559	1.118

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	171	103	181	0	0	0
normalized size	1	1.	1.42	0.86	1.51	0.	0.	0.
time (sec)	N/A	0.106	0.008	0.047	1.711	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	67	63	104	151	323	95
normalized size	1	1.	1.08	1.02	1.68	2.44	5.21	1.53
time (sec)	N/A	0.039	0.056	0.043	1.544	1.716	10.951	1.112

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	92	105	151	225	644	155
normalized size	1	1.	0.96	1.09	1.57	2.34	6.71	1.61
time (sec)	N/A	0.083	0.096	0.043	1.526	1.892	16.572	1.085

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	128	162	223	321	1127	239
normalized size	1	1.	0.99	1.26	1.73	2.49	8.74	1.85
time (sec)	N/A	0.114	0.136	0.043	1.543	1.653	26.956	1.101

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	863	863	701	631	0	0	0	0
normalized size	1	1.	0.81	0.73	0.	0.	0.	0.
time (sec)	N/A	1.207	0.805	0.636	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2192	0	0	0	0
normalized size	1	1.	0.75	4.04	0.	0.	0.	0.
time (sec)	N/A	0.607	0.329	0.713	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	231	198	383	0	0	0
normalized size	1	1.	1.52	1.3	2.52	0.	0.	0.
time (sec)	N/A	0.138	0.02	0.054	1.974	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	771	317	383	0	0	0
normalized size	1	1.	3.16	1.3	1.57	0.	0.	0.
time (sec)	N/A	0.239	11.075	0.06	2.018	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	668	668	1536	53434	0	0	0	0
normalized size	1	1.	2.3	79.99	0.	0.	0.	0.
time (sec)	N/A	0.855	21.489	2.177	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	933	933	933	682	0	0	0	0
normalized size	1	1.	1.	0.73	0.	0.	0.	0.
time (sec)	N/A	1.367	7.074	0.658	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	604	344	0	0	0	0
normalized size	1	1.	0.9	0.51	0.	0.	0.	0.
time (sec)	N/A	0.898	0.518	0.338	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	770	770	666	377	0	0	0	0
normalized size	1	1.	0.86	0.49	0.	0.	0.	0.
time (sec)	N/A	0.982	0.719	0.318	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	283	833	443	0	0	0
normalized size	1	1.	1.03	3.04	1.62	0.	0.	0.
time (sec)	N/A	0.271	0.053	0.444	1.835	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2192	0	0	0	0
normalized size	1	1.	0.75	4.04	0.	0.	0.	0.
time (sec)	N/A	0.6	0.354	0.702	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	443	4743	0	0	0	0
normalized size	1	1.	1.21	12.92	0.	0.	0.	0.
time (sec)	N/A	0.673	0.413	1.036	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	97	143	0	0	0	0
normalized size	1	1.	0.73	1.08	0.	0.	0.	0.
time (sec)	N/A	0.096	0.101	0.397	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	125	176	0	0	0	0
normalized size	1	1.	0.58	0.81	0.	0.	0.	0.
time (sec)	N/A	0.162	0.064	0.439	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	22	0	163	0	0	0	0	0
normalized size	1	0.	7.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.363	1.044	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	24	0	165	0	0	0	0	0
normalized size	1	0.	6.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.073	0.996	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	145	187	0	0	0	0
normalized size	1	1.	0.78	1.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.657	0.619	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	189	222	0	0	0	0
normalized size	1	1.	0.67	0.79	0.	0.	0.	0.
time (sec)	N/A	0.331	0.133	1.125	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	29	0	181	0	0	0	0	0
normalized size	1	0.	6.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	1.433	1.329	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	31	0	225	0	0	0	0	0
normalized size	1	0.	7.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.312	1.3	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [ 1.25 ]



Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.	21	0.238
2	A	6	5	1.	21	0.238
3	A	5	5	1.	19	0.263
4	A	5	4	1.	21	0.19
5	A	7	7	1.	21	0.333
6	A	5	5	1.	21	0.238
7	A	13	9	1.	23	0.391
8	A	11	10	1.	23	0.435
9	A	8	7	1.	21	0.333
10	A	8	7	1.	23	0.304
11	A	6	6	1.	23	0.261
12	A	10	9	1.	23	0.391
13	A	10	9	1.	23	0.391
14	A	15	10	1.	23	0.435
15	A	14	11	1.	23	0.478
16	A	10	10	1.	21	0.476
17	A	10	8	1.	23	0.348
18	A	7	8	1.	23	0.348
19	A	9	8	1.	23	0.348
20	A	16	13	1.	23	0.565
21	A	5	4	1.	12	0.333
22	A	5	4	1.	19	0.21
23	A	0	0	0.	0	0.
24	A	7	6	1.	18	0.333
25	A	7	6	1.	18	0.333
26	A	7	6	1.	16	0.375
27	A	4	3	1.	10	0.3
28	A	5	5	1.	18	0.278
29	A	8	8	1.	18	0.444
30	A	9	8	1.	18	0.444
31	A	16	13	1.	20	0.65
32	A	13	10	1.	18	0.556
33	A	6	6	1.	12	0.5
34	A	2	2	1.	20	0.1
35	A	25	25	1.	20	1.25
36	A	21	14	1.	20	0.7
37	A	15	11	1.	18	0.611
38	A	6	7	1.	12	0.583
39	A	2	2	1.	20	0.1
40	A	35	22	1.	20	1.1
41	A	6	4	1.	18	0.222
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	7	6	1.	10	0.6
45	A	7	6	1.	10	0.6
46	A	7	6	1.	8	0.75
47	A	3	3	1.	6	0.5
48	A	5	5	1.	10	0.5
49	A	7	7	1.	10	0.7
50	A	8	7	1.	10	0.7
51	A	8	7	1.	10	0.7
52	A	23	5	1.	16	0.312
53	A	17	5	1.	16	0.312
54	A	5	5	1.	14	0.357
55	A	15	7	1.	16	0.438
56	A	25	7	1.	16	0.438
57	A	31	7	1.	16	0.438
58	A	31	13	1.	18	0.722
59	A	37	16	1.	18	0.889
60	A	17	5	1.	14	0.357
61	A	17	5	1.	16	0.312
62	A	12	8	1.	19	0.421
63	A	2	2	1.	28	0.071
64	A	3	3	1.	33	0.091
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	4	4	1.	35	0.114
68	A	5	5	1.	40	0.125
69	A	0	0	0.	0	0.
70	A	0	0	0.	0	0.

# Chapter 3

## Listing of integrals

### 3.1 $\int (ce + dex)^3 (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=72

$$\frac{e^3(c + dx)^4 (a + b \tan^{-1}(c + dx))}{4d} - \frac{be^3(c + dx)^3}{12d} - \frac{be^3 \tan^{-1}(c + dx)}{4d} + \frac{1}{4}be^3x$$

[Out] (b\*e^3\*x)/4 - (b\*e^3\*(c + d\*x)^3)/(12\*d) - (b\*e^3\*ArcTan[c + d\*x])/(4\*d) + (e^3\*(c + d\*x)^4\*(a + b\*ArcTan[c + d\*x]))/(4\*d)

---

**Rubi [A]** time = 0.250649, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {5043, 12, 4852, 302, 203}

$$\frac{e^3(c + dx)^4 (a + b \tan^{-1}(c + dx))}{4d} - \frac{be^3(c + dx)^3}{12d} - \frac{be^3 \tan^{-1}(c + dx)}{4d} + \frac{1}{4}be^3x$$

Antiderivative was successfully verified.

[In] Int[(c\*e + d\*e\*x)^3\*(a + b\*ArcTan[c + d\*x]),x]

[Out] (b\*e^3\*x)/4 - (b\*e^3\*(c + d\*x)^3)/(12\*d) - (b\*e^3\*ArcTan[c + d\*x])/(4\*d) + (e^3\*(c + d\*x)^4\*(a + b\*ArcTan[c + d\*x]))/(4\*d)

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2)

), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :> Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int (ce + dex)^3 (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{e^3 \text{Subst}\left(\int x^3 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))}{4d} - \frac{(be^3) \text{Subst}\left(\int \frac{x^4}{1+x^2} dx, x, c + dx\right)}{4d} \\
 &= \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))}{4d} - \frac{(be^3) \text{Subst}\left(\int \left(-1 + x^2 + \frac{1}{1+x^2}\right) dx, x, c + dx\right)}{4d} \\
 &= \frac{1}{4} be^3 x - \frac{be^3 (c + dx)^3}{12d} + \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))}{4d} - \frac{(be^3) \text{Subst}\left(\int \frac{1}{1+x} dx, x, c + dx\right)}{4d} \\
 &= \frac{1}{4} be^3 x - \frac{be^3 (c + dx)^3}{12d} - \frac{be^3 \tan^{-1}(c + dx)}{4d} + \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))}{4d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0595514, size = 56, normalized size = 0.78

$$\frac{e^3 \left( \frac{1}{4} (c + dx)^4 (a + b \tan^{-1}(c + dx)) - \frac{1}{4} b \left( \frac{1}{3} (c + dx)^3 + \tan^{-1}(c + dx) - dx \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*e + d\*e\*x)^3\*(a + b\*ArcTan[c + d\*x]),x]

[Out] (e^3\*(-(b\*(-(d\*x) + (c + d\*x)^3/3 + ArcTan[c + d\*x]))/4 + ((c + d\*x)^4\*(a + b\*ArcTan[c + d\*x]))/4))/d

**Maple [B]** time = 0.035, size = 225, normalized size = 3.1

$$\frac{d^3 x^4 a e^3}{4} + d^2 x^3 a c e^3 + \frac{3 d x^2 a c^2 e^3}{2} + x a c^3 e^3 + \frac{a c^4 e^3}{4 d} + \frac{d^3 \arctan(dx + c) x^4 b e^3}{4} + d^2 \arctan(dx + c) x^3 b c e^3 + \frac{3 d \arctan(dx + c) b c^2 e^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*e\*x+c\*e)^3\*(a+b\*arctan(d\*x+c)),x)

```
[Out] 1/4*d^3*x^4*a*e^3+d^2*x^3*a*c*e^3+3/2*d*x^2*a*c^2*e^3+x*a*c^3*e^3+1/4/d*a*c^4*e^3+1/4*d^3*arctan(d*x+c)*x^4*b*e^3+d^2*arctan(d*x+c)*x^3*b*c*e^3+3/2*d*arctan(d*x+c)*x^2*b*c^2*e^3+arctan(d*x+c)*x*b*c^3*e^3+1/4/d*arctan(d*x+c)*b*c^4*e^3-1/12*d^2*x^3*b*e^3-1/4*d*x^2*b*c*e^3-1/4*x*b*c^2*e^3-1/12/d*b*c^3*e^3+1/4*b*e^3*x+1/4/d*b*c*e^3-1/4*b*e^3*arctan(d*x+c)/d
```

**Maxima [B]** time = 1.51803, size = 500, normalized size = 6.94

$$\frac{1}{4} ad^3 e^3 x^4 + acd^2 e^3 x^3 + \frac{3}{2} ac^2 d e^3 x^2 + \frac{3}{2} \left( x^2 \arctan(dx + c) - d \left( \frac{x}{d^2} + \frac{(c^2 - 1) \arctan\left(\frac{d^2 x + cd}{d}\right)}{d^3} - \frac{c \log(d^2 x^2 + 2cdx + 2 + 2c*d*x + c^2 + 1)}{d^3} \right) \right) * b * c^2 * d * e^3 + \frac{1}{2} * (2 * x^3 * \arctan(dx + c) - d * ((d * x^2 - 4 * c * x) / d^3 - 2 * (c^3 - 3 * c) * \arctan((d^2 * x + c * d) / d) / d^4 + (3 * c^2 - 1) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / d^4)) * b * c * d^2 * e^3 + \frac{1}{12} * (3 * x^4 * \arctan(dx + c) - d * ((d^2 * x^3 - 3 * c * d * x^2 + 3 * (3 * c^2 - 1) * x) / d^4 + 3 * (c^4 - 6 * c^2 + 1) * \arctan((d^2 * x + c * d) / d) / d^5 - 6 * (c^3 - c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / d^5)) * b * d^3 * e^3 + a * c^3 * e^3 * x + \frac{1}{2} * (2 * (d * x + c) * \arctan(dx + c) - \log((d * x + c)^2 + 1)) * b * c^3 * e^3 / d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/12*(3*x^4*arctan(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*d^3*e^3 + a*c^3*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c^3*e^3/d
```

**Fricas [B]** time = 1.5769, size = 315, normalized size = 4.38

$$\frac{3 ad^4 e^3 x^4 + (12 ac - b) d^3 e^3 x^3 + 3 (6 ac^2 - bc) d^2 e^3 x^2 + 3 (4 ac^3 - bc^2 + b) d e^3 x + 3 (bd^4 e^3 x^4 + 4 bcd^3 e^3 x^3 + 6 bc^2 d^2 e^3 x^2 + 3 cd^3 e^3 x + 3 c^4 e^3)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*e^3*x^4 + (12*a*c - b)*d^3*e^3*x^3 + 3*(6*a*c^2 - b*c)*d^2*e^3*x^2 + 3*(4*a*c^3 - b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + (b*c^4 - b)*e^3)*arctan(d*x + c))/d
```

**Sympy [A]** time = 4.43974, size = 231, normalized size = 3.21

$$\begin{cases} ac^3 e^3 x + \frac{3ac^2 d e^3 x^2}{2} + acd^2 e^3 x^3 + \frac{ad^3 e^3 x^4}{4} + \frac{bc^4 e^3 \operatorname{atan}(c+dx)}{4d} + bc^3 e^3 x \operatorname{atan}(c + dx) + \frac{3bc^2 d e^3 x^2 \operatorname{atan}(c+dx)}{2} - \frac{bc^2 e^3 x}{4} + bcd^2 e^3 x \\ c^3 e^3 x (a + b \operatorname{atan}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*d**3*e**3*x**4/4 + b*c**4*e**3*atan(c + d*x)/(4*d) + b*c**3*e**3*x*atan(c +
```

```
d*x) + 3*b*c**2*d*e**3*x**2*atan(c + d*x)/2 - b*c**2*e**3*x/4 + b*c*d**2*e
**3*x**3*atan(c + d*x) - b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atan(c + d*x)
/4 - b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atan(c + d*x)/(4*d), Ne(d, 0
)), (c**3*e**3*x*(a + b*atan(c)), True))
```

**Giac [B]** time = 1.15081, size = 317, normalized size = 4.4

$$6bd^4x^4 \arctan(dx + c)e^3 + 6ad^4x^4e^3 + 24bcd^3x^3 \arctan(dx + c)e^3 + 24acd^3x^3e^3 + 36bc^2d^2x^2 \arctan(dx + c)e^3 + 36$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(6*b*d^4*x^4*arctan(d*x + c)*e^3 + 6*a*d^4*x^4*e^3 + 24*b*c*d^3*x^3*ar
ctan(d*x + c)*e^3 + 24*a*c*d^3*x^3*e^3 + 36*b*c^2*d^2*x^2*arctan(d*x + c)*e
^3 + 36*a*c^2*d^2*x^2*e^3 - 2*b*d^3*x^3*e^3 + 24*b*c^3*d*x*arctan(d*x + c)*
e^3 + 3*pi*b*c^4*e^3*sgn(d*x + c) - 3*pi*b*c^4*e^3 + 24*a*c^3*d*x*e^3 - 6*b
*c*d^2*x^2*e^3 - 6*b*c^4*arctan(1/(d*x + c))*e^3 - 6*b*c^2*d*x*e^3 + 6*b*d*
x*e^3 - 3*pi*b*e^3*sgn(d*x + c) + 3*pi*b*e^3 + 6*b*arctan(1/(d*x + c))*e^3)
/d
```

### 3.2 $\int (ce + dex)^2 (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=67

$$\frac{e^2(c + dx)^3 (a + b \tan^{-1}(c + dx))}{3d} - \frac{be^2(c + dx)^2}{6d} + \frac{be^2 \log((c + dx)^2 + 1)}{6d}$$

[Out]  $-(b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)$

**Rubi [A]** time = 0.0547487, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {5043, 12, 4852, 266, 43}

$$\frac{e^2(c + dx)^3 (a + b \tan^{-1}(c + dx))}{3d} - \frac{be^2(c + dx)^2}{6d} + \frac{be^2 \log((c + dx)^2 + 1)}{6d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]), x]$

[Out]  $-(b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)$

#### Rule 5043

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p, x]$   $\rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(f*x)/d]^m*(a + b*ArcTan[x])^p, x], x, c + d*x]$   $;$   $\text{FreeQ}\{a, b, c, d, e, f, m, x\}$   $\&\&$   $\text{EqQ}[d*e - c*f, 0]$   $\&\&$   $\text{IGtQ}[p, 0]$

#### Rule 12

$\text{Int}[u*(v + w*x)^p, x]$   $\rightarrow \text{Dist}[u, \text{Int}[v + w*x, x], x]$   $;$   $\text{FreeQ}[u, x]$   $\&\&$   $! \text{MatchQ}[u, (b + c*x)^p]$   $;$   $\text{FreeQ}[b, c]$

#### Rule 4852

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p*(d + e*x)^m, x]$   $\rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*ArcTan[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{m+1}*(a + b*ArcTan[c*x])^{p-1}/(1 + c^2*x^2), x], x]$   $;$   $\text{FreeQ}\{a, b, c, d, m, x\}$   $\&\&$   $\text{IGtQ}[p, 0]$   $\&\&$   $(\text{EqQ}[p, 1] \parallel \text{IntegerQ}[m]) \&\&$   $\text{NeQ}[m, -1]$

#### Rule 266

$\text{Int}[x^m*(a + b*x)^n, x]$   $\rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(Simplify[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x]$   $;$   $\text{FreeQ}\{a, b, m, n, p\}$   $\&\&$   $\text{IntegerQ}[Simplify[(m+1)/n]]$

#### Rule 43

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x]$   $\rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x]$   $;$   $\text{FreeQ}\{a, b, c, d, n, x\}$   $\&\&$   $\text{NeQ}[b*c - a*d, 0]$   $\&\&$   $\text{IGtQ}[m, 0]$   $\&\&$   $(! \text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned}
\int (ce + dex)^2 (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\
&= \frac{e^2 \text{Subst}\left(\int x^2 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{3d} - \frac{(be^2) \text{Subst}\left(\int \frac{x^3}{1+x^2} dx, x, c + dx\right)}{3d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{3d} - \frac{(be^2) \text{Subst}\left(\int \frac{x}{1+x} dx, x, (c + dx)^2\right)}{6d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{3d} - \frac{(be^2) \text{Subst}\left(\int \left(1 + \frac{1}{-1-x}\right) dx, x, (c + dx)^2\right)}{6d} \\
&= -\frac{be^2 (c + dx)^2}{6d} + \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{3d} + \frac{be^2 \log(1 + (c + dx)^2)}{6d}
\end{aligned}$$

**Mathematica [A]** time = 0.016689, size = 54, normalized size = 0.81

$$\frac{e^2 \left( \frac{1}{3} (c + dx)^3 (a + b \tan^{-1}(c + dx)) - \frac{1}{6} b ((c + dx)^2 - \log((c + dx)^2 + 1)) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*e + d\*e\*x)^2\*(a + b\*ArcTan[c + d\*x]), x]

[Out] (e^2\*(((c + d\*x)^3\*(a + b\*ArcTan[c + d\*x]))/3 - (b\*((c + d\*x)^2 - Log[1 + (c + d\*x)^2]))/6))/d

**Maple [B]** time = 0.037, size = 161, normalized size = 2.4

$$\frac{d^2 x^3 a e^2}{3} + dx^2 a c e^2 + x a c^2 e^2 + \frac{a c^3 e^2}{3d} + \frac{d^2 \arctan(dx + c) x^3 b e^2}{3} + d \arctan(dx + c) x^2 b c e^2 + \arctan(dx + c) x b c^2 e^2 + \frac{b c d e^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c)), x)

[Out] 1/3\*d^2\*x^3\*a\*e^2+d\*x^2\*a\*c\*e^2+x\*a\*c^2\*e^2+1/3/d\*a\*c^3\*e^2+1/3\*d^2\*arctan(d\*x+c)\*x^3\*b\*e^2+d\*arctan(d\*x+c)\*x^2\*b\*c\*e^2+arctan(d\*x+c)\*x\*b\*c^2\*e^2+1/3/d\*arctan(d\*x+c)\*b\*c^3\*e^2-1/6\*d\*x^2\*b\*e^2-1/3\*x\*b\*c\*e^2-1/6/d\*b\*c^2\*e^2+1/6\*b\*e^2\*ln(1+(d\*x+c)^2)/d

**Maxima [B]** time = 1.51354, size = 321, normalized size = 4.79

$$\frac{1}{3} a d^2 e^2 x^3 + a c d e^2 x^2 + \left( x^2 \arctan(dx + c) - d \left( \frac{x}{d^2} + \frac{(c^2 - 1) \arctan\left(\frac{d^2 x + cd}{d}\right)}{d^3} - \frac{c \log(d^2 x^2 + 2 c dx + c^2 + 1)}{d^3} \right) \right) b c d e^2 + \frac{1}{6} b c d e^2$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{3}a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3)) * b*c*d*e^2 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c^2*e^2/d$

**Fricas [B]** time = 1.69925, size = 278, normalized size = 4.15

$$\frac{2ad^3e^2x^3 + (6ac - b)d^2e^2x^2 + 2(3ac^2 - bc)de^2x + be^2 \log(d^2x^2 + 2cdx + c^2 + 1) + 2(bd^3e^2x^3 + 3bcd^2e^2x^2 + 3bc^2de^2x + 3b^2c^2e^2)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{6}*(2*a*d^3*e^2*x^3 + (6*a*c - b)*d^2*e^2*x^2 + 2*(3*a*c^2 - b*c)*d*e^2*x + b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*d^3*e^2*x^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*arctan(d*x + c))/d$

**Sympy [A]** time = 2.47253, size = 178, normalized size = 2.66

$$\left\{ \begin{array}{l} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2 \operatorname{atan}(c+dx)}{3d} + bc^2e^2x \operatorname{atan}(c + dx) + bcde^2x^2 \operatorname{atan}(c + dx) - \frac{bce^2x}{3} + \frac{bd^2e^2x^3 \operatorname{atan}(c+dx)}{3} \\ c^2e^2x(a + b \operatorname{atan}(c)) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*\*2\*(a+b\*atan(d\*x+c)),x)

[Out] Piecewise((a\*c\*\*2\*e\*\*2\*x + a\*c\*d\*e\*\*2\*x\*\*2 + a\*d\*\*2\*e\*\*2\*x\*\*3/3 + b\*c\*\*3\*e\*\*2\*atan(c + d\*x)/(3\*d) + b\*c\*\*2\*e\*\*2\*x\*atan(c + d\*x) + b\*c\*d\*e\*\*2\*x\*\*2\*atan(c + d\*x) - b\*c\*e\*\*2\*x/3 + b\*d\*\*2\*e\*\*2\*x\*\*3\*atan(c + d\*x)/3 - b\*d\*e\*\*2\*x\*\*2/6 + b\*e\*\*2\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(6\*d), Ne(d, 0)), (c\*\*2\*e\*\*2\*x\*(a + b\*atan(c)), True))

**Giac [B]** time = 1.16889, size = 216, normalized size = 3.22

$$\frac{2bd^3x^3 \arctan(dx + c)e^2 + 2ad^3x^3e^2 + 6bcd^2x^2 \arctan(dx + c)e^2 + 6acd^2x^2e^2 + 6bc^2dx \arctan(dx + c)e^2 - 2\pi bc^2e^2}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{6}*(2*b*d^3*x^3*arctan(d*x + c)*e^2 + 2*a*d^3*x^3*e^2 + 6*b*c*d^2*x^2*arctan(d*x + c)*e^2 + 6*a*c*d^2*x^2*e^2 + 6*b*c^2*d*x*arctan(d*x + c)*e^2 - 2*pi*b*c^3*e^2*sgn(d*x + c) + 6*a*c^2*d*x*e^2 - b*d^2*x^2*e^2 + 2*b*c^3*arctan(d*x + c)*e^2 - 2*b*c*d*x*e^2 + b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d$

### 3.3 $\int (ce + dex) (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=48

$$\frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))}{2d} + \frac{be \tan^{-1}(c + dx)}{2d} - \frac{bex}{2}$$

[Out]  $-(b * e * x) / 2 + (b * e * \text{ArcTan}[c + d * x]) / (2 * d) + (e * (c + d * x)^2 * (a + b * \text{ArcTan}[c + d * x])) / (2 * d)$

**Rubi [A]** time = 0.0302586, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5043, 12, 4852, 321, 203}

$$\frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))}{2d} + \frac{be \tan^{-1}(c + dx)}{2d} - \frac{bex}{2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c * e + d * e * x) * (a + b * \text{ArcTan}[c + d * x]), x]$

[Out]  $-(b * e * x) / 2 + (b * e * \text{ArcTan}[c + d * x]) / (2 * d) + (e * (c + d * x)^2 * (a + b * \text{ArcTan}[c + d * x])) / (2 * d)$

#### Rule 5043

$\text{Int}[(a + \text{ArcTan}[c + (d * x)] * b)^p * (e + f * x)^m, x\_Symbol] :> \text{Dist}[1/d, \text{Subst}[\text{Int}[(f * x)/d]^m * (a + b * \text{ArcTan}[x])^p, x], x, c + d * x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d \* e - c \* f, 0] && IGtQ[p, 0]

#### Rule 12

$\text{Int}[a * (u), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b \* v) /; FreeQ[b, x]]

#### Rule 4852

$\text{Int}[(a + \text{ArcTan}[c * x] * b)^p * (d * x)^m, x\_Symbol] :> \text{Simp}[(d * x)^{m + 1} * (a + b * \text{ArcTan}[c * x])^p / (d * (m + 1)), x] - \text{Dist}[(b * c * p) / (d * (m + 1)), \text{Int}[(d * x)^{m + 1} * (a + b * \text{ArcTan}[c * x])^{p - 1} / (1 + c^2 * x^2), x], x] /;$  FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 321

$\text{Int}[(c * x)^m * (a + b * x^n)^p, x\_Symbol] :> \text{Simp}[(c^{n - 1} * (c * x)^{m - n + 1} * (a + b * x^n)^{p + 1}) / (b * (m + n * p + 1)), x] - \text{Dist}[(a * c^{n * (m - n + 1)}) / (b * (m + n * p + 1)), \text{Int}[(c * x)^{m - n} * (a + b * x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n \* p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 203

$\text{Int}[(a + b * x^2)^{-1}, x\_Symbol] :> \text{Simp}[(1 * \text{ArcTan}[\text{Rt}[b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[b, 2]), x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int (ce + dex) (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst} \left( \int ex (a + b \tan^{-1}(x)) dx, x, c + dx \right)}{d} \\
 &= \frac{e \text{Subst} \left( \int x (a + b \tan^{-1}(x)) dx, x, c + dx \right)}{d} \\
 &= \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))}{2d} - \frac{(be) \text{Subst} \left( \int \frac{x^2}{1+x^2} dx, x, c + dx \right)}{2d} \\
 &= -\frac{1}{2} bex + \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))}{2d} + \frac{(be) \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, c + dx \right)}{2d} \\
 &= -\frac{1}{2} bex + \frac{be \tan^{-1}(c + dx)}{2d} + \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))}{2d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0120132, size = 40, normalized size = 0.83

$$\frac{e((c + dx)^2 (a + b \tan^{-1}(c + dx)) + b(\tan^{-1}(c + dx) - dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*e + d\*e\*x)\*(a + b\*ArcTan[c + d\*x]),x]

[Out] (e\*(b\*(-(d\*x) + ArcTan[c + d\*x]) + (c + d\*x)^2\*(a + b\*ArcTan[c + d\*x]))) / (2\*d)

**Maple [B]** time = 0.036, size = 92, normalized size = 1.9

$$\frac{dx^2 ae}{2} + xace + \frac{ac^2 e}{2d} + \frac{d \arctan(dx + c) x^2 be}{2} + \arctan(dx + c) x bce + \frac{\arctan(dx + c) bc^2 e}{2d} - \frac{bex}{2} - \frac{bce}{2d} + \frac{be \arctan(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c)),x)

[Out] 1/2\*d\*x^2\*a\*e+x\*a\*c\*e+1/2/d\*a\*c^2\*e+1/2\*d\*arctan(d\*x+c)\*x^2\*b\*e+arctan(d\*x+c)\*x\*b\*c\*e+1/2/d\*arctan(d\*x+c)\*b\*c^2\*e-1/2\*b\*e\*x-1/2/d\*b\*c\*e+1/2\*b\*e\*arctan(d\*x+c)/d

**Maxima [B]** time = 1.49881, size = 162, normalized size = 3.38

$$\frac{1}{2} adex^2 + \frac{1}{2} \left( x^2 \arctan(dx + c) - d \left( \frac{x}{d^2} + \frac{(c^2 - 1) \arctan\left(\frac{d^2 x + cd}{d}\right)}{d^3} - \frac{c \log(d^2 x^2 + 2cdx + c^2 + 1)}{d^3} \right) \right) bde + acex + \frac{(2d^2 c^2 + c^2 + 1) \arctan\left(\frac{d^2 x + cd}{d}\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2}ad^2e^2x^2 + \frac{1}{2}(x^2\arctan(dx+c) - d(x/d^2 + (c^2-1)\arctan((d^2x+cd)/d)/d^3 - c\log(d^2x^2 + 2c*d*x + c^2 + 1)/d^3))*b*d*e + a*c*e*x + \frac{1}{2}(2*(dx+c)\arctan(dx+c) - \log((dx+c)^2 + 1))*b*c*e/d$

**Fricas [A]** time = 1.54816, size = 139, normalized size = 2.9

$$\frac{ad^2ex^2 + (2ac - b)dex + (bd^2ex^2 + 2bcdex + (bc^2 + b)e)\arctan(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2}(a*d^2*e*x^2 + (2*a*c - b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b*c^2 + b)*e)*\arctan(d*x + c))/d$

**Sympy [A]** time = 1.56704, size = 95, normalized size = 1.98

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e \operatorname{atan}(c+dx)}{2d} + bcex \operatorname{atan}(c + dx) + \frac{bdex^2 \operatorname{atan}(c+dx)}{2} - \frac{bex}{2} + \frac{be \operatorname{atan}(c+dx)}{2d} & \text{for } d \neq 0 \\ cex(a + b \operatorname{atan}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*atan(d\*x+c)),x)

[Out] Piecewise((a\*c\*e\*x + a\*d\*e\*x\*\*2/2 + b\*c\*\*2\*e\*atan(c + d\*x)/(2\*d) + b\*c\*e\*x\*atan(c + d\*x) + b\*d\*e\*x\*\*2\*atan(c + d\*x)/2 - b\*e\*x/2 + b\*e\*atan(c + d\*x)/(2\*d), Ne(d, 0)), (c\*e\*x\*(a + b\*atan(c)), True))

**Giac [B]** time = 1.13934, size = 178, normalized size = 3.71

$$\frac{2bd^2x^2 \arctan(dx + c)e + 2ad^2x^2e + 4bcdx \arctan(dx + c)e + \pi bc^2 \operatorname{esgn}(dx + c) - \pi bc^2e + 4acdxe - 2bc^2 \arctan\left(\frac{1}{dx}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{4}(2*b*d^2*x^2*\arctan(d*x + c)*e + 2*a*d^2*x^2*e + 4*b*c*d*x*\arctan(d*x + c)*e + \pi*b*c^2*e*\operatorname{sgn}(d*x + c) - \pi*b*c^2*e + 4*a*c*d*x*e - 2*b*c^2*\arctan(1/(d*x + c))*e - 2*b*d*x*e + \pi*b*e*\operatorname{sgn}(d*x + c) - \pi*b*e - 2*b*\arctan(1/(d*x + c))*e)/d$

$$3.4 \quad \int \frac{a+b \tan^{-1}(c+dx)}{ce+dex} dx$$

**Optimal.** Leaf size=63

$$\frac{ib\text{PolyLog}(2, -i(c+dx))}{2de} - \frac{ib\text{PolyLog}(2, i(c+dx))}{2de} + \frac{a \log(c+dx)}{de}$$

[Out] (a\*Log[c + d\*x])/(d\*e) + ((I/2)\*b\*PolyLog[2, (-I)\*(c + d\*x)]/(d\*e) - ((I/2)\*b\*PolyLog[2, I\*(c + d\*x)]/(d\*e)

**Rubi [A]** time = 0.0579499, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {5043, 12, 4848, 2391}

$$\frac{ib\text{PolyLog}(2, -i(c+dx))}{2de} - \frac{ib\text{PolyLog}(2, i(c+dx))}{2de} + \frac{a \log(c+dx)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])/(c\*e + d\*e\*x), x]

[Out] (a\*Log[c + d\*x])/(d\*e) + ((I/2)\*b\*PolyLog[2, (-I)\*(c + d\*x)]/(d\*e) - ((I/2)\*b\*PolyLog[2, I\*(c + d\*x)]/(d\*e)

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4848

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 + I\*c\*x]/x, x], x]) /; FreeQ[{a, b, c}, x]

#### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tan^{-1}(c + dx)}{ce + dex} dx &= \frac{\text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{ex} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x} dx, x, c + dx\right)}{de} \\
&= \frac{a \log(c + dx)}{de} + \frac{(ib) \text{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, c + dx\right)}{2de} - \frac{(ib) \text{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, c + dx\right)}{2de} \\
&= \frac{a \log(c + dx)}{de} + \frac{ib \text{Li}_2(-i(c + dx))}{2de} - \frac{ib \text{Li}_2(i(c + dx))}{2de}
\end{aligned}$$

**Mathematica [A]** time = 0.0216952, size = 52, normalized size = 0.83

$$\frac{\frac{1}{2}ib\text{PolyLog}(2, -i(c + dx)) - \frac{1}{2}ib\text{PolyLog}(2, i(c + dx)) + a \log(c + dx)}{de}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])/(c\*e + d\*e\*x), x]

[Out] (a\*Log[c + d\*x] + (I/2)\*b\*PolyLog[2, (-I)\*(c + d\*x)] - (I/2)\*b\*PolyLog[2, I\*(c + d\*x)])/(d\*e)

**Maple [B]** time = 0.046, size = 132, normalized size = 2.1

$$\frac{a \ln(dx + c)}{de} + \frac{b \ln(dx + c) \arctan(dx + c)}{de} + \frac{\frac{i}{2} b \ln(dx + c) \ln(1 + i(dx + c))}{de} - \frac{\frac{i}{2} b \ln(dx + c) \ln(1 - i(dx + c))}{de} + \frac{\frac{i}{2} b \text{dilog}(1 + i(dx + c))}{de} - \frac{\frac{i}{2} b \text{dilog}(1 - i(dx + c))}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e), x)

[Out] a\*ln(d\*x+c)/d/e+1/d\*b/e\*ln(d\*x+c)\*arctan(d\*x+c)+1/2\*I/d\*b/e\*ln(d\*x+c)\*ln(1+I\*(d\*x+c))-1/2\*I/d\*b/e\*ln(d\*x+c)\*ln(1-I\*(d\*x+c))+1/2\*I/d\*b/e\*dilog(1+I\*(d\*x+c))-1/2\*I/d\*b/e\*dilog(1-I\*(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2b \int \frac{\arctan(dx + c)}{2(dx + ce)} dx + \frac{a \log(dex + ce)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e), x, algorithm="maxima")

[Out] 2\*b\*integrate(1/2\*arctan(d\*x + c)/(d\*e\*x + c\*e), x) + a\*log(d\*e\*x + c\*e)/(d\*e)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arctan(dx + c) + a}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e),x, algorithm="fricas")

[Out] integral((b\*arctan(d\*x + c) + a)/(d\*e\*x + c\*e), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c+dx} dx + \int \frac{b \operatorname{atan}(c+dx)}{c+dx} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))/(d\*e\*x+c\*e),x)

[Out] (Integral(a/(c + d\*x), x) + Integral(b\*atan(c + d\*x)/(c + d\*x), x))/e

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(dx + c) + a}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e),x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)/(d\*e\*x + c\*e), x)

### 3.5 $\int \frac{a+b \tan^{-1}(c+dx)}{(ce+dex)^2} dx$

**Optimal.** Leaf size=61

$$-\frac{a+b \tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b \log(c+dx)}{de^2} - \frac{b \log((c+dx)^2+1)}{2de^2}$$

[Out]  $-\left(\frac{a+b \operatorname{ArcTan}[c+d*x]}{d*e^2*(c+d*x)}\right) + \frac{b*\operatorname{Log}[c+d*x]}{d*e^2} - \left(\frac{b*\operatorname{Log}[1+(c+d*x)^2]}{2*d*e^2}\right)$

**Rubi [A]** time = 0.0472608, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5043, 12, 4852, 266, 36, 29, 31}

$$-\frac{a+b \tan^{-1}(c+dx)}{de^2(c+dx)} + \frac{b \log(c+dx)}{de^2} - \frac{b \log((c+dx)^2+1)}{2de^2}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]`

[Out]  $-\left(\frac{a+b \operatorname{ArcTan}[c+d*x]}{d*e^2*(c+d*x)}\right) + \frac{b*\operatorname{Log}[c+d*x]}{d*e^2} - \left(\frac{b*\operatorname{Log}[1+(c+d*x)^2]}{2*d*e^2}\right)$

#### Rule 5043

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```



Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tan^{-1}(c + dx)}{(ce + dex)^2} dx &= \frac{\text{Subst}\left(\int \frac{a + b \tan^{-1}(x)}{e^2 x^2} dx, x, c + dx\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \frac{a + b \tan^{-1}(x)}{x^2} dx, x, c + dx\right)}{de^2} \\
 &= -\frac{a + b \tan^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \text{Subst}\left(\int \frac{1}{x(1+x^2)} dx, x, c + dx\right)}{de^2} \\
 &= -\frac{a + b \tan^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, (c + dx)^2\right)}{2de^2} \\
 &= -\frac{a + b \tan^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \text{Subst}\left(\int \frac{1}{x} dx, x, (c + dx)^2\right)}{2de^2} - \frac{b \text{Subst}\left(\int \frac{1}{1+x} dx, x, (c + dx)^2\right)}{2de^2} \\
 &= -\frac{a + b \tan^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \log(c + dx)}{de^2} - \frac{b \log(1 + (c + dx)^2)}{2de^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0181161, size = 50, normalized size = 0.82

$$\frac{-\frac{a + b \tan^{-1}(c + dx)}{c + dx} + b \log(c + dx) - \frac{1}{2} b \log((c + dx)^2 + 1)}{de^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2, x]`

[Out] `(-((a + b*ArcTan[c + d*x])/(c + d*x)) + b*Log[c + d*x] - (b*Log[1 + (c + d*x)^2])/2)/(d*e^2)`

**Maple [A]** time = 0.041, size = 73, normalized size = 1.2

$$-\frac{a}{de^2(dx + c)} - \frac{b \arctan(dx + c)}{de^2(dx + c)} - \frac{b \ln(1 + (dx + c)^2)}{2de^2} + \frac{b \ln(dx + c)}{de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctan(d*x+c))/(d*e*x+c*e)^2, x)`

[Out] `-1/d*a/e^2/(d*x+c)-1/d*b/e^2/(d*x+c)*arctan(d*x+c)-1/2*b*ln(1+(d*x+c)^2)/d/e^2+b*ln(d*x+c)/d/e^2`

---

**Maxima [A]** time = 0.988298, size = 124, normalized size = 2.03

$$-\frac{1}{2} \left( d \left( \frac{\log(d^2 x^2 + 2 c d x + c^2 + 1)}{d^2 e^2} - \frac{2 \log(dx + c)}{d^2 e^2} \right) + \frac{2 \arctan(dx + c)}{d^2 e^2 x + c d e^2} \right) b - \frac{a}{d^2 e^2 x + c d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e)^2,x, algorithm="maxima")

[Out] -1/2\*(d\*(log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)/(d^2\*e^2) - 2\*log(dx + c)/(d^2\*e^2)) + 2\*arctan(dx + c)/(d^2\*e^2\*x + c\*d\*e^2))\*b - a/(d^2\*e^2\*x + c\*d\*e^2)

---

**Fricas [A]** time = 1.67866, size = 184, normalized size = 3.02

$$\frac{2 b \arctan(dx + c) + (bdx + bc) \log(d^2 x^2 + 2 c d x + c^2 + 1) - 2(bdx + bc) \log(dx + c) + 2 a}{2(d^2 e^2 x + c d e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e)^2,x, algorithm="fricas")

[Out] -1/2\*(2\*b\*arctan(dx + c) + (b\*d\*x + b\*c)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) - 2\*(b\*d\*x + b\*c)\*log(dx + c) + 2\*a)/(d^2\*e^2\*x + c\*d\*e^2)

---

**Sympy [A]** time = 125.41, size = 2814, normalized size = 46.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))/(d\*e\*x+c\*e)\*\*2,x)

[Out] Piecewise((x\*(a + b\*atan(c))/(c\*\*2\*e\*\*2), Eq(d, 0)), (-9\*a\*d\*\*2\*x\*\*2/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) + 6\*sqrt(3)\*I\*a\*d\*x/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) - 6\*a/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) + 9\*b\*d\*\*3\*x\*\*3\*log(x - sqrt(3)\*I/(3\*d))/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) - 9\*b\*d\*\*3\*x\*\*3\*log(x - I/d - sqrt(3)\*I/(3\*d))/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) + 9\*I\*b\*d\*\*3\*x\*\*3\*atan(dx - sqrt(3)\*I/3)/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) - 9\*sqrt(3)\*I\*b\*d\*\*2\*x\*\*2\*log(x - sqrt(3)\*I/(3\*d))/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) - 9\*b\*d\*\*2\*x\*\*2\*atan(dx - sqrt(3)\*I/3)/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) + 9\*sqrt(3)\*I\*b\*d\*\*2\*x\*\*2\*log(x - I/d - sqrt(3)\*I/(3\*d))/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) - 9\*b\*d\*\*2\*x\*\*2\*atan(dx - sqrt(3)\*I/3)/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) + 6\*sqrt(3)\*I\*b\*d\*x\*atan(dx - sqrt(3)\*I/3)/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) - 2\*sqrt(3)\*I\*b\*log(x - sqrt(3)\*I/(3\*d))/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2) + 2\*sqrt(3)\*I\*b\*log(x - I/d - sqrt(3)\*I/(3\*d))/(9\*d\*\*4\*e\*\*2\*x\*\*3 - 9\*sqrt(3)\*I\*d\*\*3\*e\*\*2\*x\*\*2 - 2\*sqrt(3)\*I\*d\*e\*\*2)

```

*2 - 2*sqrt(3)*I*d**2) - 6*b*atan(d*x - sqrt(3)*I/3)/(9*d**4*e**2*x**3 -
9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*e**2) + 2*sqrt(3)*b*atan(d*x - s
qrt(3)*I/3)/(9*d**4*e**2*x**3 - 9*sqrt(3)*I*d**3*e**2*x**2 - 2*sqrt(3)*I*d*
e**2), Eq(c, -sqrt(3)*I/3)), (-3*sqrt(3)*I*a*d**3*x**3/(9*d**4*e**2*x**3 +
9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 6*sqrt(3)*I*a*d*x/(9*d**
4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 4*a/(9*d**
4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) + 9*b*d**3*x
**3*log(x + sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2
+ 2*sqrt(3)*I*d*e**2) - 9*b*d**3*x**3*log(x - I/d + sqrt(3)*I/(3*d))/(9*d*
**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) + 9*I*b*d**
3*x**3*atan(d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x*
**2 + 2*sqrt(3)*I*d*e**2) + 9*sqrt(3)*I*b*d**2*x**2*log(x + sqrt(3)*I/(3*d))
/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 9*s
qrt(3)*I*b*d**2*x**2*log(x - I/d + sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*s
qrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 9*sqrt(3)*b*d**2*x**2*atan(
d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(
3)*I*d*e**2) - 9*b*d**2*x**2*atan(d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*
sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 6*sqrt(3)*I*b*d*x*atan(d*x
+ sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*
I*d*e**2) + 2*sqrt(3)*I*b*log(x + sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*sq
rt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 2*sqrt(3)*I*b*log(x - I/d +
sqrt(3)*I/(3*d))/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)
*I*d*e**2) - 6*b*atan(d*x + sqrt(3)*I/3)/(9*d**4*e**2*x**3 + 9*sqrt(3)*I*d*
**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2) - 2*sqrt(3)*b*atan(d*x + sqrt(3)*I/3)/(9
*d**4*e**2*x**3 + 9*sqrt(3)*I*d**3*e**2*x**2 + 2*sqrt(3)*I*d*e**2), Eq(c, s
qrt(3)*I/3)), (zoo*a*x, Eq(c, -d*x)), (-2*a*c**2/(6*c**3*d*e**2 + 6*c**2*d*
**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) + 4*a*c*d*x/(6*c**3*d*e**2 + 6*c**2
*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 2*a/(6*c**3*d*e**2 + 6*c**2*d*
**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) + 6*b*c**3*log(c/d + x)/(6*c**3*d*e
**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 3*b*c**3*log(c**2/
d**2 + 2*c*x/d + x**2 + d**(-2))/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*
d*e**2 + 2*d**2*e**2*x) + 6*b*c**2*d*x*log(c/d + x)/(6*c**3*d*e**2 + 6*c**2
*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 3*b*c**2*d*x*log(c**2/d**2 + 2
*c*x/d + x**2 + d**(-2))/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 +
2*d**2*e**2*x) - 6*b*c**2*atan(c + d*x)/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*
x + 2*c*d*e**2 + 2*d**2*e**2*x) + 2*b*c*log(c/d + x)/(6*c**3*d*e**2 + 6*c**
2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - b*c*log(c**2/d**2 + 2*c*x/d +
x**2 + d**(-2))/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*
e**2*x) + 2*b*d*x*log(c/d + x)/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*
e**2 + 2*d**2*e**2*x) - b*d*x*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(6*
c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x) - 2*b*atan(c
+ d*x)/(6*c**3*d*e**2 + 6*c**2*d**2*e**2*x + 2*c*d*e**2 + 2*d**2*e**2*x),
True))

```

**Giac [A]** time = 1.08692, size = 100, normalized size = 1.64

$$-\frac{1}{2}b\left(\frac{e^{(-2)}\log\left(\frac{e^2}{(dx+ce)^2}+1\right)}{d}+\frac{2\arctan(dx+c)e^{(-1)}}{(dx+ce)d}\right)-\frac{ae^{(-1)}}{(dx+ce)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*(e^(-2)*log(e^2/(d*x*e + c*e)^2 + 1)/d + 2*arctan(d*x + c)*e^(-1)/((
d*x*e + c*e)*d)) - a*e^(-1)/((d*x*e + c*e)*d)
```

### 3.6 $\int \frac{a+b \tan^{-1}(c+dx)}{(ce+dex)^3} dx$

**Optimal.** Leaf size=63

$$-\frac{a+b \tan^{-1}(c+dx)}{2de^3(c+dx)^2} - \frac{b}{2de^3(c+dx)} - \frac{b \tan^{-1}(c+dx)}{2de^3}$$

[Out]  $-b/(2*d*e^3*(c+d*x)) - (b*ArcTan[c+d*x])/(2*d*e^3) - (a+b*ArcTan[c+d*x])/(2*d*e^3*(c+d*x)^2)$

**Rubi [A]** time = 0.0446105, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {5043, 12, 4852, 325, 203}

$$-\frac{a+b \tan^{-1}(c+dx)}{2de^3(c+dx)^2} - \frac{b}{2de^3(c+dx)} - \frac{b \tan^{-1}(c+dx)}{2de^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])/(c\*e + d\*e\*x)^3, x]

[Out]  $-b/(2*d*e^3*(c+d*x)) - (b*ArcTan[c+d*x])/(2*d*e^3) - (a+b*ArcTan[c+d*x])/(2*d*e^3*(c+d*x)^2)$

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcTan[c\*x])^(p-1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_.))^ (m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^ (p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^ (-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tan^{-1}(c + dx)}{(ce + dex)^3} dx &= \frac{\text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{e^3 x^3} dx, x, c + dx\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^3} dx, x, c + dx\right)}{de^3} \\
 &= -\frac{a + b \tan^{-1}(c + dx)}{2de^3(c + dx)^2} + \frac{b \text{Subst}\left(\int \frac{1}{x^2(1+x^2)} dx, x, c + dx\right)}{2de^3} \\
 &= -\frac{b}{2de^3(c + dx)} - \frac{a + b \tan^{-1}(c + dx)}{2de^3(c + dx)^2} - \frac{b \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c + dx\right)}{2de^3} \\
 &= -\frac{b}{2de^3(c + dx)} - \frac{b \tan^{-1}(c + dx)}{2de^3} - \frac{a + b \tan^{-1}(c + dx)}{2de^3(c + dx)^2}
 \end{aligned}$$

**Mathematica [C]** time = 0.0131616, size = 51, normalized size = 0.81

$$\frac{b(c + dx)\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -(c + dx)^2\right) + a + b \tan^{-1}(c + dx)}{2de^3(c + dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])/(c\*e + d\*e\*x)^3,x]

[Out] -(a + b\*ArcTan[c + d\*x] + b\*(c + d\*x)\*Hypergeometric2F1[-1/2, 1, 1/2, -(c + d\*x)^2])/(2\*d\*e^3\*(c + d\*x)^2)

**Maple [A]** time = 0.041, size = 71, normalized size = 1.1

$$-\frac{a}{2de^3(dx + c)^2} - \frac{b \arctan(dx + c)}{2de^3(dx + c)^2} - \frac{b \arctan(dx + c)}{2de^3} - \frac{b}{2de^3(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e)^3,x)

[Out] -1/2/d\*a/e^3/(d\*x+c)^2-1/2/d\*b/e^3/(d\*x+c)^2\*arctan(d\*x+c)-1/2\*b\*arctan(d\*x+c)/d/e^3-1/2\*b/d/e^3/(d\*x+c)

**Maxima [B]** time = 1.48335, size = 162, normalized size = 2.57

$$-\frac{1}{2} \left( d \left( \frac{1}{d^3 e^3 x + cd^2 e^3} + \frac{\arctan\left(\frac{d^2 x + cd}{d}\right)}{d^2 e^3} \right) + \frac{\arctan(dx + c)}{d^3 e^3 x^2 + 2cd^2 e^3 x + c^2 de^3} \right) b - \frac{a}{2(d^3 e^3 x^2 + 2cd^2 e^3 x + c^2 de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e)^3,x, algorithm="maxima")

[Out]  $-1/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + \arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a \arctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*b - 1/2*a/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)$

**Fricas [A]** time = 1.68845, size = 162, normalized size = 2.57

$$\frac{bdx + bc + (bd^2x^2 + 2bcdx + bc^2 + b) \arctan(dx + c) + a}{2(d^3e^3x^2 + 2cd^2e^3x + c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e)^3,x, algorithm="fricas")

[Out]  $-1/2*(b*d*x + b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*\arctan(d*x + c) + a)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)$

**Sympy [A]** time = 5.35019, size = 314, normalized size = 4.98

$$\left\{ \frac{a}{\frac{x(a+b \operatorname{atan}(c))}{c^3 e^3}} - \frac{bc^2 \operatorname{atan}(c+dx)}{2c^2 de^3 + 4cd^2 e^3 x + 2d^3 e^3 x^2} - \frac{2bcdx \operatorname{atan}(c+dx)}{2c^2 de^3 + 4cd^2 e^3 x + 2d^3 e^3 x^2} - \frac{bc}{2c^2 de^3 + 4cd^2 e^3 x + 2d^3 e^3 x^2} - \frac{bd^2 x^2 \operatorname{atan}(c+dx)}{2c^2 de^3 + 4cd^2 e^3 x + 2d^3 e^3 x^2} - \frac{a}{2c^2 de^3 + 4cd^2 e^3 x + 2d^3 e^3 x^2} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))/(d\*e\*x+c\*e)\*\*3,x)

[Out] Piecewise((-a/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2) - b\*c\*\*2\*atan(c + d\*x)/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2) - 2\*b\*c\*d\*x\*atan(c + d\*x)/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2) - b\*c/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2) - b\*d\*\*2\*x\*\*2\*atan(c + d\*x)/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2) - b\*d\*x/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2) - b\*atan(c + d\*x)/(2\*c\*\*2\*d\*e\*\*3 + 4\*c\*d\*\*2\*e\*\*3\*x + 2\*d\*\*3\*e\*\*3\*x\*\*2), Ne(d, 0)), (x\*(a + b\*atan(c))/(c\*\*3\*e\*\*3), True))

**Giac [A]** time = 1.12582, size = 113, normalized size = 1.79

$$\frac{bd^2x^2 \arctan(dx + c) + 2bcdx \arctan(dx + c) + bc^2 \arctan(dx + c) + bdx + bc + b \arctan(dx + c) + a}{2(d^3x^2e^3 + 2cd^2xe^3 + c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(d\*e\*x+c\*e)^3,x, algorithm="giac")

[Out]  $-1/2*(b*d^2*x^2*\arctan(d*x + c) + 2*b*c*d*x*\arctan(d*x + c) + b*c^2*\arctan(d*x + c) + b*d*x + b*c + b*\arctan(d*x + c) + a)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)$

### 3.7 $\int (ce + dex)^3 (a + b \tan^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=157

$$\frac{e^3(c+dx)^4(a+b\tan^{-1}(c+dx))^2}{4d} - \frac{be^3(c+dx)^3(a+b\tan^{-1}(c+dx))}{6d} - \frac{e^3(a+b\tan^{-1}(c+dx))^2}{4d} + \frac{1}{2}abe^3x + \frac{b^2e^3}{1}$$

[Out] (a\*b\*e^3\*x)/2 + (b^2\*e^3\*(c + d\*x)^2)/(12\*d) + (b^2\*e^3\*(c + d\*x)\*ArcTan[c + d\*x])/(2\*d) - (b\*e^3\*(c + d\*x)^3\*(a + b\*ArcTan[c + d\*x]))/(6\*d) - (e^3\*(a + b\*ArcTan[c + d\*x])^2)/(4\*d) + (e^3\*(c + d\*x)^4\*(a + b\*ArcTan[c + d\*x])^2)/(4\*d) - (b^2\*e^3\*Log[1 + (c + d\*x)^2])/(3\*d)

**Rubi [A]** time = 0.22059, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {5043, 12, 4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{e^3(c+dx)^4(a+b\tan^{-1}(c+dx))^2}{4d} - \frac{be^3(c+dx)^3(a+b\tan^{-1}(c+dx))}{6d} - \frac{e^3(a+b\tan^{-1}(c+dx))^2}{4d} + \frac{1}{2}abe^3x + \frac{b^2e^3}{1}$$

Antiderivative was successfully verified.

[In] Int[(c\*e + d\*e\*x)^3\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] (a\*b\*e^3\*x)/2 + (b^2\*e^3\*(c + d\*x)^2)/(12\*d) + (b^2\*e^3\*(c + d\*x)\*ArcTan[c + d\*x])/(2\*d) - (b\*e^3\*(c + d\*x)^3\*(a + b\*ArcTan[c + d\*x]))/(6\*d) - (e^3\*(a + b\*ArcTan[c + d\*x])^2)/(4\*d) + (e^3\*(c + d\*x)^4\*(a + b\*ArcTan[c + d\*x])^2)/(4\*d) - (b^2\*e^3\*Log[1 + (c + d\*x)^2])/(3\*d)

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4916

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_))^(m\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcTan[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[  
Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b,  
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Int  
[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n},  
x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le  
Q[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 4846

Int[((a\_) + ArcTan[(c\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] := Simp[x\*(a + b\*Ar  
cTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2  
\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveConten  
t[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 4884

Int[((a\_) + ArcTan[(c\_)\*(x\_)])\*(b\_)^(p\_)/((d\_) + (e\_)\*(x\_)^2), x\_Symbo  
l] := Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b,  
c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int (ce + dex)^3 (a + b \tan^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
 &= \frac{e^3 \text{Subst}\left(\int x^3 (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
 &= \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))^2}{4d} - \frac{(be^3) \text{Subst}\left(\int \frac{x^4 (a + b \tan^{-1}(x))}{1+x^2} dx, x, c + dx\right)}{2d} \\
 &= \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))^2}{4d} - \frac{(be^3) \text{Subst}\left(\int x^2 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{2d} \\
 &= -\frac{be^3 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{6d} + \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))^2}{4d} + \frac{(b^2 e^3 (c + dx)^3 (a + b \tan^{-1}(c + dx)))^2}{6d} \\
 &= \frac{1}{2} abe^3 x - \frac{be^3 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{6d} - \frac{e^3 (a + b \tan^{-1}(c + dx))^2}{4d} + \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))^2}{4d} \\
 &= \frac{1}{2} abe^3 x + \frac{b^2 e^3 (c + dx) \tan^{-1}(c + dx)}{2d} - \frac{be^3 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{6d} - \frac{e^3 (c + dx)^4 (a + b \tan^{-1}(c + dx))^2}{4d} \\
 &= \frac{1}{2} abe^3 x + \frac{b^2 e^3 (c + dx)^2}{12d} + \frac{b^2 e^3 (c + dx) \tan^{-1}(c + dx)}{2d} - \frac{be^3 (c + dx)^3 (a + b \tan^{-1}(c + dx))}{6d}
 \end{aligned}$$



**Mathematica [A]** time = 0.0980365, size = 216, normalized size = 1.38

$$e^3 \left( (c + dx) \left( 3a^2(c + dx)^3 - 2ab(c^2 + 2cdx + d^2x^2 - 3) + b^2(c + dx) \right) + 2b \tan^{-1}(c + dx) \left( 3a(6c^2d^2x^2 + 4c^3dx + c^4 + \dots) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c\*e + d\*e\*x)^3\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] (e^3\*((c + d\*x)\*(b^2\*(c + d\*x) + 3\*a^2\*(c + d\*x)^3 - 2\*a\*b\*(-3 + c^2 + 2\*c\*d\*x + d^2\*x^2)) + 2\*b\*(-(b\*(-3\*c + c^3 - 3\*d\*x + 3\*c^2\*d\*x + 3\*c\*d^2\*x^2 + d^3\*x^3)) + 3\*a\*(-1 + c^4 + 4\*c^3\*d\*x + 6\*c^2\*d^2\*x^2 + 4\*c\*d^3\*x^3 + d^4\*x^4))\*ArcTan[c + d\*x] + 3\*b^2\*(-1 + c^4 + 4\*c^3\*d\*x + 6\*c^2\*d^2\*x^2 + 4\*c\*d^3\*x^3 + d^4\*x^4)\*ArcTan[c + d\*x]^2 - 4\*b^2\*Log[1 + (c + d\*x)^2]))/(12\*d)

**Maple [B]** time = 0.046, size = 543, normalized size = 3.5

$$\frac{b^2c^2e^3}{12d} + \frac{a^2c^4e^3}{4d} - \frac{dx^2abce^3}{2} - \frac{d \arctan(dx + c)x^2b^2ce^3}{2} + \frac{d^3 \arctan(dx + c)x^4abe^3}{2} + \frac{\arctan(dx + c)abc^4e^3}{2d} + d^2(a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*e\*x+c\*e)^3\*(a+b\*arctan(d\*x+c))^2,x)

[Out] 1/12/d\*b^2\*c^2\*e^3+1/4/d\*a^2\*c^4\*e^3-1/2\*d\*x^2\*a\*b\*c\*e^3-1/2\*d\*arctan(d\*x+c)\*x^2\*b^2\*c\*e^3+1/2\*d^3\*arctan(d\*x+c)\*x^4\*a\*b\*e^3+1/2/d\*arctan(d\*x+c)\*a\*b\*c^4\*e^3+d^2\*arctan(d\*x+c)^2\*x^3\*b^2\*c\*e^3+2\*d^2\*arctan(d\*x+c)\*x^3\*a\*b\*c\*e^3+3\*d\*arctan(d\*x+c)\*x^2\*a\*b\*c^2\*e^3-1/2\*x\*a\*b\*c^2\*e^3+1/4\*d^3\*x^4\*a^2\*e^3-1/4/d\*e^3\*b^2\*arctan(d\*x+c)^2+1/2\*arctan(d\*x+c)\*x\*b^2\*e^3+1/12\*d\*x^2\*b^2\*e^3+1/6\*x\*b^2\*c\*e^3+x\*a^2\*c^3\*e^3-1/2\*arctan(d\*x+c)\*x\*b^2\*c^2\*e^3-1/6\*d^2\*x^3\*a\*b\*e^3+d^2\*x^3\*a^2\*c\*e^3+3/2\*d\*x^2\*a^2\*c^2\*e^3+1/4\*d^3\*arctan(d\*x+c)^2\*x^4\*b^2\*e^3-1/6\*d^2\*arctan(d\*x+c)\*x^3\*b^2\*e^3-1/6/d\*arctan(d\*x+c)\*b^2\*c^3\*e^3-1/2/d\*e^3\*a\*b\*arctan(d\*x+c)+1/4/d\*arctan(d\*x+c)^2\*b^2\*c^4\*e^3+1/2/d\*arctan(d\*x+c)\*b^2\*c\*e^3+1/2/d\*a\*b\*c\*e^3-1/3\*b^2\*e^3\*ln(1+(d\*x+c)^2)/d+arctan(d\*x+c)^2\*x\*b^2\*c^3\*e^3+3/2\*d\*arctan(d\*x+c)^2\*x^2\*b^2\*c^2\*e^3+2\*arctan(d\*x+c)\*x\*a\*b\*c^3\*e^3+1/2\*a\*b\*e^3\*x-1/6/d\*a\*b\*c^3\*e^3

**Maxima [B]** time = 5.51433, size = 806, normalized size = 5.13

$$\frac{1}{4} a^2 d^3 e^3 x^4 + a^2 c d^2 e^3 x^3 + \frac{3}{2} a^2 c^2 d e^3 x^2 + 3 \left( x^2 \arctan(dx + c) - d \left( \frac{x}{d^2} + \frac{(c^2 - 1) \arctan\left(\frac{d^2 x + cd}{d}\right)}{d^3} - \frac{c \log(d^2 x^2 + 2cdx + c^2 + 1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^3\*(a+b\*arctan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*a^2\*d^3\*e^3\*x^4 + a^2\*c\*d^2\*e^3\*x^3 + 3/2\*a^2\*c^2\*d\*e^3\*x^2 + 3\*(x^2\*arctan(d\*x + c) - d\*(x/d^2 + (c^2 - 1)\*arctan((d^2\*x + c\*d)/d)/d^3 - c\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)/d^3))\*a\*b\*c^2\*d\*e^3 + (2\*x^3\*arctan(d\*x + c) - d\*((d\*x^2 - 4\*c\*x)/d^3 - 2\*(c^3 - 3\*c)\*arctan((d^2\*x + c\*d)/d)/d^4 + (3\*c^2 - 1)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)/d^4))\*a\*b\*c\*d^2\*e^3 + 1/6\*(3\*x^4\*arctan(d\*x + c) - d\*((d^2\*x^3 - 3\*c\*d\*x^2 + 3\*(3\*c^2 - 1)\*x)/d^4 + 3\*(c^4 - 6\*c^2 + 1)\*arctan((d^2\*x + c\*d)/d)/d^5 - 6\*(c^3 - c)\*log(d^2\*x^2 + 2\*c\*d\*x + c

$$\begin{aligned} & \wedge^2 + 1)/d^5)) * a * b * d^3 * e^3 + a^2 * c^3 * e^3 * x + (2 * (d * x + c) * \arctan(d * x + c) - \\ & \log((d * x + c)^2 + 1)) * a * b * c^3 * e^3 / d + 1/12 * (b^2 * d^2 * e^3 * x^2 + 2 * b^2 * c * d * e^3 \\ & * x - 4 * b^2 * e^3 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) + 3 * (b^2 * d^4 * e^3 * x^4 + 4 * b^2 \\ & * c * d^3 * e^3 * x^3 + 6 * b^2 * c^2 * d^2 * e^3 * x^2 + 4 * b^2 * c^3 * d * e^3 * x + (b^2 * c^4 - b^2 \\ & * e^3) * \arctan(d * x + c)^2 - 2 * (b^2 * d^3 * e^3 * x^3 + 3 * b^2 * c * d^2 * e^3 * x^2 + 3 * (b \\ & ^2 * c^2 - b^2) * d * e^3 * x + (b^2 * c^3 - 3 * b^2 * c) * e^3) * \arctan(d * x + c)) / d \end{aligned}$$

**Fricas [B]** time = 1.79675, size = 703, normalized size = 4.48

$$3a^2d^4e^3x^4 + 2(6a^2c - ab)d^3e^3x^3 + (18a^2c^2 - 6abc + b^2)d^2e^3x^2 + 2(6a^2c^3 - 3abc^2 + b^2c + 3ab)de^3x - 4b^2e^3 \log(d^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^3\*(a+b\*arctan(d\*x+c))^2,x, algorithm="fricas")

[Out] 1/12\*(3\*a^2\*d^4\*e^3\*x^4 + 2\*(6\*a^2\*c - a\*b)\*d^3\*e^3\*x^3 + (18\*a^2\*c^2 - 6\*a\*b\*c + b^2)\*d^2\*e^3\*x^2 + 2\*(6\*a^2\*c^3 - 3\*a\*b\*c^2 + b^2\*c + 3\*a\*b)\*d\*e^3\*x - 4\*b^2\*e^3\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) + 3\*(b^2\*d^4\*e^3\*x^4 + 4\*b^2\*c\*d^3\*e^3\*x^3 + 6\*b^2\*c^2\*d^2\*e^3\*x^2 + 4\*b^2\*c^3\*d\*e^3\*x + (b^2\*c^4 - b^2)\*e^3)\*arctan(d\*x + c)^2 + 2\*(3\*a\*b\*d^4\*e^3\*x^4 + (12\*a\*b\*c - b^2)\*d^3\*e^3\*x^3 + 3\*(6\*a\*b\*c^2 - b^2\*c)\*d^2\*e^3\*x^2 + 3\*(4\*a\*b\*c^3 - b^2\*c^2 + b^2)\*d\*e^3\*x + (3\*a\*b\*c^4 - b^2\*c^3 + 3\*b^2\*c - 3\*a\*b)\*e^3)\*arctan(d\*x + c))/d

**Sympy [A]** time = 11.7642, size = 575, normalized size = 3.66

$$\begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3 \operatorname{atan}(c+dx)}{2d} + 2abc^3e^3x \operatorname{atan}(c+dx) + 3abc^2de^3x^2 \operatorname{atan}(c+dx) - \frac{abc^2e^3}{2} \\ c^3e^3x(a+b \operatorname{atan}(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*\*3\*(a+b\*atan(d\*x+c))\*\*2,x)

[Out] Piecewise((a\*\*2\*c\*\*3\*e\*\*3\*x + 3\*a\*\*2\*c\*\*2\*d\*e\*\*3\*x\*\*2/2 + a\*\*2\*c\*d\*\*2\*e\*\*3\*x\*\*3 + a\*\*2\*d\*\*3\*e\*\*3\*x\*\*4/4 + a\*b\*c\*\*4\*e\*\*3\*atan(c + d\*x)/(2\*d) + 2\*a\*b\*c\*\*3\*e\*\*3\*x\*atan(c + d\*x) + 3\*a\*b\*c\*\*2\*d\*e\*\*3\*x\*\*2\*atan(c + d\*x) - a\*b\*c\*\*2\*e\*\*3\*x/2 + 2\*a\*b\*c\*d\*\*2\*e\*\*3\*x\*\*3\*atan(c + d\*x) - a\*b\*c\*d\*e\*\*3\*x\*\*2/2 + a\*b\*d\*\*3\*e\*\*3\*x\*\*4\*atan(c + d\*x)/2 - a\*b\*d\*\*2\*e\*\*3\*x\*\*3/6 + a\*b\*e\*\*3\*x/2 - a\*b\*e\*\*3\*atan(c + d\*x)/(2\*d) + b\*\*2\*c\*\*4\*e\*\*3\*atan(c + d\*x)\*\*2/(4\*d) + b\*\*2\*c\*\*3\*e\*\*3\*x\*atan(c + d\*x)\*\*2 - b\*\*2\*c\*\*3\*e\*\*3\*atan(c + d\*x)/(6\*d) + 3\*b\*\*2\*c\*\*2\*d\*e\*\*3\*x\*\*2\*atan(c + d\*x)\*\*2/2 - b\*\*2\*c\*\*2\*e\*\*3\*x\*atan(c + d\*x)/2 + b\*\*2\*c\*d\*\*2\*e\*\*3\*x\*\*3\*atan(c + d\*x)\*\*2 - b\*\*2\*c\*d\*e\*\*3\*x\*\*2\*atan(c + d\*x)/2 + b\*\*2\*c\*e\*\*3\*x/6 + b\*\*2\*c\*e\*\*3\*atan(c + d\*x)/(2\*d) + b\*\*2\*d\*\*3\*e\*\*3\*x\*\*4\*atan(c + d\*x)\*\*2/4 - b\*\*2\*d\*\*2\*e\*\*3\*x\*\*3\*atan(c + d\*x)/6 + b\*\*2\*d\*e\*\*3\*x\*\*2/12 + b\*\*2\*e\*\*3\*x\*atan(c + d\*x)/2 - b\*\*2\*e\*\*3\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(3\*d) - b\*\*2\*e\*\*3\*atan(c + d\*x)\*\*2/(4\*d), Ne(d, 0)), (c\*\*3\*e\*\*3\*x\*(a + b\*atan(c))\*\*2, True))

**Giac [B]** time = 1.37464, size = 784, normalized size = 4.99

$$3b^2d^4x^4 \arctan(dx + c)^2 e^3 + 6abd^4x^4 \arctan(dx + c) e^3 + 12b^2cd^3x^3 \arctan(dx + c)^2 e^3 + 3a^2d^4x^4 e^3 + 24abcd^3x^3 \arctan(dx + c) e^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^3\*(a+b\*arctan(d\*x+c))^2,x, algorithm="giac")

[Out] 
$$\frac{1}{12} \cdot (3b^2d^4x^4 \arctan(dx+c)^2 e^3 + 6ab d^4x^4 \arctan(dx+c) e^3 + 12b^2c d^3x^3 \arctan(dx+c)^2 e^3 + 3a^2 d^4x^4 e^3 + 24ab c d^3x^3 \arctan(dx+c) e^3 + 18b^2c^2 d^2x^2 \arctan(dx+c)^2 e^3 + 12a^2c d^3x^3 e^3 + 36ab c^2 d^2x^2 \arctan(dx+c) e^3 - 2b^2 d^3x^3 \arctan(dx+c) e^3 + 12b^2c^3 d x \arctan(dx+c)^2 e^3 + 18a^2c^2 d^2x^2 e^3 - 2ab d^3x^3 e^3 + 24ab c^3 d x \arctan(dx+c) e^3 - 6b^2c d^2x^2 \arctan(dx+c) e^3 + 3b^2c^4 \arctan(dx+c)^2 e^3 + 3\pi a b c^4 e^3 \operatorname{sgn}(dx+c) - 3\pi a b c^4 e^3 + 12a^2c^3 d x e^3 - 6ab c d^2x^2 e^3 - 6b^2c^2 d x \arctan(dx+c) e^3 - 6ab c^4 \arctan(1/(dx+c)) e^3 - \pi b^2c^3 e^3 \operatorname{sgn}(dx+c) + \pi b^2c^3 e^3 - 6ab c^2 d x e^3 + b^2 d^2x^2 e^3 + 2b^2c^3 \arctan(1/(dx+c)) e^3 + 2b^2c d x e^3 + 6b^2 d x \arctan(dx+c) e^3 + 3\pi b^2c e^3 \operatorname{sgn}(dx+c) - 3\pi b^2c e^3 + 6ab d x e^3 - 3b^2 \arctan(dx+c)^2 e^3 - 6b^2c \arctan(1/(dx+c)) e^3 - 3\pi a b e^3 \operatorname{sgn}(dx+c) + 3\pi a b e^3 + 6ab \arctan(1/(dx+c)) e^3 - 4b^2 e^3 \log(d^2x^2 + 2c d x + c^2 + 1)) / d$$

### 3.8 $\int (ce + dex)^2 (a + b \tan^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=183

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d} + \frac{e^2(c+dx)^3(a+b\tan^{-1}(c+dx))^2}{3d} - \frac{be^2(c+dx)^2(a+b\tan^{-1}(c+dx))}{3d} - \frac{ie^2(a+btan^{-1}(c+dx))}{3d}$$

[Out] (b^2\*e^2\*x)/3 - (b^2\*e^2\*ArcTan[c + d\*x])/(3\*d) - (b\*e^2\*(c + d\*x)^2\*(a + b\*ArcTan[c + d\*x]))/(3\*d) - ((I/3)\*e^2\*(a + b\*ArcTan[c + d\*x])^2)/d + (e^2\*(c + d\*x)^3\*(a + b\*ArcTan[c + d\*x])^2)/(3\*d) - (2\*b\*e^2\*(a + b\*ArcTan[c + d\*x])\*Log[2/(1 + I\*(c + d\*x))])/(3\*d) - ((I/3)\*b^2\*e^2\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/d

**Rubi [A]** time = 0.221134, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {5043, 12, 4852, 4916, 321, 203, 4920, 4854, 2402, 2315}

$$-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d} + \frac{e^2(c+dx)^3(a+b\tan^{-1}(c+dx))^2}{3d} - \frac{be^2(c+dx)^2(a+b\tan^{-1}(c+dx))}{3d} - \frac{ie^2(a+btan^{-1}(c+dx))}{3d}$$

Antiderivative was successfully verified.

[In] Int[(c\*e + d\*e\*x)^2\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] (b^2\*e^2\*x)/3 - (b^2\*e^2\*ArcTan[c + d\*x])/(3\*d) - (b\*e^2\*(c + d\*x)^2\*(a + b\*ArcTan[c + d\*x]))/(3\*d) - ((I/3)\*e^2\*(a + b\*ArcTan[c + d\*x])^2)/d + (e^2\*(c + d\*x)^3\*(a + b\*ArcTan[c + d\*x])^2)/(3\*d) - (2\*b\*e^2\*(a + b\*ArcTan[c + d\*x])\*Log[2/(1 + I\*(c + d\*x))])/(3\*d) - ((I/3)\*b^2\*e^2\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/d

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4916

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_.))^ (m\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcTan[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcTan[c\*x])^p)/(d +

$e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{GtQ}[m, 1]$

### Rule 321

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 203

$\text{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

### Rule 4920

$\text{Int}[(a_*) + \text{ArcTan}(c_*)*(x_*)*(b_*)^{(p_*)}*(x_*)]/((d_*) + (e_*)*(x_*)^2), x\_Symbol] \rightarrow -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^{(p+1)})/(b*e^{(p+1)}), x] - \text{Dist}[1/(c*d), \text{Int}[(a + b*\text{ArcTan}[c*x])^p/(1 - c*x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[e, c^2*d] \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 4854

$\text{Int}[(a_*) + \text{ArcTan}(c_*)*(x_*)*(b_*)^{(p_*)}]/((d_*) + (e_*)*(x_*)^2), x\_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcTan}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]/e, x] + \text{Dist}[(b*c*p)/e, \text{Int}[(a + b*\text{ArcTan}[c*x])^{(p-1)}*\text{Log}[2/(1 + (e*x)/d)]/(1 + c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[c^2*d^2 + e^2, 0]$

### Rule 2402

$\text{Int}[\text{Log}[(c_*)/((d_*) + (e_*)*(x_*)^2)]/((f_*) + (g_*)*(x_*)^2), x\_Symbol] \rightarrow -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c_*)*(x_*)]/((d_*) + (e_*)*(x_*)^2), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x\} \ \&\& \ \text{EqQ}[e + c*d, 0]$

### Rubi steps

$$\begin{aligned}
\int (ce + dex)^2 (a + b \tan^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{e^2 \text{Subst}\left(\int x^2 (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))^2}{3d} - \frac{(2be^2) \text{Subst}\left(\int \frac{x^3 (a + b \tan^{-1}(x))}{1+x^2} dx, x, c + dx\right)}{3d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))^2}{3d} - \frac{(2be^2) \text{Subst}\left(\int x (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{3d} \\
&= -\frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))}{3d} - \frac{ie^2 (a + b \tan^{-1}(c + dx))^2}{3d} + \frac{e^2 (c + dx)^3}{3d} \\
&= \frac{1}{3} b^2 e^2 x - \frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))}{3d} - \frac{ie^2 (a + b \tan^{-1}(c + dx))^2}{3d} + \frac{e^2 (c + dx)^3}{3d} \\
&= \frac{1}{3} b^2 e^2 x - \frac{b^2 e^2 \tan^{-1}(c + dx)}{3d} - \frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))}{3d} - \frac{ie^2 (a + b \tan^{-1}(c + dx))^2}{3d} \\
&= \frac{1}{3} b^2 e^2 x - \frac{b^2 e^2 \tan^{-1}(c + dx)}{3d} - \frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))}{3d} - \frac{ie^2 (a + b \tan^{-1}(c + dx))^2}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.36222, size = 163, normalized size = 0.89

$$\frac{e^2 \left( b^2 \left( i \text{PolyLog}\left(2, -e^{2i \tan^{-1}(c+dx)}\right) + (c + dx)^3 \tan^{-1}(c + dx)^2 - (c + dx)^2 \tan^{-1}(c + dx) + i \tan^{-1}(c + dx)^2 - \tan^{-1}(c + dx) \right) \right)}{3d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*e + d\*e\*x)^2\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] (e^2\*(a^2\*(c + d\*x)^3 + a\*b\*(-(c + d\*x)^2 + 2\*(c + d\*x)^3\*ArcTan[c + d\*x] + Log[1 + (c + d\*x)^2]) + b^2\*(c + d\*x - ArcTan[c + d\*x] - (c + d\*x)^2\*ArcTan[c + d\*x] + I\*ArcTan[c + d\*x]^2 + (c + d\*x)^3\*ArcTan[c + d\*x]^2 - 2\*ArcTan[c + d\*x]\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) + I\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]))/(3\*d)

**Maple [B]** time = 0.123, size = 593, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c))^2,x)

[Out] 1/6\*I/d\*e^2\*b^2\*ln(d\*x+c+I)\*ln(1/2\*I\*(d\*x+c-I))-1/6\*I/d\*e^2\*b^2\*ln(1+(d\*x+c)^2)\*ln(d\*x+c+I)-1/6\*I/d\*e^2\*b^2\*ln(d\*x+c-I)\*ln(-1/2\*I\*(d\*x+c+I))+1/6\*I/d\*e^2\*b^2\*ln(1+(d\*x+c)^2)\*ln(d\*x+c-I)+2/3\*d^2\*arctan(d\*x+c)\*x^3\*a\*b\*e^2-1/3\*d\*arctan(d\*x+c)\*x^2\*b^2\*e^2+1/3/d\*arctan(d\*x+c)^2\*b^2\*c^3\*e^2-1/3/d\*arctan(d\*x+c)\*b^2\*c^2\*e^2+1/3/d\*e^2\*b^2\*arctan(d\*x+c)\*ln(1+(d\*x+c)^2)-2/3\*x\*a\*b\*c\*e^2+1/3/d\*e^2\*a\*b\*ln(1+(d\*x+c)^2)+1/3\*d^2\*arctan(d\*x+c)^2\*x^3\*b^2\*e^2-1/12\*I/

$$d^2 e^{2b} \ln(dx+c-I)^2 + 1/12 I/d^2 e^{2b} \ln(dx+c+I)^2 - 1/6 I/d^2 e^{2b} \operatorname{dilog}(-1/2 I*(dx+c+I)) + 1/6 I/d^2 e^{2b} \operatorname{dilog}(1/2 I*(dx+c-I)) + 1/3 b^2 e^{2x} - 1/3 d^2 a^2 b^2 c^2 e^{2x} + 1/3 d^2 x^3 a^2 e^{2x} + 2d \arctan(dx+c) x^2 a^2 b^2 c^2 e^{2x} + 1/3 d^2 b^2 c^2 e^{2x} + 1/3 d^2 a^2 c^3 e^{2x} + d^2 x^2 a^2 c^2 e^{2x} - 2/3 \arctan(dx+c) x^2 b^2 c^2 e^{2x} + \arctan(dx+c)^2 x^2 b^2 c^2 e^{2x} - 1/3 d^2 x^2 a^2 b^2 e^{2x} + 2 \arctan(dx+c) x^2 a^2 b^2 c^2 e^{2x} + 2/3 d \arctan(dx+c) a^2 b^2 c^3 e^{2x} + d \arctan(dx+c)^2 x^2 b^2 c^2 e^{2x} - 1/3 b^2 e^{2x} \arctan(dx+c)/d$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c))^2,x, algorithm="maxima")

[Out]  $\frac{3}{4} b^2 c^4 e^{2x} \arctan(dx+c)^2 \arctan\left(\frac{d^2 x + cd}{d}\right)/d - \frac{1}{4} (3 \arctan(dx+c) \arctan\left(\frac{d^2 x + cd}{d}\right)^2/d - \arctan\left(\frac{d^2 x + cd}{d}\right)^3/d) b^2 c^4 e^{2x} + \frac{1}{3} a^2 d^2 e^{2x} x^3 + 36 b^2 d^4 e^{2x} \int \frac{1}{48 x^4 \arctan(dx+c)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 3 b^2 d^4 e^{2x} \int \frac{1}{48 x^4 \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 144 b^2 c^2 d^3 e^{2x} \int \frac{1}{48 x^3 \arctan(dx+c)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 4 b^2 d^4 e^{2x} \int \frac{1}{48 x^4 \log(d^2 x^2 + 2cdx + c^2 + 1) / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 12 b^2 c^2 d^3 e^{2x} \int \frac{1}{48 x^3 \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 216 b^2 c^2 d^2 e^{2x} \int \frac{1}{48 x^2 \arctan(dx+c)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 16 b^2 c^2 d^3 e^{2x} \int \frac{1}{48 x^3 \log(d^2 x^2 + 2cdx + c^2 + 1) / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 18 b^2 c^2 d^2 e^{2x} \int \frac{1}{48 x^2 \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 144 b^2 c^3 d e^{2x} \int \frac{1}{48 x \arctan(dx+c)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 24 b^2 c^2 d^2 e^{2x} \int \frac{1}{48 x^2 \log(d^2 x^2 + 2cdx + c^2 + 1) / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 12 b^2 c^3 d e^{2x} \int \frac{1}{48 x \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 12 b^2 c^3 d e^{2x} \int \frac{1}{48 x \log(d^2 x^2 + 2cdx + c^2 + 1) / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 3 b^2 c^4 e^{2x} \int \frac{1}{48 \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + a^2 c^2 d e^{2x} x^2 + \frac{3}{4} b^2 c^2 e^{2x} \arctan(dx+c)^2 \arctan\left(\frac{d^2 x + cd}{d}\right)/d - 8 b^2 d^3 e^{2x} \int \frac{1}{48 x^3 \arctan(dx+c) / (d^2 x^2 + 2cdx + c^2 + 1)} dx - 24 b^2 c^2 d^2 e^{2x} \int \frac{1}{48 x^2 \arctan(dx+c) / (d^2 x^2 + 2cdx + c^2 + 1)} dx - 24 b^2 c^2 d e^{2x} \int \frac{1}{48 x \arctan(dx+c) / (d^2 x^2 + 2cdx + c^2 + 1)} dx - \frac{1}{4} (3 \arctan(dx+c) \arctan\left(\frac{d^2 x + cd}{d}\right)^2/d - \arctan\left(\frac{d^2 x + cd}{d}\right)^3/d) b^2 c^2 e^{2x} + 2(x^2 \arctan(dx+c) - d(x/d^2 + (c^2 - 1) \arctan\left(\frac{d^2 x + cd}{d}\right))/d^3 - c \log(d^2 x^2 + 2cdx + c^2 + 1)/d^3) a^2 b^2 c^2 d e^{2x} + \frac{1}{3} (2x^3 \arctan(dx+c) - d((dx^2 - 4cx)/d^3 - 2(c^3 - 3c) \arctan\left(\frac{d^2 x + cd}{d}\right))/d^4 + (3c^2 - 1) \log(d^2 x^2 + 2cdx + c^2 + 1)/d^4) a^2 b^2 d^2 e^{2x} + a^2 c^2 e^{2x} x + 36 b^2 d^2 e^{2x} \int \frac{1}{48 x^2 \arctan(dx+c)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 3 b^2 d^2 e^{2x} \int \frac{1}{48 x^2 \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 72 b^2 c^2 d e^{2x} \int \frac{1}{48 x \arctan(dx+c)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 6 b^2 c^2 d e^{2x} \int \frac{1}{48 x \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + 3 b^2 c^2 e^{2x} \int \frac{1}{48 \log(d^2 x^2 + 2cdx + c^2 + 1)^2 / (d^2 x^2 + 2cdx + c^2 + 1)} dx + (2(dx+c) \arctan(dx+c) - \log((dx+c)^2 + 1)) a^2 b^2 c^2 e^{2x} / d + \frac{1}{12} (b^2 d^2 e^{2x} x^3 + 3 b^2 c^2 d e^{2x} x^2 + 3 b^2 c^2 e^{2x} x) \arctan(dx+c)^2 - \frac{1}{48} (b^2 d^2 e^{2x} x^3 + 3 b^2 c^2 d e^{2x} x^2 + 3 b^2 c^2 e^{2x} x) \log(d^2 x^2 + 2cdx + c^2 + 1)^2$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral( $a^2 d^2 e^2 x^2 + 2 a^2 c d e^2 x + a^2 c^2 e^2 + (b^2 d^2 e^2 x^2 + 2 b^2 c d e^2 x + b^2 c^2 e^2) \arctan(dx + c)^2 + 2 (a b d^2 e^2 x^2 + 2 a b c d e^2 x +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c))^2,x, algorithm="fricas")

[Out] integral(a^2\*d^2\*e^2\*x^2 + 2\*a^2\*c\*d\*e^2\*x + a^2\*c^2\*e^2 + (b^2\*d^2\*e^2\*x^2 + 2\*b^2\*c\*d\*e^2\*x + b^2\*c^2\*e^2)\*arctan(d\*x + c)^2 + 2\*(a\*b\*d^2\*e^2\*x^2 + 2\*a\*b\*c\*d\*e^2\*x + a\*b\*c^2\*e^2)\*arctan(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$e^2 \left( \int a^2 c^2 dx + \int a^2 d^2 x^2 dx + \int b^2 c^2 \operatorname{atan}^2(c + dx) dx + \int 2abc^2 \operatorname{atan}(c + dx) dx + \int 2a^2 c dx dx + \int b^2 d^2 x^2 \operatorname{atan}^2(c + dx) dx \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*\*2\*(a+b\*atan(d\*x+c))\*\*2,x)

[Out] e\*\*2\*(Integral(a\*\*2\*c\*\*2, x) + Integral(a\*\*2\*d\*\*2\*x\*\*2, x) + Integral(b\*\*2\*c\*\*2\*atan(c + d\*x)\*\*2, x) + Integral(2\*a\*b\*c\*\*2\*atan(c + d\*x), x) + Integral(2\*a\*\*2\*c\*d\*x, x) + Integral(b\*\*2\*d\*\*2\*x\*\*2\*atan(c + d\*x)\*\*2, x) + Integral(2\*a\*b\*d\*\*2\*x\*\*2\*atan(c + d\*x), x) + Integral(2\*b\*\*2\*c\*d\*x\*atan(c + d\*x)\*\*2, x) + Integral(4\*a\*b\*c\*d\*x\*atan(c + d\*x), x))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \arctan(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((d\*e\*x + c\*e)^2\*(b\*arctan(d\*x + c) + a)^2, x)



### 3.9 $\int (ce + dex) (a + b \tan^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=95

$$\frac{e(c+dx)^2(a+b\tan^{-1}(c+dx))^2}{2d} + \frac{e(a+b\tan^{-1}(c+dx))^2}{2d} - abex + \frac{b^2e\log((c+dx)^2+1)}{2d} - \frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}$$

[Out]  $-(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[1 + (c + d*x)^2])/(2*d)$

**Rubi [A]** time = 0.119431, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5043, 12, 4852, 4916, 4846, 260, 4884}

$$\frac{e(c+dx)^2(a+b\tan^{-1}(c+dx))^2}{2d} + \frac{e(a+b\tan^{-1}(c+dx))^2}{2d} - abex + \frac{b^2e\log((c+dx)^2+1)}{2d} - \frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2, x]$

[Out]  $-(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[1 + (c + d*x)^2])/(2*d)$

#### Rule 5043

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p, x] := \text{Dist}[1/d, \text{Subst}[\text{Int}[(f*x)/d]^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

$\text{Int}[a*(u + b*v)^p, x] := \text{Dist}[a, \text{Int}[u + b*v, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p*(d + e*x)^m, x] := \text{Simp}[(d*x)^{m+1}*(a + b*ArcTan[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{m+1}*(a + b*ArcTan[c*x])^{p-1}/(1 + c^2*x^2), x], x] /;$  FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4916

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p*(f + g*x)^m/(d + e*x^2), x] := \text{Dist}[f^2/e, \text{Int}[(f*x)^{m-2}*(a + b*ArcTan[c*x])^p, x] - \text{Dist}[(d*f^2)/e, \text{Int}[(f*x)^{m-2}*(a + b*ArcTan[c*x])^p/(d + e*x^2), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

#### Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int (ce + dex) (a + b \tan^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int ex (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\ &= \frac{e \text{Subst}\left(\int x (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\ &= \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{(be) \text{Subst}\left(\int \frac{x^2(a + b \tan^{-1}(x))}{1 + x^2} dx, x, c + dx\right)}{d} \\ &= \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{(be) \text{Subst}\left(\int (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= -abex + \frac{e(a + b \tan^{-1}(c + dx))^2}{2d} + \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{(b^2e) \text{Subst}\left(\int \frac{1}{1 + x^2} dx, x, c + dx\right)}{d} \\ &= -abex - \frac{b^2e(c + dx) \tan^{-1}(c + dx)}{d} + \frac{e(a + b \tan^{-1}(c + dx))^2}{2d} + \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} \\ &= -abex - \frac{b^2e(c + dx) \tan^{-1}(c + dx)}{d} + \frac{e(a + b \tan^{-1}(c + dx))^2}{2d} + \frac{e(c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0576755, size = 107, normalized size = 1.13

$$\frac{e(2b \tan^{-1}(c + dx) (a(c^2 + 2cdx + d^2x^2 + 1) - b(c + dx)) + a(c + dx)(ac + adx - 2b) + b^2(c^2 + 2cdx + d^2x^2 + 1) \tan^{-1}(c + dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e*(a*(c + d*x)*(-2*b + a*c + a*d*x) + 2*b*(-(b*(c + d*x)) + a*(1 + c^2 + 2*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^2 + b^2*Log[1 + (c + d*x)^2])/(2*d)
```

**Maple [B]** time = 0.046, size = 220, normalized size = 2.3

$$\frac{a^2x^2de}{2} + xa^2ce + \frac{a^2c^2e}{2d} + \frac{d(\arctan(dx + c))^2x^2b^2e}{2} + (\arctan(dx + c))^2xb^2ce + \frac{(\arctan(dx + c))^2b^2c^2e}{2d} + \frac{eb^2(\arctan(dx + c))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x)`

[Out]  $\frac{1}{2}a^2x^2d^2e+x^2a^2c^2e+\frac{1}{2}d^2a^2c^2e+\frac{1}{2}d^2e\arctan(d*x+c)^2*x^2*b^2e+a\arctan(d*x+c)^2*x*b^2c^2e+\frac{1}{2}d^2e\arctan(d*x+c)^2*b^2c^2e+\frac{1}{2}d^2e*b^2\arctan(d*x+c)^2-\arctan(d*x+c)*x*b^2e-\frac{1}{d}\arctan(d*x+c)*b^2c^2e+\frac{1}{2}b^2e*\ln(1+(d*x+c)^2)/d+d\arctan(d*x+c)*x^2*a*b^2e+2\arctan(d*x+c)*x*a*b*c^2e+\frac{1}{d}\arctan(d*x+c)*a*b*c^2e+\frac{1}{d^2}e*a*b*\arctan(d*x+c)-a*b^2e*x-\frac{1}{d}a*b^2c^2e$

**Maxima [B]** time = 5.20193, size = 294, normalized size = 3.09

$$\frac{1}{2}a^2dex^2 + \left( x^2 \arctan(dx + c) - d \left( \frac{x}{d^2} + \frac{(c^2 - 1) \arctan\left(\frac{d^2x + cd}{d}\right)}{d^3} - \frac{c \log(d^2x^2 + 2cdx + c^2 + 1)}{d^3} \right) \right) abde + a^2cex + \frac{(2}{$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}a^2d^2e*x^2 + (x^2*\arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*\arctan((d^2*x + c*d)/d)/d^3 - c*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*d^2e + a^2*c^2e*x + (2*(d*x + c)*\arctan(d*x + c) - \log((d*x + c)^2 + 1))*a*b*c^2e/d + 1/2*(b^2*e*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 + b^2)*e)*\arctan(d*x + c)^2 - 2*(b^2*d^2*e*x + b^2*c^2e)*\arctan(d*x + c))/d$

**Fricas [A]** time = 1.72326, size = 335, normalized size = 3.53

$$\frac{a^2d^2ex^2 + 2(a^2c - ab)dex + b^2e \log(d^2x^2 + 2cdx + c^2 + 1) + (b^2d^2ex^2 + 2b^2cdex + (b^2c^2 + b^2)e) \arctan(dx + c)^2 + \dots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{2}*(a^2*d^2*e*x^2 + 2*(a^2*c - a*b)*d^2e*x + b^2e*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 + b^2)*e)*\arctan(d*x + c)^2 + 2*(a*b*d^2*e*x^2 + (2*a*b*c - b^2)*d^2e*x + (a*b*c^2 - b^2*c + a*b)*e)*\arctan(d*x + c))/d$

**Sympy [A]** time = 3.04498, size = 240, normalized size = 2.53

$$\left\{ \begin{array}{l} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e \operatorname{atan}(c+dx)}{d} + 2abcex \operatorname{atan}(c + dx) + abdex^2 \operatorname{atan}(c + dx) - abex + \frac{abe \operatorname{atan}(c+dx)}{d} + \frac{b^2c^2e \operatorname{atan}^2(c+dx)}{2d} \\ cex(a + b \operatorname{atan}(c))^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)*(a+b*atan(d*x+c))^2,x)`

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atan(c + d*x)/d + 2*a*
b*c*e*x*atan(c + d*x) + a*b*d*e*x**2*atan(c + d*x) - a*b*e*x + a*b*e*atan(c
+ d*x)/d + b**2*c**2*e*atan(c + d*x)**2/(2*d) + b**2*c*e*x*atan(c + d*x)**
2 - b**2*c*e*atan(c + d*x)/d + b**2*d*e*x**2*atan(c + d*x)**2/2 - b**2*e*x*
atan(c + d*x) + b**2*e*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*d) + b*
*2*e*atan(c + d*x)**2/(2*d), Ne(d, 0)), (c*e*x*(a + b*atan(c))**2, True))
```

**Giac [B]** time = 1.23824, size = 294, normalized size = 3.09

$$b^2 d^2 x^2 \arctan(dx + c)^2 e + 2 a b d^2 x^2 \arctan(dx + c) e + 2 b^2 c d x \arctan(dx + c)^2 e + a^2 d^2 x^2 e + 4 a b c d x \arctan(dx + c) e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*arctan(d*x + c)^2*e + 2*a*b*d^2*x^2*arctan(d*x + c)*e + 2*
b^2*c*d*x*arctan(d*x + c)^2*e + a^2*d^2*x^2*e + 4*a*b*c*d*x*arctan(d*x + c)
*e + b^2*c^2*arctan(d*x + c)^2*e + 2*a^2*c*d*x*e + 2*a*b*c^2*arctan(d*x + c)
)*e - 2*b^2*d*x*arctan(d*x + c)*e - 2*a*b*d*x*e - 2*b^2*c*arctan(d*x + c)*e
+ b^2*arctan(d*x + c)^2*e + 2*a*b*arctan(d*x + c)*e + b^2*e*log(d^2*x^2 +
2*c*d*x + c^2 + 1))/d
```

$$3.10 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{ce+dex} dx$$

**Optimal.** Leaf size=183

$$\frac{ibPolyLog\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de} + \frac{ibPolyLog\left(2, -1 + \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de} - \frac{b^2PolyLog[3, 1 - 2/(1 + I*(c + d*x))]}{(2*d*e)} + \frac{b^2PolyLog[3, -1 + 2/(1 + I*(c + d*x))]}{(2*d*e)}$$

[Out] (2\*(a + b\*ArcTan[c + d\*x])^2\*ArcTanh[1 - 2/(1 + I\*(c + d\*x))])/(d\*e) - (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/(d\*e) + (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, -1 + 2/(1 + I\*(c + d\*x))])/(d\*e) - (b^2\*PolyLog[3, 1 - 2/(1 + I\*(c + d\*x))])/(2\*d\*e) + (b^2\*PolyLog[3, -1 + 2/(1 + I\*(c + d\*x))])/(2\*d\*e)

**Rubi [A]** time = 0.338879, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {5043, 12, 4850, 4988, 4884, 4994, 6610}

$$\frac{ibPolyLog\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de} + \frac{ibPolyLog\left(2, -1 + \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de} - \frac{b^2PolyLog[3, 1 - 2/(1 + I*(c + d*x))]}{(2*d*e)} + \frac{b^2PolyLog[3, -1 + 2/(1 + I*(c + d*x))]}{(2*d*e)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x), x]

[Out] (2\*(a + b\*ArcTan[c + d\*x])^2\*ArcTanh[1 - 2/(1 + I\*(c + d\*x))])/(d\*e) - (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/(d\*e) + (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, -1 + 2/(1 + I\*(c + d\*x))])/(d\*e) - (b^2\*PolyLog[3, 1 - 2/(1 + I\*(c + d\*x))])/(2\*d\*e) + (b^2\*PolyLog[3, -1 + 2/(1 + I\*(c + d\*x))])/(2\*d\*e)

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4850

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcTan[c\*x])^p\*ArcTanh[1 - 2/(1 + I\*c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 + I\*c\*x)])/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

#### Rule 4988

Int[(ArcTanh[u\_]\*((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[1 + u]\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && Eq

$Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]$

#### Rule 4884

$\text{Int}[(a + \text{ArcTan}[c*x])*(b + (d + e*x^2))^p, x] \rightarrow \text{Simp}[(a + b*\text{ArcTan}[c*x])^{p+1}/(b*c*d*(p+1)), x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[e, c^2*d] \ \&\& \ \text{NeQ}[p, -1]$

#### Rule 4994

$\text{Int}[(\text{Log}[u]*(a + \text{ArcTan}[c*x])*(b + (d + e*x^2))^p), x] \rightarrow -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^p*\text{PolyLog}[2, 1 - u])/(2*c*d), x] + \text{Dist}[(b*p*I)/2, \text{Int}[(a + b*\text{ArcTan}[c*x])^{p-1}*\text{PolyLog}[2, 1 - u])/(d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[e, c^2*d] \ \&\& \ \text{EqQ}[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]$

#### Rule 6610

$\text{Int}[u*\text{PolyLog}[n, v], x] \rightarrow \text{With}\{w = \text{DerivativeDivides}[v, u*v, x]\}, \text{Simp}[w*\text{PolyLog}[n + 1, v], x] /; \text{!FalseQ}[w] /; \text{FreeQ}[n, x]$

#### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan^{-1}(c + dx))^2}{ce + dex} dx &= \frac{\text{Subst}\left(\int \frac{(a + b \tan^{-1}(x))^2}{ex} dx, x, c + dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(a + b \tan^{-1}(x))^2}{x} dx, x, c + dx\right)}{de} \\ &= \frac{2(a + b \tan^{-1}(c + dx))^2 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{(4b) \text{Subst}\left(\int \frac{(a + b \tan^{-1}(x)) \tanh^{-1}\left(1 - \frac{2}{1+ix}\right)}{1+x^2} dx\right)}{de} \\ &= \frac{2(a + b \tan^{-1}(c + dx))^2 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{(2b) \text{Subst}\left(\int \frac{(a + b \tan^{-1}(x)) \log\left(2 - \frac{2}{1+ix}\right)}{1+x^2} dx\right)}{de} \\ &= \frac{2(a + b \tan^{-1}(c + dx))^2 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib(a + b \tan^{-1}(c + dx)) \text{Li}_2\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} \\ &= \frac{2(a + b \tan^{-1}(c + dx))^2 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib(a + b \tan^{-1}(c + dx)) \text{Li}_2\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} \end{aligned}$$

**Mathematica [A]** time = 0.0626482, size = 170, normalized size = 0.93

$$\frac{2ib \text{PolyLog}\left(2, -\frac{c+dx+i}{c+dx-i}\right)(a + b \tan^{-1}(c + dx)) - 2ib \text{PolyLog}\left(2, \frac{c+dx+i}{c+dx-i}\right)(a + b \tan^{-1}(c + dx)) + b^2 \text{PolyLog}\left(3, -\frac{c+dx+i}{c+dx-i}\right)}{2de}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x), x]

[Out] (4\*(a + b\*ArcTan[c + d\*x])^2\*ArcTanh[(I + c + d\*x)/(-I + c + d\*x)] + (2\*I)\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, -((I + c + d\*x)/(-I + c + d\*x))] - (2\*I)\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, (I + c + d\*x)/(-I + c + d\*x)] + b^2



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(dx + c)^2 + 2ab \arctan(dx + c) + a^2}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e),x, algorithm="fricas")

[Out] integral((b^2\*arctan(d\*x + c)^2 + 2\*a\*b\*arctan(d\*x + c) + a^2)/(d\*e\*x + c\*e), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c+dx} dx + \int \frac{b^2 \operatorname{atan}^2(c+dx)}{c+dx} dx + \int \frac{2ab \operatorname{atan}(c+dx)}{c+dx} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))^2/(d\*e\*x+c\*e),x)

[Out] (Integral(a\*\*2/(c + d\*x), x) + Integral(b\*\*2\*atan(c + d\*x)\*\*2/(c + d\*x), x) + Integral(2\*a\*b\*atan(c + d\*x)/(c + d\*x), x))/e

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^2}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e),x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)^2/(d\*e\*x + c\*e), x)



$$3.11 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^2} dx$$

**Optimal.** Leaf size=119

$$-\frac{ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{(a+b \tan^{-1}(c+dx))^2}{de^2(c+dx)} - \frac{i(a+b \tan^{-1}(c+dx))^2}{de^2} + \frac{2b \log\left(2 - \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de^2}$$

[Out]  $((-I)*(a + b*\text{ArcTan}[c + d*x])^2)/(d*e^2) - (a + b*\text{ArcTan}[c + d*x])^2/(d*e^2*(c + d*x)) + (2*b*(a + b*\text{ArcTan}[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)$

**Rubi [A]** time = 0.186854, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {5043, 12, 4852, 4924, 4868, 2447}

$$-\frac{ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{(a+b \tan^{-1}(c+dx))^2}{de^2(c+dx)} - \frac{i(a+b \tan^{-1}(c+dx))^2}{de^2} + \frac{2b \log\left(2 - \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTan}[c + d*x])^2/(c*e + d*e*x)^2, x]$

[Out]  $((-I)*(a + b*\text{ArcTan}[c + d*x])^2)/(d*e^2) - (a + b*\text{ArcTan}[c + d*x])^2/(d*e^2*(c + d*x)) + (2*b*(a + b*\text{ArcTan}[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)$

#### Rule 5043

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p, x] := \text{Dist}[1/d, \text{Subst}[\text{Int}[(f*x)/d]^m*(a + b*\text{ArcTan}[x])^p, x], x, c + d*x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

$\text{Int}[(a + b*\text{ArcTan}[c + d*x])^p, x] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + e\*x)^p] /;

#### Rule 4852

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p, x] := \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcTan}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{m+1}*(a + b*\text{ArcTan}[c*x])^{p-1}/(1 + c^2*x^2), x], x] /;$  FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4924

$\text{Int}[(a + \text{ArcTan}[(c + d*x)]*(b + e*x))^p, x] := -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^{p+1})/(b*d*(p+1)), x] + \text{Dist}[I/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^p/(x*(I + c*x)), x], x] /;$  FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && GtQ[p, 0]

#### Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.))), x_
Symbol] := Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u)
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan^{-1}(c + dx))^2}{(ce + dex)^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{e^2 x^2} dx, x, c + dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^2} dx, x, c + dx\right)}{de^2} \\ &= -\frac{(a + b \tan^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{(2b) \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x(1+x^2)} dx, x, c + dx\right)}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^2}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{(2ib) \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x(i+x)} dx, x, c + dx\right)}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^2}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{2b(a + b \tan^{-1}(c + dx)) \log\left(2 - \frac{1}{1+i}\right)}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^2}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{2b(a + b \tan^{-1}(c + dx)) \log\left(2 - \frac{1}{1+i}\right)}{de^2} \end{aligned}$$

**Mathematica [A]** time = 0.190932, size = 135, normalized size = 1.13

$$\frac{-ib^2(c + dx)\text{PolyLog}\left(2, e^{2i \tan^{-1}(c+dx)}\right) + a\left(2b(c + dx) \log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right) - a\right) + 2b \tan^{-1}(c + dx)\left(-a + b(c + dx) \log\left(1 - \frac{1}{1+i}\right)\right)}{de^2(c + dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2, x]
```

```
[Out] ((-I)*b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(-a + b*(c
+ d*x)*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c
+ d*x)/Sqrt[1 + (c + d*x)^2]]) - I*b^2*(c + d*x)*PolyLog[2, E^((2*I)*ArcTan
[c + d*x])])/(d*e^2*(c + d*x))
```

**Maple [B]** time = 0.125, size = 471, normalized size = 4.

$$-\frac{a^2}{de^2(dx + c)} - \frac{b^2(\arctan(dx + c))^2}{de^2(dx + c)} - \frac{b^2 \arctan(dx + c) \ln(1 + (dx + c)^2)}{de^2} + 2 \frac{b^2 \ln(dx + c) \arctan(dx + c)}{de^2} - \frac{ib^2 \ln(1 - \frac{1}{1+i})}{de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x)`

[Out] 
$$\begin{aligned} & -1/d*a^2/e^2/(d*x+c) - 1/d*b^2/e^2/(d*x+c)*\arctan(d*x+c)^2 - 1/d*b^2/e^2*\arctan \\ & (d*x+c)*\ln(1+(d*x+c)^2) + 2/d*b^2/e^2*\ln(d*x+c)*\arctan(d*x+c) - I/d*b^2/e^2*\ln \\ & (d*x+c)*\ln(1-I*(d*x+c)) - I/d*b^2/e^2*\operatorname{dilog}(1-I*(d*x+c)) + 1/2*I/d*b^2/e^2*\ln(d* \\ & x+c-I)*\ln(-1/2*I*(d*x+c+I)) + 1/2*I/d*b^2/e^2*\operatorname{dilog}(-1/2*I*(d*x+c+I)) + 1/2*I/d \\ & *b^2/e^2*\ln(1+(d*x+c)^2)*\ln(d*x+c+I) - 1/2*I/d*b^2/e^2*\operatorname{dilog}(1/2*I*(d*x+c-I)) \\ & + 1/4*I/d*b^2/e^2*\ln(d*x+c-I)^2 - 1/2*I/d*b^2/e^2*\ln(1+(d*x+c)^2)*\ln(d*x+c-I) - \\ & 1/4*I/d*b^2/e^2*\ln(d*x+c+I)^2 + I/d*b^2/e^2*\ln(d*x+c)*\ln(1+I*(d*x+c)) - 1/2*I/d \\ & *b^2/e^2*\ln(d*x+c+I)*\ln(1/2*I*(d*x+c-I)) + I/d*b^2/e^2*\operatorname{dilog}(1+I*(d*x+c)) - 2/d \\ & *a*b/e^2/(d*x+c)*\arctan(d*x+c) - 1/d*a*b/e^2*\ln(1+(d*x+c)^2) + 2/d*a*b/e^2*\ln(d \\ & *x+c) \end{aligned}$$

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \arctan(dx+c)^2 + 2ab \arctan(dx+c) + a^2}{d^2 e^2 x^2 + 2cde^2 x + c^2 e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")`

[Out] `integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2} dx + \int \frac{b^2 \operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2} dx + \int \frac{2ab \operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2} dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**2,x)`

[Out] `(Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atan(c + d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atan(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2), x))/e**2`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^2}{(dex + ce)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

$$3.12 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^3} dx$$

**Optimal.** Leaf size=117

$$\frac{b(a+b \tan^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b \tan^{-1}(c+dx))^2}{2de^3(c+dx)^2} - \frac{(a+b \tan^{-1}(c+dx))^2}{2de^3} + \frac{b^2 \log(c+dx)}{de^3} - \frac{b^2 \log((c+dx)^2+1)}{2de^3}$$

[Out]  $-\frac{(b*(a + b*ArcTan[c + d*x]))}{(d*e^3*(c + d*x))} - \frac{(a + b*ArcTan[c + d*x])^2}{(2*d*e^3)} - \frac{(a + b*ArcTan[c + d*x])^2}{(2*d*e^3*(c + d*x)^2)} + \frac{(b^2*Log[c + d*x])}{(d*e^3)} - \frac{(b^2*Log[1 + (c + d*x)^2])}{(2*d*e^3)}$

**Rubi [A]** time = 0.152844, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {5043, 12, 4852, 4918, 266, 36, 29, 31, 4884}

$$\frac{b(a+b \tan^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b \tan^{-1}(c+dx))^2}{2de^3(c+dx)^2} - \frac{(a+b \tan^{-1}(c+dx))^2}{2de^3} + \frac{b^2 \log(c+dx)}{de^3} - \frac{b^2 \log((c+dx)^2+1)}{2de^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x)^3,x]

[Out]  $-\frac{(b*(a + b*ArcTan[c + d*x]))}{(d*e^3*(c + d*x))} - \frac{(a + b*ArcTan[c + d*x])^2}{(2*d*e^3)} - \frac{(a + b*ArcTan[c + d*x])^2}{(2*d*e^3*(c + d*x)^2)} + \frac{(b^2*Log[c + d*x])}{(d*e^3)} - \frac{(b^2*Log[1 + (c + d*x)^2])}{(2*d*e^3)}$

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4918

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_))^(m\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTan[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbo
l] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^2}{(ce + dex)^3} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{e^3 x^3} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^3} dx, x, c + dx\right)}{de^3} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^2(1+x^2)} dx, x, c + dx\right)}{de^3} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^2} dx, x, c + dx\right)}{de^3} - \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{1+x^2} dx, x, c + dx\right)}{de^3} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c + dx\right)}{de^3} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c + dx\right)}{de^3} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c + dx\right)}{de^3} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b^2 \log(c + dx)}{de^3}
\end{aligned}$$

**Mathematica [A]** time = 0.108117, size = 194, normalized size = 1.66

$$\frac{a^2 + 2b \tan^{-1}(c + dx) (a(c^2 + 2cdx + d^2x^2 + 1) + b(c + dx)) + 2abc + 2abdx + b^2c^2 \log(c^2 + 2cdx + d^2x^2 + 1) + b^2d^2x^2}{de^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x)^3,x]

[Out]  $-(a^2 + 2*a*b*c + 2*a*b*d*x + 2*b*(b*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x] + b^2*c^2*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 2*b^2*c*d*x*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + b^2*d^2*x^2*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*d*e^3*(c + d*x)^2)$

**Maple [A]** time = 0.053, size = 182, normalized size = 1.6

$$\frac{a^2}{2de^3(dx+c)^2} - \frac{b^2(\arctan(dx+c))^2}{2de^3(dx+c)^2} - \frac{b^2(\arctan(dx+c))^2}{2de^3} - \frac{b^2\arctan(dx+c)}{de^3(dx+c)} - \frac{b^2\ln(1+(dx+c)^2)}{2de^3} + \frac{b^2\ln(a^2 + b^2(dx+c)^2)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^3,x)

[Out]  $-1/2/d*a^2/e^3/(d*x+c)^2 - 1/2/d*b^2/e^3/(d*x+c)^2*arctan(d*x+c)^2 - 1/2/d*b^2/e^3*arctan(d*x+c)^2 - 1/d*b^2/e^3*arctan(d*x+c)/(d*x+c) - 1/2*b^2*ln(1+(d*x+c)^2)/d/e^3 + b^2*ln(d*x+c)/d/e^3 - 1/d*a*b/e^3/(d*x+c)^2*arctan(d*x+c) - 1/d*a*b/e^3*arctan(d*x+c) - 1/d*a*b/e^3/(d*x+c)$

**Maxima [B]** time = 1.62124, size = 362, normalized size = 3.09

$$-\left(d\left(\frac{1}{d^3e^3x + cd^2e^3} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right) + \frac{\arctan(dx+c)}{d^3e^3x^2 + 2cd^2e^3x + c^2de^3}\right)ab - \frac{1}{2}\left(2d\left(\frac{1}{d^3e^3x + cd^2e^3} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^3,x, algorithm="maxima")

[Out]  $-(d*(1/(d^3*e^3*x + c*d^2*e^3) + \arctan((d^2*x + c*d)/d)/(d^2*e^3)) + \arctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a*b - 1/2*(2*d*(1/(d^3*e^3*x + c*d^2*e^3) + \arctan((d^2*x + c*d)/d)/(d^2*e^3))*\arctan(d*x + c) - (\arctan(d*x + c)^2 - \log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*\log(d*x + c))/(d*e^3))*b^2 - 1/2*b^2*\arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)$

**Fricas [A]** time = 1.87855, size = 470, normalized size = 4.02

$$\frac{2abdx + 2abc + (b^2d^2x^2 + 2b^2cdx + b^2c^2 + b^2)\arctan(dx+c)^2 + a^2 + 2(abd^2x^2 + abc^2 + b^2c + (2abc + b^2)dx + a^2)}{2(d^3e^3x^2 + 2cd^2e^3x + c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^3,x, algorithm="fricas")

```
[Out] -1/2*(2*a*b*d*x + 2*a*b*c + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*arc
tan(d*x + c)^2 + a^2 + 2*(a*b*d^2*x^2 + a*b*c^2 + b^2*c + (2*a*b*c + b^2)*d
*x + a*b)*arctan(d*x + c) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x +
c))/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

**Sympy [A]** time = 6.51563, size = 986, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a
*b*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 4*a*b*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*
x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*d**2*x**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x*
*2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b*
*2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) -
b**2*c**2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(2*c**2*d*e**3 + 4*c*d
**2*e**3*x + 2*d**3*e**3*x**2) - b**2*c**2*atan(c + d*x)**2/(2*c**2*d*e**3
+ 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d
*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*d*x*log(c**2/d**2 +
2*c*x/d + x**2 + d**(-2))/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x*
*2) - 2*b**2*c*d*x*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d*
**3*e**3*x**2) - 2*b**2*c*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2
*d**3*e**3*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*
e**3*x + 2*d**3*e**3*x**2) - b**2*d**2*x**2*log(c**2/d**2 + 2*c*x/d + x**2
+ d**(-2))/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*d**2
*x**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 2*b**2*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*
x**2) - b**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**
3*x**2), Ne(d, 0)), (x*(a + b*atan(c))**2/(c**3*e**3), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^2}{(dex + ce)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```



$$3.13 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^4} dx$$

**Optimal.** Leaf size=194

$$\frac{ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)}{3de^4} - \frac{b(a+b \tan^{-1}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b \tan^{-1}(c+dx))^2}{3de^4(c+dx)^3} + \frac{i(a+b \tan^{-1}(c+dx))^2}{3de^4} - \frac{2b \log}{3de^4}$$

[Out]  $-b^2/(3*d*e^4*(c+d*x)) - (b^2*ArcTan[c+d*x])/(3*d*e^4) - (b*(a+b*ArcTan[c+d*x]))/(3*d*e^4*(c+d*x)^2) + ((I/3)*(a+b*ArcTan[c+d*x])^2)/(d*e^4) - (a+b*ArcTan[c+d*x])^2/(3*d*e^4*(c+d*x)^3) - (2*b*(a+b*ArcTan[c+d*x])*Log[2-2/(1-I*(c+d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2,-1+2/(1-I*(c+d*x))])/(d*e^4)$

**Rubi [A]** time = 0.257446, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {5043, 12, 4852, 4918, 325, 203, 4924, 4868, 2447}

$$\frac{ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)}{3de^4} - \frac{b(a+b \tan^{-1}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b \tan^{-1}(c+dx))^2}{3de^4(c+dx)^3} + \frac{i(a+b \tan^{-1}(c+dx))^2}{3de^4} - \frac{2b \log}{3de^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x)^4, x]

[Out]  $-b^2/(3*d*e^4*(c+d*x)) - (b^2*ArcTan[c+d*x])/(3*d*e^4) - (b*(a+b*ArcTan[c+d*x]))/(3*d*e^4*(c+d*x)^2) + ((I/3)*(a+b*ArcTan[c+d*x])^2)/(d*e^4) - (a+b*ArcTan[c+d*x])^2/(3*d*e^4*(c+d*x)^3) - (2*b*(a+b*ArcTan[c+d*x])*Log[2-2/(1-I*(c+d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2,-1+2/(1-I*(c+d*x))])/(d*e^4)$

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcTan[c\*x])^(p-1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4918

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_.))^ (m\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTan[c\*x])^p, x],

$x] - \text{Dist}[e/(d*f^2), \text{Int}[(f*x)^{(m+2)}*(a + b*\text{ArcTan}[c*x])^p/(d + e*x^2), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 325

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_) + (b_*)*(x_)^{(n_*)})^{(p_)}, x\_Symbol] := \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), \text{Int}[(c*x)^{(m+n)}*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 203

$\text{Int}[(a_) + (b_*)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 4924

$\text{Int}[(a_) + \text{ArcTan}[(c_*)*(x_)]*(b_*)^{(p_)}/((x_)*((d_) + (e_*)*(x_)^2)), x\_Symbol] := -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^{(p+1)})/(b*d*(p+1)), x] + \text{Dist}[I/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^p/(x*(I + c*x)), x], x] /;$  FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && GtQ[p, 0]

### Rule 4868

$\text{Int}[(a_) + \text{ArcTan}[(c_*)*(x_)]*(b_*)^{(p_)}/((x_)*((d_) + (e_*)*(x_))), x\_Symbol] := \text{Simp}[(a + b*\text{ArcTan}[c*x])^p*\text{Log}[2 - 2/(1 + (e*x)/d)]/d, x] - \text{Dist}[(b*c*p)/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^{(p-1)}*\text{Log}[2 - 2/(1 + (e*x)/d)]/(1 + c^2*x^2), x], x] /;$  FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

### Rule 2447

$\text{Int}[\text{Log}[u_]*(Pq_)^{(m_)}, x\_Symbol] := \text{With}[\{C = \text{FullSimplify}[(Pq^m*(1-u))/D[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1-u], x] /;$  FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^2}{(ce + dex)^4} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{e^4 x^4} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^4} dx, x, c + dx\right)}{de^4} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{3de^4(c + dx)^3} + \frac{(2b) \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^3(1+x^2)} dx, x, c + dx\right)}{3de^4} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{3de^4(c + dx)^3} + \frac{(2b) \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^3} dx, x, c + dx\right)}{3de^4} - \frac{(2b) \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{1+x^2} dx, x, c + dx\right)}{3de^4} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{3de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^2}{3de^4} - \frac{(a + b \tan^{-1}(c + dx))^2}{3de^4(c + dx)^3} - \frac{(2ib) \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{1+x^2} dx, x, c + dx\right)}{3de^4} \\
&= -\frac{b^2}{3de^4(c + dx)} - \frac{b(a + b \tan^{-1}(c + dx))}{3de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^2}{3de^4} - \frac{(a + b \tan^{-1}(c + dx))^2}{3de^4(c + dx)^3} \\
&= -\frac{b^2}{3de^4(c + dx)} - \frac{b^2 \tan^{-1}(c + dx)}{3de^4} - \frac{b(a + b \tan^{-1}(c + dx))}{3de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^2}{3de^4}
\end{aligned}$$

**Mathematica [A]** time = 0.606005, size = 163, normalized size = 0.84

$$\frac{-ib^2 \text{PolyLog}\left(2, e^{2i \tan^{-1}(c+dx)}\right) + \frac{a^2}{(c+dx)^3} + \frac{ab}{(c+dx)^2} + 2ab \log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right) + b \tan^{-1}(c + dx) \left(\frac{2a}{(c+dx)^3} + \frac{b}{(c+dx)^2} + 2b \log\left(\frac{c+dx}{\sqrt{(c+dx)^2+1}}\right)\right)}{3de^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x)^4, x]

[Out]  $-(a*b + a^2/(c + d*x)^3 + (a*b)/(c + d*x)^2 + b^2/(c + d*x) + b^2*(-I + (c + d*x)^{-3})*\text{ArcTan}[c + d*x]^2 + b*\text{ArcTan}[c + d*x]*(b + (2*a)/(c + d*x)^3 + b/(c + d*x)^2 + 2*b*\text{Log}[1 - E^((2*I)*\text{ArcTan}[c + d*x])]) + 2*a*b*\text{Log}[(c + d*x)/\text{Sqrt}[1 + (c + d*x)^2]] - I*b^2*\text{PolyLog}[2, E^((2*I)*\text{ArcTan}[c + d*x])])/(3*d*e^4)$

**Maple [B]** time = 0.125, size = 547, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^4, x)

[Out]  $-1/3/d*a^2/e^4/(d*x+c)^3 - 1/3/d*b^2/e^4/(d*x+c)^3*\text{arctan}(d*x+c)^2 + 1/3/d*b^2/e^4*\text{arctan}(d*x+c)*\ln(1+(d*x+c)^2) - 1/3/d*b^2/e^4*\text{arctan}(d*x+c)/(d*x+c)^2 - 2/3/d*b^2/e^4*\ln(d*x+c)*\text{arctan}(d*x+c) - 1/6*I/d*b^2/e^4*\ln(d*x+c+I)*\ln(1+(d*x+c)^2) - 1/6*I/d*b^2/e^4*\ln(d*x+c-I)*\ln(-1/2*I*(d*x+c+I)) - 1/3*I/d*b^2/e^4*dilog(1+I*(d*x+c)) + 1/3*I/d*b^2/e^4*dilog(1-I*(d*x+c)) + 1/3*I/d*b^2/e^4*\ln(d*x+c)*\ln(1-I*(d*x+c)) + 1/6*I/d*b^2/e^4*dilog(1/2*I*(d*x+c-I)) + 1/12*I/d*b^2/e^4*\ln(d$

$*x+c+I)^2-1/12*I/d*b^2/e^4*\ln(d*x+c-I)^2-1/3*b^2*\arctan(d*x+c)/d/e^4-1/3*b^2/d/e^4/(d*x+c)+1/6*I/d*b^2/e^4*\ln(d*x+c+I)*\ln(1/2*I*(d*x+c-I))-1/3*I/d*b^2/e^4*\ln(d*x+c)*\ln(1+I*(d*x+c))-1/6*I/d*b^2/e^4*\operatorname{dilog}(-1/2*I*(d*x+c+I))+1/6*I/d*b^2/e^4*\ln(d*x+c-I)*\ln(1+(d*x+c)^2)-2/3/d*a*b/e^4/(d*x+c)^3*\arctan(d*x+c)+1/3/d*a*b/e^4*\ln(1+(d*x+c)^2)-1/3/d*a*b/e^4/(d*x+c)^2-2/3/d*a*b/e^4*\ln(d*x+c)$

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^4,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \arctan(dx+c)^2 + 2ab \arctan(dx+c) + a^2}{d^4 e^4 x^4 + 4cd^3 e^4 x^3 + 6c^2 d^2 e^4 x^2 + 4c^3 d e^4 x + c^4 e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^4,x, algorithm="fricas")

[Out] integral((b^2\*arctan(d\*x + c)^2 + 2\*a\*b\*arctan(d\*x + c) + a^2)/(d^4\*e^4\*x^4 + 4\*c\*d^3\*e^4\*x^3 + 6\*c^2\*d^2\*e^4\*x^2 + 4\*c^3\*d\*e^4\*x + c^4\*e^4), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{b^2 \operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{2ab \operatorname{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))^2/(d\*e\*x+c\*e)\*\*4,x)

[Out] (Integral(a\*\*2/(c\*\*4 + 4\*c\*\*3\*d\*x + 6\*c\*\*2\*d\*\*2\*x\*\*2 + 4\*c\*d\*\*3\*x\*\*3 + d\*\*4\*x\*\*4), x) + Integral(b\*\*2\*atan(c + d\*x)\*\*2/(c\*\*4 + 4\*c\*\*3\*d\*x + 6\*c\*\*2\*d\*\*2\*x\*\*2 + 4\*c\*d\*\*3\*x\*\*3 + d\*\*4\*x\*\*4), x) + Integral(2\*a\*b\*atan(c + d\*x)/(c\*\*4 + 4\*c\*\*3\*d\*x + 6\*c\*\*2\*d\*\*2\*x\*\*2 + 4\*c\*d\*\*3\*x\*\*3 + d\*\*4\*x\*\*4), x))/e\*\*4

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx+c) + a)^2}{(dex+ce)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

$$3.14 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{(ce+dex)^5} dx$$

**Optimal.** Leaf size=170

$$\frac{b(a+b \tan^{-1}(c+dx))}{2de^5(c+dx)} - \frac{b(a+b \tan^{-1}(c+dx))}{6de^5(c+dx)^3} - \frac{(a+b \tan^{-1}(c+dx))^2}{4de^5(c+dx)^4} + \frac{(a+b \tan^{-1}(c+dx))^2}{4de^5} - \frac{b^2}{12de^5(c+dx)^2}$$

[Out]  $-b^2/(12*d*e^5*(c+d*x)^2) - (b*(a+b*ArcTan[c+d*x]))/(6*d*e^5*(c+d*x)^3) + (b*(a+b*ArcTan[c+d*x]))/(2*d*e^5*(c+d*x)) + (a+b*ArcTan[c+d*x])^2/(4*d*e^5) - (a+b*ArcTan[c+d*x])^2/(4*d*e^5*(c+d*x)^4) - (2*b^2*Log[c+d*x])/(3*d*e^5) + (b^2*Log[1+(c+d*x)^2])/(3*d*e^5)$

**Rubi [A]** time = 0.226746, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {5043, 12, 4852, 4918, 266, 44, 36, 29, 31, 4884}

$$\frac{b(a+b \tan^{-1}(c+dx))}{2de^5(c+dx)} - \frac{b(a+b \tan^{-1}(c+dx))}{6de^5(c+dx)^3} - \frac{(a+b \tan^{-1}(c+dx))^2}{4de^5(c+dx)^4} + \frac{(a+b \tan^{-1}(c+dx))^2}{4de^5} - \frac{b^2}{12de^5(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x)^5,x]

[Out]  $-b^2/(12*d*e^5*(c+d*x)^2) - (b*(a+b*ArcTan[c+d*x]))/(6*d*e^5*(c+d*x)^3) + (b*(a+b*ArcTan[c+d*x]))/(2*d*e^5*(c+d*x)) + (a+b*ArcTan[c+d*x])^2/(4*d*e^5) - (a+b*ArcTan[c+d*x])^2/(4*d*e^5*(c+d*x)^4) - (2*b^2*Log[c+d*x])/(3*d*e^5) + (b^2*Log[1+(c+d*x)^2])/(3*d*e^5)$

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcTan[c\*x])^(p-1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4918

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_.))^(m\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTan[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m+2)\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^2}{(ce + dex)^5} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{e^5 x^5} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^5} dx, x, c + dx\right)}{de^5} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{4de^5(c + dx)^4} + \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^4(1+x^2)} dx, x, c + dx\right)}{2de^5} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{4de^5(c + dx)^4} + \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^4} dx, x, c + dx\right)}{2de^5} - \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^2(1+x^2)} dx, x, c + dx\right)}{2de^5} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{6de^5(c + dx)^3} - \frac{(a + b \tan^{-1}(c + dx))^2}{4de^5(c + dx)^4} - \frac{b \text{Subst}\left(\int \frac{a+b \tan^{-1}(x)}{x^2} dx, x, c + dx\right)}{2de^5} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{6de^5(c + dx)^3} + \frac{b(a + b \tan^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tan^{-1}(c + dx))^2}{4de^5} - \frac{(a + b \tan^{-1}(c + dx))}{4de^5} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))}{6de^5(c + dx)^3} + \frac{b(a + b \tan^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tan^{-1}(c + dx))^2}{4de^5} - \frac{(a + b \tan^{-1}(c + dx))}{4de^5} \\
&= -\frac{b^2}{12de^5(c + dx)^2} - \frac{b(a + b \tan^{-1}(c + dx))}{6de^5(c + dx)^3} + \frac{b(a + b \tan^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tan^{-1}(c + dx))}{4de^5} \\
&= -\frac{b^2}{12de^5(c + dx)^2} - \frac{b(a + b \tan^{-1}(c + dx))}{6de^5(c + dx)^3} + \frac{b(a + b \tan^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tan^{-1}(c + dx))}{4de^5}
\end{aligned}$$

**Mathematica [A]** time = 0.270523, size = 245, normalized size = 1.44

$$\frac{3a^2 - 2b \tan^{-1}(c + dx) (3a (6c^2 d^2 x^2 + 4c^3 dx + c^4 + 4cd^3 x^3 + d^4 x^4 - 1) + b (9c^2 dx + 3c^3 + 9cd^2 x^2 - c + 3d^3 x^3 - dx))}{(ce + dex)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^2/(c\*e + d\*e\*x)^5,x]

[Out]  $-\frac{(3a^2 + 2ab(c + dx) + b^2(c + dx)^2 - 6ab(c + dx)^3 - 2b(b(-c + 3c^3 - dx + 9c^2 dx + 9cd^2 x^2 + 3d^3 x^3) + 3a(-1 + c^4 + 4c^3 dx + 6c^2 d^2 x^2 + 4cd^3 x^3 + d^4 x^4)) \text{ArcTan}[c + dx] - 3b^2(-1 + c^4 + 4c^3 dx + 6c^2 d^2 x^2 + 4cd^3 x^3 + d^4 x^4) \text{ArcTan}[c + dx]^2 + 8b^2(c + dx)^4 \text{Log}[c + dx] - 4b^2(c + dx)^4 \text{Log}[1 + c^2 + 2cdx + d^2 x^2])}{(12de^5(c + dx)^4)}$

**Maple [A]** time = 0.054, size = 242, normalized size = 1.4

$$-\frac{a^2}{4de^5(dx+c)^4} - \frac{b^2(\arctan(dx+c))^2}{4de^5(dx+c)^4} + \frac{b^2(\arctan(dx+c))^2}{4de^5} - \frac{b^2 \arctan(dx+c)}{6de^5(dx+c)^3} + \frac{b^2 \arctan(dx+c)}{2de^5(dx+c)} + \frac{b^2 \ln(1 + \dots)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^5,x)



[Out]  $-1/4/d*a^2/e^5/(d*x+c)^4-1/4/d*b^2/e^5/(d*x+c)^4*\arctan(d*x+c)^2+1/4/d*b^2/e^5*\arctan(d*x+c)^2-1/6/d*b^2/e^5*\arctan(d*x+c)/(d*x+c)^3+1/2/d*b^2/e^5*\arctan(d*x+c)/(d*x+c)+1/3*b^2*\ln(1+(d*x+c)^2)/d/e^5-1/12*b^2/d/e^5/(d*x+c)^2-2/3*b^2*\ln(d*x+c)/d/e^5-1/2/d*a*b/e^5/(d*x+c)^4*\arctan(d*x+c)+1/2/d*a*b/e^5*\arctan(d*x+c)-1/6/d*a*b/e^5/(d*x+c)^3+1/2/d*a*b/e^5/(d*x+c)$

**Maxima [B]** time = 1.8787, size = 721, normalized size = 4.24

$$\frac{1}{6} \left( d \left( \frac{3d^2x^2 + 6cdx + 3c^2 - 1}{d^5e^5x^3 + 3cd^4e^5x^2 + 3c^2d^3e^5x + c^3d^2e^5} + \frac{3 \arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5} \right) - \frac{3 \arctan(dx+c)}{d^5e^5x^4 + 4cd^4e^5x^3 + 6c^2d^3e^5x^2 + 4c^3d^2e^5x + c^4d^2e^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^5,x, algorithm="maxima")

[Out]  $1/6*(d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*\arctan((d^2*x + c*d)/d)/(d^2*e^5)) - 3*\arctan(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d^2*e^5))*a*b + 1/12*(2*d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*\arctan((d^2*x + c*d)/d)/(d^2*e^5))*\arctan(d*x + c) - (3*(d^2*x^2 + 2*c*d*x + c^2)*\arctan(d*x + c)^2 - 4*(d^2*x^2 + 2*c*d*x + c^2)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 8*(d^2*x^2 + 2*c*d*x + c^2)*\log(d*x + c) + 1)*d^2/(d^5*e^5*x^2 + 2*c*d^4*e^5*x + c^2*d^3*e^5))*b^2 - 1/4*b^2*\arctan(d*x + c)^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d^2*e^5) - 1/4*a^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d^2*e^5)$

**Fricas [B]** time = 2.09413, size = 937, normalized size = 5.51

$$6abd^3x^3 + 6abc^3 + (18abc - b^2)d^2x^2 - b^2c^2 - 2abc + 2(9abc^2 - b^2c - ab)dx + 3(b^2d^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(d\*e\*x+c\*e)^5,x, algorithm="fricas")

[Out]  $1/12*(6*a*b*d^3*x^3 + 6*a*b*c^3 + (18*a*b*c - b^2)*d^2*x^2 - b^2*c^2 - 2*a*b*c + 2*(9*a*b*c^2 - b^2*c - a*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*\arctan(d*x + c)^2 - 3*a^2 + 2*(3*a*b*d^4*x^4 + 3*(4*a*b*c + b^2)*d^3*x^3 + 3*a*b*c^4 + 3*b^2*c^3 + 9*(2*a*b*c^2 + b^2*c)*d^2*x^2 - b^2*c + (12*a*b*c^3 + 9*b^2*c^2 - b^2)*d*x - 3*a*b)*\arctan(d*x + c) + 4*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 8*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\log(d*x + c))/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d^2*e^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**5,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.49382, size = 1902, normalized size = 11.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] 1/24*(6*pi^2*b^2*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)*sgn(1/(d*x*e + c*e))*sgn(d)^2 + 3*pi^2*b^2*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*sgn(1/(d*x*e + c*e))*sgn(d)^2 + 6*pi^2*b^2*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)^2 + 12*pi*a*b*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 6*pi*b^2*e*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e) + 6*pi^2*b^2*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi) - 12*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi) - 6*pi^2*b^2*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 3*pi^2*b^2 - 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1)) + 6*b^2*arctan((d*x*e + c*e)*e^(-1))^2 - 12*pi*b^2*e*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e) - 3*pi^2*b^2*e^4*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*e^4*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 2*pi*b^2*e^3*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^3 - 12*a*b*arctan(e/(d*x*e + c*e)) - 6*pi*b^2*e/(d*x*e + c*e) + 12*b^2*arctan((d*x*e + c*e)*e^(-1))*e/(d*x*e + c*e) - 6*pi^2*b^2*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)^2/(d*x*e + c*e)^4 + 8*b^2*log(e^2/(d*x*e + c*e)^2 + 1) + 6*pi*a*b*e^4*sgn(1/(d*x*e + c*e))*sgn(d)^2/(d*x*e + c*e)^4 + 12*a*b*e/(d*x*e + c*e) - 6*pi^2*b^2*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^4 + 12*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^4 + 4*pi*b^2*e^3*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^3 - 3*pi^2*b^2*e^4/(d*x*e + c*e)^4 + 6*pi*b^2*arctan((d*x*e + c*e)*e^(-1))*e^4/(d*x*e + c*e)^4 - 6*b^2*arctan((d*x*e + c*e)*e^(-1))^2*e^4/(d*x*e + c*e)^4 + 2*pi*b^2*e^3/(d*x*e + c*e)^3 - 4*b^2*arctan((d*x*e + c*e)*e^(-1))*e^3/(d*x*e + c*e)^3 - 2*b^2*e^2/(d*x*e + c*e)^2 + 12*pi*a*b*e^4*floor(-1/2*(pi*sgn(1/(d*x*e + c*e))*sgn(d)^2 - 2*arctan((d*x*e + c*e)*e^(-1)))/pi)/(d*x*e + c*e)^4 + 6*pi*a*b*e^4/(d*x*e + c*e)^4 - 12*a*b*arctan((d*x*e + c*e)*e^(-1))*e^4/(d*x*e + c*e)^4 - 4*a*b*e^3/(d*x*e + c*e)^3 - 6*a^2*e^4/(d*x*e + c*e)^4)*e^(-5)/d
```

### 3.15 $\int (ce + dex)^2 (a + b \tan^{-1}(c + dx))^3 dx$

**Optimal.** Leaf size=271

$$\frac{ib^2e^2\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{d} - \frac{b^3e^2\text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)}{2d} + ab^2e^2x - \frac{be^2(a + b \tan^{-1}(c + dx))^2}{2d}$$

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/d
```

**Rubi [A]** time = 0.438706, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {5043, 12, 4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$\frac{ib^2e^2\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{d} - \frac{b^3e^2\text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)}{2d} + ab^2e^2x - \frac{be^2(a + b \tan^{-1}(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3, x]
```

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/d
```

#### Rule 5043

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_)])*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 4916

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_)]/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

#### Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

#### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

#### Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

#### Rule 4920

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

#### Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

#### Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

#### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

#### Rubi steps

$$\begin{aligned}
\int (ce + dex)^2 (a + b \tan^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b \tan^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{e^2 \text{Subst}\left(\int x^2 (a + b \tan^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))^3}{3d} - \frac{(be^2) \text{Subst}\left(\int \frac{x^3 (a + b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tan^{-1}(c + dx))^3}{3d} - \frac{(be^2) \text{Subst}\left(\int x (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= -\frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{ie^2 (a + b \tan^{-1}(c + dx))^3}{3d} + \frac{e^2 (c + dx)^3}{3d} \\
&= -\frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{ie^2 (a + b \tan^{-1}(c + dx))^3}{3d} + \frac{e^2 (c + dx)^3}{3d} \\
&= ab^2 e^2 x - \frac{be^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{be^2 (c + dx)^2 (a + b \tan^{-1}(c + dx))^2}{2d} \\
&= ab^2 e^2 x + \frac{b^3 e^2 (c + dx) \tan^{-1}(c + dx)}{d} - \frac{be^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{be^2 (c + dx)^2}{2d} \\
&= ab^2 e^2 x + \frac{b^3 e^2 (c + dx) \tan^{-1}(c + dx)}{d} - \frac{be^2 (a + b \tan^{-1}(c + dx))^2}{2d} - \frac{be^2 (c + dx)^2}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.529795, size = 349, normalized size = 1.29

$$\frac{e^2 \left( 6ab^2 \left( i \text{PolyLog} \left( 2, -e^{2i \tan^{-1}(c+dx)} \right) \right) + (c + dx)^3 \tan^{-1}(c + dx)^2 - (c + dx)^2 \tan^{-1}(c + dx) + i \tan^{-1}(c + dx)^2 - \tan^{-1}(c + dx) \right)}{6d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*e + d\*e\*x)^2\*(a + b\*ArcTan[c + d\*x])^3,x]

[Out] (e^2\*(-3\*a^2\*b\*(c + d\*x)^2 + 2\*a^3\*(c + d\*x)^3 + 6\*a^2\*b\*(c + d\*x)^3\*ArcTan[c + d\*x] + 3\*a^2\*b\*Log[1 + (c + d\*x)^2] + 6\*a\*b^2\*(c + d\*x - ArcTan[c + d\*x]) - (c + d\*x)^2\*ArcTan[c + d\*x] + I\*ArcTan[c + d\*x]^2 + (c + d\*x)^3\*ArcTan[c + d\*x]^2 - 2\*ArcTan[c + d\*x]\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])] + I\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) + b^3\*(6\*(c + d\*x)\*ArcTan[c + d\*x] - 3\*(1 + (c + d\*x)^2)\*ArcTan[c + d\*x]^2 + (2\*I)\*ArcTan[c + d\*x]^3 - 2\*(c + d\*x)\*ArcTan[c + d\*x]^3 + 2\*(c + d\*x)\*(1 + (c + d\*x)^2)\*ArcTan[c + d\*x]^3 - 6\*ArcTan[c + d\*x]^2\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])] + 6\*Log[1/Sqrt[1 + (c + d\*x)^2]] + (6\*I)\*ArcTan[c + d\*x]\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])] - 3\*PolyLog[3, -E^((2\*I)\*ArcTan[c + d\*x])]))/(6\*d)

**Maple [C]** time = 0.947, size = 3242, normalized size = 12.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x+c*e)^2*(a+b*\arctan(d*x+c))^3,x)$

[Out] 
$$\begin{aligned} & -1/4*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*c\text{sgn}(I*(1+I \\ & *(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*x*b^3*e^ \\ & 2-1/8/d*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3*b^ \\ & 3*c*e^2+1/8/d*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I \\ & *(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3*b^3*c*e^2+3*d*\arctan(d*x+c)*x^2*a^2*b*c \\ & *e^2+3*d*\arctan(d*x+c)^2*x^2*a*b^2*c*e^2-1/8*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi} \\ & *c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3+1/4*I/d*e^2*b^3*\arctan(d*x+c \\ & )^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3+1/4*I/d*e^2*b^3*\arctan(d*x+c \\ & )^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+ \\ & 1)^2)^3-1/8*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x+c \\ & )^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3-1/8/d*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn} \\ & (I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^ \\ & 2)+1))^2*b^3*c*e^2-1/4/d*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+ \\ & c)^2)+I)*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x \\ & +c)^2)+I)^2*b^3*c*e^2+1/4/d*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+ \\ & d*x+c)^2)+1)^2)^2*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*b^3*c*e^2+1/4*I \\ & /d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*c\text{sgn}(I* \\ & (1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}+1/8/d*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*( \\ & d*x+c))^2/(1+(d*x+c)^2)+I)^2*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+ \\ & I*(d*x+c))^2/(1+(d*x+c)^2)+I)*b^3*c*e^2-1/4*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}* \\ & c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/ \\ & ((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-1/4*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}*c \\ & \text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2* \\ & c\text{sgn}(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)-1/2*I/d*e^2*b^3*\arctan(d*x+c)^2 \\ & *\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2*c\text{sgn}(I*(1+I*(d*x+c))/(1+(d*x+c) \\ & ^2)^{(1/2)}+1/4*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d \\ & *x+c)^2)+1))*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-1/8*I/d*e^2*b^3* \\ & \arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*c\text{sgn}(I*((1+I \\ & *(d*x+c))^2/(1+(d*x+c)^2)+1)^2)-1/8*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*( \\ & 1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2* \\ & I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)+1/4*I/d*e^2*b^3*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}( \\ & I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2 \\ & *I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2+d*\arctan(d*x+c)^3*x^2*b^3*c*e^2+\arctan \\ & (d*x+c)^3*x*b^3*c^2*e^2+1/3/d*\arctan(d*x+c)^3*b^3*c^3*e^2-1/2*d*\arctan(d*x \\ & +c)^2*x^2*b^3*e^2+1/3*d^2*\arctan(d*x+c)^3*x^3*b^3*e^2-1/d*e^2*a*b^2*\arctan( \\ & d*x+c)+1/2/d*e^2*a^2*b*\ln(1+(d*x+c)^2)-1/d*e^2*b^3*\arctan(d*x+c)^2*\ln(2)+1/ \\ & 2/d*e^2*b^3*\arctan(d*x+c)^2*\ln(1+(d*x+c)^2)-1/d*e^2*b^3*\arctan(d*x+c)^2*\ln( \\ & (1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}+1/d*\arctan(d*x+c)*b^3*c*e^2+1/4*I/d*e^2* \\ & b^3*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*c\text{sgn}(I*(1+I*(d \\ & *x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*c\text{sgn}(I/((1+I*(d \\ & *x+c))^2/(1+(d*x+c)^2)+1)^2)-1/2/d*a^2*b*c^2*e^2+1/d*a*b^2*c*e^2-1/2/d*\arct \\ & \arctan(d*x+c)^2*b^3*c^2*e^2-\arctan(d*x+c)^2*x*b^3*c*e^2+1/3*I/d*e^2*b^3*\arctan( \\ & d*x+c)^3-I/d*e^2*b^3*\arctan(d*x+c)-1/2*d*x^2*a^2*b*e^2+d*x^2*a^3*c*e^2+x*a^ \\ & 3*c^2*e^2+1/3*d^2*x^3*a^3*e^2+\arctan(d*x+c)*x*b^3*e^2-1/2/d*e^2*b^3*\arctan( \\ & d*x+c)^2-1/2/d*e^2*b^3*\text{polylog}(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*e^2*b^ \\ & 3*\ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)+1/2*I/d*e^2*a*b^2*\ln(1+(d*x+c)^2)*\ln( \\ & d*x+c-I)-1/2*I/d*e^2*a*b^2*\ln(d*x+c-I)*\ln(-1/2*I*(d*x+c+I))-1/8*\arctan(d*x+ \\ & c)^2*\text{Pi}*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)*c\text{sgn}(I*((1+I*(d*x+c))^2 \\ & /((1+(d*x+c)^2)+1))^2*x*b^3*e^2-1/2*I/d*e^2*a*b^2*\ln(1+(d*x+c)^2)*\ln(d*x+c+I \\ & )+1/2*I/d*e^2*a*b^2*\ln(d*x+c+I)*\ln(1/2*I*(d*x+c-I))-x*a^2*b*c*e^2+3*\arctan( \\ & d*x+c)*x*a^2*b*c^2*e^2+3*\arctan(d*x+c)^2*x*a*b^2*c^2*e^2-2*\arctan(d*x+c)*x* \\ & a*b^2*c*e^2-1/4*I/d*e^2*a*b^2*\ln(d*x+c-I)^2-1/2*I/d*e^2*a*b^2*\text{dilog}(-1/2*I* \\ & (d*x+c+I))+1/4*I/d*e^2*a*b^2*\ln(d*x+c+I)^2+1/2*I/d*e^2*a*b^2*\text{dilog}(1/2*I*(d \\ & *x+c-I))+1/4*\arctan(d*x+c)^2*\text{Pi}*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2) \\ & ^2*c\text{sgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*x*b^3*e^2+1/8*\arctan(d*x+c)^2* \\ & \text{Pi}*c\text{sgn}(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*c\text{sgn}(I*(1+I*(d*x+c))^4/(1+(d*x \\ & +c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I))*x*b^3*e^2+1/3/d*a^3*c^3*e^2+I \end{aligned}$$

$$\begin{aligned} & /d * e^2 * b^3 * \arctan(dx+c) * \text{polylog}(2, -(1+I*(dx+c))^2 / (1+(dx+c)^2)) + 1/d * e^2 * \\ & a * b^2 * \arctan(dx+c) * \ln(1+(dx+c)^2) + 1/d * \arctan(dx+c) * a^2 * b * c^3 * e^2 + 1/d * \arctan(dx+c) \\ & ^2 * a * b^2 * c^3 * e^2 - 1/d * \arctan(dx+c) * a * b^2 * c^2 * e^2 + d^2 * \arctan(dx+c) \\ & ) * x^3 * a^2 * b * e^2 + d^2 * \arctan(dx+c)^2 * x^3 * a * b^2 * e^2 - d * \arctan(dx+c) * x^2 * a * b^2 \\ & * e^2 - 1/8 * \arctan(dx+c)^2 * \text{Pi} * \text{csgn}(I * ((1+I*(dx+c))^2 / (1+(dx+c)^2) + 1)^2)^3 * x \\ & * b^3 * e^2 + 1/8 * \arctan(dx+c)^2 * \text{Pi} * \text{csgn}(I * (1+I*(dx+c))^4 / (1+(dx+c)^2)^2 + 2 * I * \\ & (1+I*(dx+c))^2 / (1+(dx+c)^2) + I)^3 * x * b^3 * e^2 + a * b^2 * e^2 * x \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 7/8 * b^3 * c^4 * e^2 * \arctan(dx+c)^3 * \arctan((d^2*x+c*d)/d) / d + 3 * a * b^2 * c^4 * e \\ & ^2 * \arctan(dx+c)^2 * \arctan((d^2*x+c*d)/d) / d - (3 * \arctan(dx+c) * \arctan( \\ & (d^2*x+c*d)/d)^2 / d - \arctan((d^2*x+c*d)/d)^3 / d) * a * b^2 * c^4 * e^2 - 7/32 * (6 \\ & * \arctan(dx+c)^2 * \arctan((d^2*x+c*d)/d)^2 / d - 4 * \arctan(dx+c) * \arctan( \\ & (d^2*x+c*d)/d)^3 / d + \arctan((d^2*x+c*d)/d)^4 / d) * b^3 * c^4 * e^2 + 1/3 * a^3 * d^ \\ & ^2 * e^2 * x^3 + 7/8 * b^3 * c^2 * e^2 * \arctan(dx+c)^3 * \arctan((d^2*x+c*d)/d) / d + 2 \\ & 8 * b^3 * d^4 * e^2 * \text{integrate}(1/32 * x^4 * \arctan(dx+c)^3 / (d^2 * x^2 + 2 * c * d * x + c^2 \\ & + 1), x) + 3 * b^3 * d^4 * e^2 * \text{integrate}(1/32 * x^4 * \arctan(dx+c) * \log(d^2 * x^2 + \\ & 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 96 * a * b^2 * d^4 * e^2 * \text{i} \\ & \text{ntegrate}(1/32 * x^4 * \arctan(dx+c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 112 \\ & * b^3 * c * d^3 * e^2 * \text{integrate}(1/32 * x^3 * \arctan(dx+c)^3 / (d^2 * x^2 + 2 * c * d * x + c^ \\ & 2 + 1), x) + 4 * b^3 * d^4 * e^2 * \text{integrate}(1/32 * x^4 * \arctan(dx+c) * \log(d^2 * x^2 + \\ & 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 12 * b^3 * c * d^3 * e^2 * \text{i} \\ & \text{ntegrate}(1/32 * x^3 * \arctan(dx+c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^ \\ & 2 + 2 * c * d * x + c^2 + 1), x) + 384 * a * b^2 * c * d^3 * e^2 * \text{integrate}(1/32 * x^3 * \arctan( \\ & dx+c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 168 * b^3 * c^2 * d^2 * e^2 * \text{integr} \\ & \text{ate}(1/32 * x^2 * \arctan(dx+c)^3 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 16 * b^3 * c * d \\ & ^3 * e^2 * \text{integrate}(1/32 * x^3 * \arctan(dx+c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / \\ & (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 18 * b^3 * c^2 * d^2 * e^2 * \text{integrate}(1/32 * x^2 * \text{a} \\ & \text{rctan}(dx+c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 \\ & + 1), x) + 576 * a * b^2 * c^2 * d^2 * e^2 * \text{integrate}(1/32 * x^2 * \arctan(dx+c)^2 / (d^2 * \\ & x^2 + 2 * c * d * x + c^2 + 1), x) + 112 * b^3 * c^3 * d * e^2 * \text{integrate}(1/32 * x * \arctan(d \\ & x+c)^3 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 24 * b^3 * c^2 * d^2 * e^2 * \text{integrate}(1 \\ & /32 * x^2 * \arctan(dx+c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x \\ & + c^2 + 1), x) + 12 * b^3 * c^3 * d * e^2 * \text{integrate}(1/32 * x * \arctan(dx+c) * \log(d^2 \\ & * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 384 * a * b^2 * c \\ & ^3 * d * e^2 * \text{integrate}(1/32 * x * \arctan(dx+c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), \\ & x) + 12 * b^3 * c^3 * d * e^2 * \text{integrate}(1/32 * x * \arctan(dx+c) * \log(d^2 * x^2 + 2 * c * d * \\ & x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 3 * b^3 * c^4 * e^2 * \text{integrate}(1/ \\ & 32 * \arctan(dx+c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + \\ & c^2 + 1), x) + a^3 * c * d * e^2 * x^2 + 3 * a * b^2 * c^2 * e^2 * \arctan(dx+c)^2 * \arctan( \\ & (d^2 * x+c*d)/d) / d - 4 * b^3 * d^3 * e^2 * \text{integrate}(1/32 * x^3 * \arctan(dx+c)^2 / (d^2 \\ & * x^2 + 2 * c * d * x + c^2 + 1), x) + b^3 * d^3 * e^2 * \text{integrate}(1/32 * x^3 * \log(d^2 * x^2 \\ & + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) - 12 * b^3 * c * d^2 * e^2 \\ & * \text{integrate}(1/32 * x^2 * \arctan(dx+c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 3 \\ & * b^3 * c * d^2 * e^2 * \text{integrate}(1/32 * x^2 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x \\ & ^2 + 2 * c * d * x + c^2 + 1), x) - 12 * b^3 * c^2 * d * e^2 * \text{integrate}(1/32 * x * \arctan(dx \\ & + c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 3 * b^3 * c^2 * d * e^2 * \text{integrate}(1/32 * x \\ & * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) - (3 * \\ & \arctan(dx+c) * \arctan((d^2 * x+c*d)/d)^2 / d - \arctan((d^2 * x+c*d)/d)^3 / d) * \\ & a * b^2 * c^2 * e^2 - 7/32 * (6 * \arctan(dx+c)^2 * \arctan((d^2 * x+c*d)/d)^2 / d - 4 * a \end{aligned}$$

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rctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b
^3*c^2*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x +
c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*c*d*e^2 + 1/2*
(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2
*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b
*d^2*e^2 + a^3*c^2*e^2*x + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*e^2*integrate(1/32*x*arctan(d
*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)
*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/24*(b^3*d^2*e^
2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)^3 - 1/32*(b^3*
d^2*e^2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral( $a^3 d^2 e^2 x^2 + 2 a^3 c d e^2 x + a^3 c^2 e^2 + (b^3 d^2 e^2 x^2 + 2 b^3 c d e^2 x + b^3 c^2 e^2)$  arctan(dx + c)<sup>3</sup> + 3( $ab^2 d^2 e^2 x^2 + 2 ab^2 c d e^2 x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)^2\*(a+b\*arctan(d\*x+c))^3,x, algorithm="fricas")

[Out] integral( $a^3 d^2 e^2 x^2 + 2 a^3 c d e^2 x + a^3 c^2 e^2 + (b^3 d^2 e^2 x^2 + 2 b^3 c d e^2 x + b^3 c^2 e^2)$  arctan(dx + c)<sup>3</sup> + 3( $a b^2 d^2 e^2 x^2 + 2 a b^2 c d e^2 x + a b^2 c^2 e^2$ ) arctan(dx + c)<sup>2</sup> + 3( $a^2 b d^2 e^2 x^2 + 2 a^2 b c d e^2 x + a^2 b c^2 e^2$ ) arctan(dx + c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$e^2 \left( \int a^3 c^2 dx + \int a^3 d^2 x^2 dx + \int b^3 c^2 \operatorname{atan}^3(c + dx) dx + \int 3ab^2 c^2 \operatorname{atan}^2(c + dx) dx + \int 3a^2 bc^2 \operatorname{atan}(c + dx) dx + \int \right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*\*2\*(a+b\*atan(d\*x+c))\*\*3,x)

[Out]  $e^{**2} * (\operatorname{Integral}(a^{**3} * c^{**2}, x) + \operatorname{Integral}(a^{**3} * d^{**2} * x^{**2}, x) + \operatorname{Integral}(b^{**3} * c^{**2} * \operatorname{atan}(c + d*x)^{**3}, x) + \operatorname{Integral}(3 * a * b^{**2} * c^{**2} * \operatorname{atan}(c + d*x)^{**2}, x) + \operatorname{Integral}(3 * a^{**2} * b * c^{**2} * \operatorname{atan}(c + d*x), x) + \operatorname{Integral}(2 * a^{**3} * c * d * x, x) + \operatorname{Integral}(b^{**3} * d^{**2} * x^{**2} * \operatorname{atan}(c + d*x)^{**3}, x) + \operatorname{Integral}(3 * a * b^{**2} * d^{**2} * x^{**2} * \operatorname{atan}(c + d*x)^{**2}, x) + \operatorname{Integral}(3 * a^{**2} * b * d^{**2} * x^{**2} * \operatorname{atan}(c + d*x), x) + \operatorname{Integral}(2 * b^{**3} * c * d * x * \operatorname{atan}(c + d*x)^{**3}, x) + \operatorname{Integral}(6 * a * b^{**2} * c * d * x * \operatorname{atan}(c + d*x)^{**2}, x) + \operatorname{Integral}(6 * a^{**2} * b * c * d * x * \operatorname{atan}(c + d*x), x))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \arctan(dx + c) + a)^3 dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arctan(d*x + c) + a)^3, x)
```

### 3.16 $\int (ce + dex) \left( a + b \tan^{-1}(c + dx) \right)^3 dx$

**Optimal.** Leaf size=164

$$\frac{3ib^3e\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{2d} - \frac{3b^2e \log\left(\frac{2}{1+i(c+dx)}\right) \left(a + b \tan^{-1}(c + dx)\right)}{d} - \frac{3ibe \left(a + b \tan^{-1}(c + dx)\right)^2}{2d} - \frac{3be(c + dx)}{d}$$

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
```

**Rubi [A]** time = 0.243152, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5043, 12, 4852, 4916, 4846, 4920, 4854, 2402, 2315, 4884}

$$\frac{3ib^3e\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{2d} - \frac{3b^2e \log\left(\frac{2}{1+i(c+dx)}\right) \left(a + b \tan^{-1}(c + dx)\right)}{d} - \frac{3ibe \left(a + b \tan^{-1}(c + dx)\right)^2}{2d} - \frac{3be(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3, x]
```

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
```

#### Rule 5043

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^ (m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^ (m_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
```

$e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{GtQ}[p, 0] \&\& \text{GtQ}[m, 1]$

#### Rule 4846

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p, x\_Symbol] \text{ :> } \text{Simp}[x*(a + b*\text{ArcTan}[c*x])^p, x] - \text{Dist}[b*c*p, \text{Int}[(x*(a + b*\text{ArcTan}[c*x])^{p-1})/(1 + c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{IGtQ}[p, 0]$

#### Rule 4920

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p*(x)/(d + e*x^2), x\_Symbol] \text{ :> } -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^{p+1})/(b*e*(p+1)), x] - \text{Dist}[1/(c*d), \text{Int}[(a + b*\text{ArcTan}[c*x])^p/(I - c*x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[e, c^2*d] \&\& \text{IGtQ}[p, 0]$

#### Rule 4854

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p/(d + e*x), x\_Symbol] \text{ :> } -\text{Simp}[(a + b*\text{ArcTan}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]/e, x] + \text{Dist}[(b*c*p)/e, \text{Int}[(a + b*\text{ArcTan}[c*x])^{p-1}*\text{Log}[2/(1 + (e*x)/d)]/(1 + c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[c^2*d^2 + e^2, 0]$

#### Rule 2402

$\text{Int}[\text{Log}[c/(d + e*x)]/(f + g*x^2), x\_Symbol] \text{ :> } -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

#### Rule 2315

$\text{Int}[\text{Log}[c*x]/(d + e*x), x\_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x\} \&\& \text{EqQ}[e + c*d, 0]$

#### Rule 4884

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p/(d + e*x^2), x\_Symbol] \text{ :> } \text{Simp}[(a + b*\text{ArcTan}[c*x])^{p+1}/(b*c*d*(p+1)), x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\} \&\& \text{EqQ}[e, c^2*d] \&\& \text{NeQ}[p, -1]$

#### Rubi steps



$$\begin{aligned}
& x+c-I)) - 3/4 * I / d * e * b^3 * \ln(d*x+c-I) * \ln(-1/2 * I * (d*x+c+I)) - 3/2 / d * a^2 * b * c * e + 3/2 / \\
& d * e * b^3 * \arctan(d*x+c) * \ln(1+(d*x+c)^2) + 3/4 * I / d * e * b^3 * \operatorname{dilog}(1/2 * I * (d*x+c-I)) - \\
& 3 * \arctan(d*x+c) * x * a * b^2 * e + 1/2 * d * \arctan(d*x+c)^3 * x^2 * b^3 * e - 3/2 / d * \arctan(d*x+c) \\
& ^2 * b^3 * c * e + \arctan(d*x+c)^3 * x * b^3 * c * e - 3/8 * I / d * e * b^3 * \ln(d*x+c-I)^2 - 3/4 * I / d * \\
& e * b^3 * \operatorname{dilog}(-1/2 * I * (d*x+c+I)) + 3/8 * I / d * e * b^3 * \ln(d*x+c+I)^2 + 1/2 / d * \arctan(d*x+c) \\
& ^3 * b^3 * c^2 * e + 3/2 / d * e * a * b^2 * \ln(1+(d*x+c)^2) + 3/2 / d * e * a^2 * b * \arctan(d*x+c) + 3/ \\
& 2 / d * e * a * b^2 * \arctan(d*x+c)^2 + 3/2 / d * \arctan(d*x+c)^2 * a * b^2 * c^2 * e + 3/2 * d * \arctan(d*x+c) \\
& ^2 * x^2 * a * b^2 * e + 3 * \arctan(d*x+c)^2 * x * a * b^2 * c * e - 3 / d * \arctan(d*x+c) * a * b^2 * \\
& c * e + 3/2 / d * \arctan(d*x+c) * a^2 * b * c^2 * e + 3/2 * d * \arctan(d*x+c) * x^2 * a^2 * b * e + 1/2 / d * e \\
& * b^3 * \arctan(d*x+c)^3 + 1/2 / d * a^3 * c^2 * e - 3/2 * e * x * a^2 * b + x * a^3 * c * e + 1/2 * d * x^2 * a^3 * \\
& e - 3/2 * \arctan(d*x+c)^2 * x * b^3 * e
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c))^3,x, algorithm="maxima")

[Out]  $1/2 * a^3 * d * e * x^2 + 3/2 * (x^2 * \arctan(d*x + c) - d * (x/d^2 + (c^2 - 1) * \arctan((d^2 * x + c * d)/d) / d^3 - c * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / d^3)) * a^2 * b * d * e + a^3 * c * e * x + 3/2 * (2 * (d*x + c) * \arctan(d*x + c) - \log((d*x + c)^2 + 1)) * a^2 * b * c * e / d + 1/32 * (8 * (b^3 * d^2 * e * x^2 + 2 * b^3 * c * d * e * x + (b^3 * c^2 + b^3) * e) * \arctan(d*x + c)^3 + 12 * (a * b^2 * d^2 * e * x^2 + (2 * a * b^2 * c - b^3) * d * e * x) * \arctan(d*x + c)^2 - 3 * (a * b^2 * d^2 * e * x^2 + (2 * a * b^2 * c - b^3) * d * e * x) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 + 4 * (4 * b^3 * c^3 * e * \arctan(d*x + c)^3 * \arctan((d^2 * x + c * d)/d) / d + 18 * a * b^2 * c^3 * e * \arctan(d*x + c)^2 * \arctan((d^2 * x + c * d)/d) / d - 6 * (3 * \arctan(d*x + c) * \arctan((d^2 * x + c * d)/d)^2 / d - \arctan((d^2 * x + c * d)/d)^3 / d) * a * b^2 * c^3 * e - (6 * \arctan(d*x + c)^2 * \arctan((d^2 * x + c * d)/d)^2 / d - 4 * \arctan(d*x + c) * \arctan((d^2 * x + c * d)/d)^3 / d + \arctan((d^2 * x + c * d)/d)^4 / d) * b^3 * c^3 * e - 3 * b^3 * c^2 * e * \arctan(d*x + c)^2 * \arctan((d^2 * x + c * d)/d) / d + 4 * b^3 * c * e * \arctan(d*x + c)^3 * \arctan((d^2 * x + c * d)/d) / d + 128 * b^3 * d^3 * e * \operatorname{integrate}(1/32 * x^3 * \arctan(d*x + c)^3 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 576 * a * b^2 * d^3 * e * \operatorname{integrate}(1/32 * x^3 * \arctan(d*x + c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 384 * b^3 * c * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \arctan(d*x + c)^3 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 48 * a * b^2 * d^3 * e * \operatorname{integrate}(1/32 * x^3 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 1728 * a * b^2 * c * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \arctan(d*x + c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 384 * b^3 * c^2 * d * e * \operatorname{integrate}(1/32 * x * \arctan(d*x + c)^3 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 96 * a * b^2 * d^3 * e * \operatorname{integrate}(1/32 * x^3 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 144 * a * b^2 * c * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 1728 * a * b^2 * c^2 * d * e * \operatorname{integrate}(1/32 * x * \arctan(d*x + c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 288 * a * b^2 * c * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 144 * a * b^2 * c^2 * d * e * \operatorname{integrate}(1/32 * x * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 192 * a * b^2 * c^2 * d * e * \operatorname{integrate}(1/32 * x * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + 48 * a * b^2 * c^3 * e * \operatorname{integrate}(1/32 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) + (3 * \arctan(d*x + c) * \arctan((d^2 * x + c * d)/d)^2 / d - \arctan((d^2 * x + c * d)/d)^3 / d) * b^3 * c^2 * e + 18 * a * b^2 * c * e * \arctan(d*x + c)^2 * \arctan((d^2 * x + c * d)/d) / d - 96 * b^3 * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \arctan(d*x + c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) - 24 * b^3 * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) - 192 * a * b^2 * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \arctan(d*x + c) / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) - 92 * b^3 * c * d * e * \operatorname{integrate}(1/32 * x * \arctan(d*x + c)^2 / (d^2 * x^2 + 2 * c * d * x + c^2 + 1), x) - 96 * b^3 * d^2 * e * \operatorname{integrate}(1/32 * x^2 * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) / ($

$$d^2x^2 + 2c dx + c^2 + 1), x) - 48b^3c d e \int \frac{1}{32} x \log(d^2x^2 + 2c dx + c^2 + 1)^2 / (d^2x^2 + 2c dx + c^2 + 1), x) - 384a b^2 c d e \int \frac{1}{32} x \arctan(dx + c) / (d^2x^2 + 2c dx + c^2 + 1), x) - 96b^3 c d e \int \frac{1}{32} x \log(d^2x^2 + 2c dx + c^2 + 1) / (d^2x^2 + 2c dx + c^2 + 1), x) - 24b^3 c^2 e \int \frac{1}{32} \log(d^2x^2 + 2c dx + c^2 + 1)^2 / (d^2x^2 + 2c dx + c^2 + 1), x) - 6(3 \arctan(dx + c) \arctan((d^2x + c d) / d)^2 / d - \arctan((d^2x + c d) / d)^3 / d) a b^2 c e - (6 \arctan(dx + c)^2 \arctan((d^2x + c d) / d)^2 / d - 4 \arctan(dx + c) \arctan((d^2x + c d) / d)^3 / d + \arctan((d^2x + c d) / d)^4 / d) b^3 c e - 3b^3 e \arctan(dx + c)^2 \arctan((d^2x + c d) / d) / d + 128b^3 d e \int \frac{1}{32} x \arctan(dx + c)^3 / (d^2x^2 + 2c dx + c^2 + 1), x) + 576a b^2 d e \int \frac{1}{32} x \arctan(dx + c)^2 / (d^2x^2 + 2c dx + c^2 + 1), x) + 48a b^2 d e \int \frac{1}{32} x \log(d^2x^2 + 2c dx + c^2 + 1)^2 / (d^2x^2 + 2c dx + c^2 + 1), x) + 192b^3 d e \int \frac{1}{32} x \arctan(dx + c) / (d^2x^2 + 2c dx + c^2 + 1), x) + 48a b^2 c e \int \frac{1}{32} \log(d^2x^2 + 2c dx + c^2 + 1)^2 / (d^2x^2 + 2c dx + c^2 + 1), x) + (3 \arctan(dx + c) \arctan((d^2x + c d) / d)^2 / d - \arctan((d^2x + c d) / d)^3 / d) b^3 e - 24b^3 e \int \frac{1}{32} \log(d^2x^2 + 2c dx + c^2 + 1)^2 / (d^2x^2 + 2c dx + c^2 + 1), x) d) / d$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int (a^3 dx + a^3 ce + (b^3 dx + b^3 ce) \arctan(dx + c)^3 + 3(ab^2 dx + ab^2 ce) \arctan(dx + c)^2 + 3(a^2 b dx + a^2 b ce) \arctan(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(a^3\*d\*e\*x + a^3\*c\*e + (b^3\*d\*e\*x + b^3\*c\*e)\*arctan(d\*x + c)^3 + 3\*(a\*b^2\*d\*e\*x + a\*b^2\*c\*e)\*arctan(d\*x + c)^2 + 3\*(a^2\*b\*d\*e\*x + a^2\*b\*c\*e)\*arctan(d\*x + c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e \left( \int a^3 c dx + \int a^3 dx dx + \int b^3 c \operatorname{atan}^3(c + dx) dx + \int 3ab^2 c \operatorname{atan}^2(c + dx) dx + \int 3a^2 b c \operatorname{atan}(c + dx) dx + \int b^3 dx a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*atan(d\*x+c))\*\*3,x)

[Out] e\*(Integral(a\*\*3\*c, x) + Integral(a\*\*3\*d\*x, x) + Integral(b\*\*3\*c\*atan(c + d\*x)\*\*3, x) + Integral(3\*a\*b\*\*2\*c\*atan(c + d\*x)\*\*2, x) + Integral(3\*a\*\*2\*b\*c\*atan(c + d\*x), x) + Integral(b\*\*3\*d\*x\*atan(c + d\*x)\*\*3, x) + Integral(3\*a\*b\*\*2\*d\*x\*atan(c + d\*x)\*\*2, x) + Integral(3\*a\*\*2\*b\*d\*x\*atan(c + d\*x), x))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \arctan(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*e\*x+c\*e)\*(a+b\*arctan(d\*x+c))^3,x, algorithm="giac")

```
[Out] integrate((d*e*x + c*e)*(b*arctan(d*x + c) + a)^3, x)
```

$$3.17 \quad \int \frac{(a+b \tan^{-1}(c+dx))^3}{ce+dex} dx$$

**Optimal.** Leaf size=279

$$\frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2de} + \frac{3b^2 \text{PolyLog}\left(3, -1 + \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2de} - \frac{3ib \text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2de}$$

[Out] (2\*(a + b\*ArcTan[c + d\*x])^3\*ArcTanh[1 - 2/(1 + I\*(c + d\*x))])/(d\*e) - (((3\*I)/2)\*b\*(a + b\*ArcTan[c + d\*x])^2\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/(d\*e) + (((3\*I)/2)\*b\*(a + b\*ArcTan[c + d\*x])^2\*PolyLog[2, -1 + 2/(1 + I\*(c + d\*x))])/(d\*e) - (3\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[3, 1 - 2/(1 + I\*(c + d\*x))])/(2\*d\*e) + (3\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[3, -1 + 2/(1 + I\*(c + d\*x))])/(2\*d\*e) + (((3\*I)/4)\*b^3\*PolyLog[4, 1 - 2/(1 + I\*(c + d\*x))])/(d\*e) - (((3\*I)/4)\*b^3\*PolyLog[4, -1 + 2/(1 + I\*(c + d\*x))])/(d\*e)

**Rubi [A]** time = 0.458166, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {5043, 12, 4850, 4988, 4884, 4994, 4998, 6610}

$$\frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2de} + \frac{3b^2 \text{PolyLog}\left(3, -1 + \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2de} - \frac{3ib \text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^3/(c\*e + d\*e\*x), x]

[Out] (2\*(a + b\*ArcTan[c + d\*x])^3\*ArcTanh[1 - 2/(1 + I\*(c + d\*x))])/(d\*e) - (((3\*I)/2)\*b\*(a + b\*ArcTan[c + d\*x])^2\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/(d\*e) + (((3\*I)/2)\*b\*(a + b\*ArcTan[c + d\*x])^2\*PolyLog[2, -1 + 2/(1 + I\*(c + d\*x))])/(d\*e) - (3\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[3, 1 - 2/(1 + I\*(c + d\*x))])/(2\*d\*e) + (3\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[3, -1 + 2/(1 + I\*(c + d\*x))])/(2\*d\*e) + (((3\*I)/4)\*b^3\*PolyLog[4, 1 - 2/(1 + I\*(c + d\*x))])/(d\*e) - (((3\*I)/4)\*b^3\*PolyLog[4, -1 + 2/(1 + I\*(c + d\*x))])/(d\*e)

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4850

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^(p\_)/(x\_), x\_Symbol] :> Simp[2\*(a + b\*ArcTan[c\*x])^p\*ArcTanh[1 - 2/(1 + I\*c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 + I\*c\*x))]/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

#### Rule 4988



```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

#### Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

#### Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

#### Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*PolyLog[k_, u_]/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

#### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^3}{ce + dex} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{ex} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{x} dx, x, c + dx\right)}{de} \\
&= \frac{2(a + b \tan^{-1}(c + dx))^3 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{(6b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2 \tanh^{-1}\left(1 - \frac{2}{1+ix}\right)}{1+x^2} dx\right)}{de} \\
&= \frac{2(a + b \tan^{-1}(c + dx))^3 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{(3b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2 \log\left(2 - \frac{2}{1+ix}\right)}{1+x^2} dx\right)}{de} \\
&= \frac{2(a + b \tan^{-1}(c + dx))^3 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a + b \tan^{-1}(c + dx))^2 \text{Li}_2\left(1 - \frac{2}{1+i(c+dx)}\right)}{2de} \\
&= \frac{2(a + b \tan^{-1}(c + dx))^3 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a + b \tan^{-1}(c + dx))^2 \text{Li}_2\left(1 - \frac{2}{1+i(c+dx)}\right)}{2de} \\
&= \frac{2(a + b \tan^{-1}(c + dx))^3 \tanh^{-1}\left(1 - \frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a + b \tan^{-1}(c + dx))^2 \text{Li}_2\left(1 - \frac{2}{1+i(c+dx)}\right)}{2de}
\end{aligned}$$

**Mathematica [A]** time = 0.0917491, size = 252, normalized size = 0.9

$$6b^2 \text{PolyLog}\left(3, -\frac{c+dx+i}{c+dx-i}\right)(a + b \tan^{-1}(c + dx)) - 6b^2 \text{PolyLog}\left(3, \frac{c+dx+i}{c+dx-i}\right)(a + b \tan^{-1}(c + dx)) + 6ib \text{PolyLog}\left(2, -\frac{c+dx+i}{c+dx-i}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^3/(c\*e + d\*e\*x), x]

[Out] (8\*(a + b\*ArcTan[c + d\*x])^3\*ArcTanh[(I + c + d\*x)/(-I + c + d\*x)] + (6\*I)\*b\*(a + b\*ArcTan[c + d\*x])^2\*PolyLog[2, -((I + c + d\*x)/(-I + c + d\*x))] - (6\*I)\*b\*(a + b\*ArcTan[c + d\*x])^2\*PolyLog[2, (I + c + d\*x)/(-I + c + d\*x)] + 6\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[3, -((I + c + d\*x)/(-I + c + d\*x))] - 6\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[3, (I + c + d\*x)/(-I + c + d\*x)] - (3\*I)\*b^3\*PolyLog[4, -((I + c + d\*x)/(-I + c + d\*x))] + (3\*I)\*b^3\*PolyLog[4, (I + c + d\*x)/(-I + c + d\*x)]/(4\*d\*e)

**Maple [C]** time = 0.377, size = 2894, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e), x)

[Out] 3/2\*I/d\*a\*b^2/e\*Pi\*csgn(I\*((1+I\*(d\*x+c))^2/(1+(d\*x+c)^2)-1)/((1+I\*(d\*x+c))^2/(1+(d\*x+c)^2)+1))\*csgn(((1+I\*(d\*x+c))^2/(1+(d\*x+c)^2)-1)/((1+I\*(d\*x+c))^2/(1+(d\*x+c)^2)+1))\*arctan(d\*x+c)^2-3/2\*I/d\*a\*b^2/e\*Pi\*csgn(I/((1+I\*(d\*x+c))

$$\begin{aligned} &^2/(1+(d*x+c)^2+1))*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^2+3/2*I/d*a*b^2/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*\text{csgn}(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1))*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))*\arctan(d*x+c)^2+3/d*a*b^2/e*\arctan(d*x+c)^2*\ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+3/d*a^2*b/e*\ln(d*x+c)*\arctan(d*x+c)-3*I/d*b^3/e*\arctan(d*x+c)^2*\text{polylog}(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+3/2*I/d*b^3/e*\arctan(d*x+c)^2*\text{polylog}(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I/d*b^3/e*\arctan(d*x+c)^2*\text{polylog}(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+1/2*I/d*b^3/e*Pi*\arctan(d*x+c)^3+3/2*I/d*a^2*b/e*\text{dilog}(1+I*(d*x+c))-3/2*I/d*a^2*b/e*\text{dilog}(1-I*(d*x+c))+3/d*a*b^2/e*\ln(d*x+c)*\arctan(d*x+c)^2-3/d*a*b^2/e*\arctan(d*x+c)^2*\ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+3/d*a*b^2/e*\arctan(d*x+c)^2*\ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+3/2*I/d*a*b^2/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^3*\arctan(d*x+c)^2-3/2*I/d*a*b^2/e*Pi*\text{csgn}(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^2+3/2*I/d*a*b^2/e*Pi*\text{csgn}(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^3*\arctan(d*x+c)^2-1/2*I/d*b^3/e*Pi*\text{csgn}(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1))*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^3-1/2*I/d*b^3/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))*\text{csgn}(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^3+1/d*a^3/e*\ln(d*x+c)-3/4*I/d*b^3/e*\text{polylog}(4,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+6*I/d*b^3/e*\text{polylog}(4,(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+6*I/d*b^3/e*\text{polylog}(4,-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+6/d*b^3/e*\arctan(d*x+c)*\text{polylog}(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+1/d*b^3/e*\arctan(d*x+c)^3*\ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+6/d*b^3/e*\arctan(d*x+c)*\text{polylog}(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))-3/2/d*b^3/e*\arctan(d*x+c)*\text{polylog}(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/d*b^3/e*\ln(d*x+c)*\arctan(d*x+c)^3-1/d*b^3/e*\arctan(d*x+c)^3*\ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))-3/2/d*a*b^2/e*\text{polylog}(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+6/d*a*b^2/e*\text{polylog}(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+6/d*a*b^2/e*\text{polylog}(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+1/d*b^3/e*\arctan(d*x+c)^3*\ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))-3/2*I/d*a*b^2/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^2-3/2*I/d*a*b^2/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^2+3/2*I/d*a*b^2/e*Pi*\arctan(d*x+c)^2+3/2*I/d*a^2*b/e*\ln(d*x+c)*\ln(1+I*(d*x+c))-3/2*I/d*a^2*b/e*\ln(d*x+c)*\ln(1-I*(d*x+c))-1/2*I/d*b^3/e*Pi*\text{csgn}(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^2*\arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*\text{csgn}(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^3*\arctan(d*x+c)^3+1/2*I/d*b^3/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))^3*\arctan(d*x+c)^3-6*I/d*a*b^2/e*\arctan(d*x+c)*\text{polylog}(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+3*I/d*a*b^2/e*\arctan(d*x+c)*\text{polylog}(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)}))+1/2*I/d*b^3/e*Pi*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*\text{csgn}(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1))*\text{csgn}(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2+1)))*\arctan(d*x+c)^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(dx + ce)}{de} + \int \frac{28b^3 \arctan(dx + c)^3 + 3b^3 \arctan(dx + c) \log(d^2x^2 + 2cdx + c^2 + 1)^2 + 96ab^2 \arctan(dx + c)}{32(dx + ce)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e),x, algorithm="maxima")

[Out]  $a^3 \log(dex + ce) / (dex + ce) + \int (1/32 * (28 * b^3 * \arctan(dx + c)^3 + 3 * b^3 * \arctan(dx + c) * \log(d^2 * x^2 + 2 * c * dx + c^2 + 1)^2 + 96 * a * b^2 * \arctan(dx + c)^2 + 96 * a^2 * b * \arctan(dx + c))) / (dex + ce), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(dx + c)^3 + 3ab^2 \arctan(dx + c)^2 + 3a^2b \arctan(dx + c) + a^3}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e),x, algorithm="fricas")

[Out]  $\text{integral}((b^3 * \arctan(dx + c)^3 + 3 * a * b^2 * \arctan(dx + c)^2 + 3 * a^2 * b * \arctan(dx + c) + a^3) / (dex + ce), x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c+dx} dx + \int \frac{b^3 \operatorname{atan}^3(c+dx)}{c+dx} dx + \int \frac{3ab^2 \operatorname{atan}^2(c+dx)}{c+dx} dx + \int \frac{3a^2b \operatorname{atan}(c+dx)}{c+dx} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))\*\*3/(d\*e\*x+c\*e),x)

[Out]  $(\text{Integral}(a^{**3} / (c + dx), x) + \text{Integral}(b^{**3} * \operatorname{atan}(c + dx)^{**3} / (c + dx), x) + \text{Integral}(3 * a * b^{**2} * \operatorname{atan}(c + dx)^{**2} / (c + dx), x) + \text{Integral}(3 * a^{**2} * b * \operatorname{atan}(c + dx) / (c + dx), x)) / e$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^3}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e),x, algorithm="giac")

[Out] integrate((b\*arctan(dx + c) + a)^3/(d\*e\*x + c\*e), x)

$$3.18 \quad \int \frac{(a+b \tan^{-1}(c+dx))^3}{(ce+dex)^2} dx$$

**Optimal.** Leaf size=163

$$\frac{3ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de^2} + \frac{3b^3 \text{PolyLog}\left(3, -1 + \frac{2}{1-i(c+dx)}\right)}{2de^2} - \frac{(a+b \tan^{-1}(c+dx))^3}{de^2(c+dx)}$$

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```

**Rubi [A]** time = 0.299623, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {5043, 12, 4852, 4924, 4868, 4884, 4992, 6610}

$$\frac{3ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de^2} + \frac{3b^3 \text{PolyLog}\left(3, -1 + \frac{2}{1-i(c+dx)}\right)}{2de^2} - \frac{(a+b \tan^{-1}(c+dx))^3}{de^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2, x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```

#### Rule 5043

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^ (m
_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

#### Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)/((x_.)*((d_.) + (e_.)*(x_.)^2)),
x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
```

$e\}, x]$  && EqQ[e, c^2\*d] && GtQ[p, 0]

### Rule 4868

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((x\_.)\*((d\_.) + (e\_.)\*(x\_.))), x\_Symbol] := Simp[((a + b\*ArcTan[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/((1 + c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

### Rule 4884

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

### Rule 4992

Int[(Log[u\_]\*((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(I\*(a + b\*ArcTan[c\*x])^p\*PolyLog[2, 1 - u])/((2\*c\*d), x] - Dist[(b\*p\*I)/2, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/((d + e\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && EqQ[(1 - u)^2 - (1 - (2\*I)/(I + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan^{-1}(c + dx))^3}{(ce + dex)^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{e^2 x^2} dx, x, c + dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{x^2} dx, x, c + dx\right)}{de^2} \\ &= -\frac{(a + b \tan^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{(3b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x(1+x^2)} dx, x, c + dx\right)}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{(3ib) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x(i+x)} dx, x, c + dx\right)}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{3b(a + b \tan^{-1}(c + dx))^2 \log(2 - \frac{c+dx}{i+c+dx})}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{3b(a + b \tan^{-1}(c + dx))^2 \log(2 - \frac{c+dx}{i+c+dx})}{de^2} \\ &= -\frac{i(a + b \tan^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tan^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{3b(a + b \tan^{-1}(c + dx))^2 \log(2 - \frac{c+dx}{i+c+dx})}{de^2} \end{aligned}$$

**Mathematica [A]** time = 0.449064, size = 263, normalized size = 1.61

$$6ab^2 \left( \tan^{-1}(c + dx) \left( \left( -\frac{1}{c+dx} - i \right) \tan^{-1}(c + dx) + 2 \log \left( 1 - e^{2i \tan^{-1}(c+dx)} \right) \right) - i \text{PolyLog} \left( 2, e^{2i \tan^{-1}(c+dx)} \right) \right) + 2b^3 \left( 3i \tan^{-1}(c + dx) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^3/(c\*e + d\*e\*x)^2,x]

[Out]  $\frac{(-2a^3)}{(c + dx)} - \frac{(6a^2b \text{ArcTan}[c + dx])}{(c + dx)} + 6a^2b \text{Log}[c + dx] - 3a^2b \text{Log}[1 + c^2 + 2cdx + d^2x^2] + 6ab^2 (\text{ArcTan}[c + dx] * ((-1 - (c + dx)^{-1}) \text{ArcTan}[c + dx] + 2 \text{Log}[1 - E^{(2I) \text{ArcTan}[c + dx]}])) - I \text{PolyLog}[2, E^{(2I) \text{ArcTan}[c + dx]}]) + 2b^3 ((-I/8) \text{Pi}^3 + I \text{ArcTan}[c + dx]^3 - \text{ArcTan}[c + dx]^3 / (c + dx) + 3 \text{ArcTan}[c + dx]^2 \text{Log}[1 - E^{(-2I) \text{ArcTan}[c + dx]}]) + (3I) \text{ArcTan}[c + dx] \text{PolyLog}[2, E^{(-2I) \text{ArcTan}[c + dx]}]) + (3 \text{PolyLog}[3, E^{(-2I) \text{ArcTan}[c + dx]}]) / 2) / (2d^2e^2)$

**Maple [C]** time = 0.4, size = 2696, normalized size = 16.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e)^2,x)

[Out]  $\frac{3}{2} I \frac{d^3 b^3}{e^2 \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)-1))} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)+1)) \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)) \text{arctan}(d*x+c)^2 - \frac{3}{4} I \frac{d^3 b^3}{e^2} \text{arctan}(d*x+c)^2 \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2) \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)) \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2 - \frac{1}{d^3 a^3 e^2} \frac{d^3 b^3}{e^2} \text{arctan}(d*x+c)^2 \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2) \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2 + \frac{3}{4} I \frac{d^3 b^3}{e^2} \text{arctan}(d*x+c)^2 \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)+1))^2 \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2) \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2 - \frac{3}{4} I \frac{d^3 b^3}{e^2} \text{arctan}(d*x+c)^2 \text{Pi} \text{csgn}(I(1+I(d*x+c))/(1+(d*x+c)^2)^{(1/2)})^2 \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)) + \frac{3}{4} I \frac{d^3 b^3}{e^2} \text{arctan}(d*x+c)^2 \text{Pi} \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)) \text{csgn}(I(1+I(d*x+c))^2/(1+(d*x+c)^2)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2 - \frac{3}{2} I \frac{d^3 b^3}{e^2} \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)+1)) \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)) \text{arctan}(d*x+c)^2 + \frac{3}{2} I \frac{d^3 b^3}{e^2} \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)) \text{csgn}(((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1))^2 \text{arctan}(d*x+c)^2 + \frac{3}{2} I \frac{d^3 b^3}{e^2} \text{Pi} \text{csgn}(I((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1)) \text{csgn}(((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+I(d*x+c))^2/(1+(d*x+c)^2)+1))^2 \text{arctan}(d*x+c)^2 + \frac{6}{d^3 b^3 e^2} \text{polylog}(3, -(1+I(d*x+c))/(1+(d*x+c)^2)^{(1/2)}) + \frac{6}{d^3 b^3 e^2} \text{polylog}(3, (1+I(d*x+c))/(1+(d*x+c)^2)^{(1/2)}) - \frac{3}{d^3 a^2 b^2 e^2} \text{arctan}(d*x+c) \ln(1+(d*x+c)^2) + \frac{6}{d^3 a^2 b^2 e^2} \text{arctan}(d*x+c) \ln(d*x+c) - \frac{3}{d^3 a^2 b^2 e^2} \text{arctan}(d*x+c)^2 + \frac{3}{2} I \frac{d^3 b^3}{e^2} \text{Pi} \text{arctan}(d*x+c)^2 - \frac{6}{d^3 b^3 e^2} \text{arctan}(d*x+c) \text{polylog}(2, -(1+I(d*x+c))/(1+(d*x+c)^2)^{(1/2)}) - \frac{6}{d^3 b^3 e^2} \text{arctan}(d*x+c) \text{polylog}(2, (1+I(d*x+c))/(1+(d*x+c)^2)^{(1/2)}) +$

$$\begin{aligned}
& 3*I/d*a*b^2/e^2*dilog(1+I*(d*x+c))-3*I/d*a*b^2/e^2*dilog(1-I*(d*x+c))+3/4*I/ \\
& /d*a*b^2/e^2*\ln(d*x+c-I)^2+3/2*I/d*a*b^2/e^2*dilog(-1/2*I*(d*x+c+I))-3/4*I/ \\
& d*a*b^2/e^2*\ln(d*x+c+I)^2-3/2*I/d*a*b^2/e^2*dilog(1/2*I*(d*x+c-I))-3/2/d*a^ \\
& 2*b/e^2*\ln(1+(d*x+c)^2)+3/d*a^2*b/e^2*\ln(d*x+c)+3/d*b^3/e^2*\arctan(d*x+c)^2 \\
& *ln(2)+3/d*b^3/e^2*\arctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+ \\
& 3/d*b^3/e^2*\arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/d*b^3 \\
& /e^2*\arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/d*b^3/e^2/(d*x \\
& +c)*\arctan(d*x+c)^3-3/d*b^3/e^2*\arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+ \\
& c)^2)-1)-3/2/d*b^3/e^2*\arctan(d*x+c)^2*ln(1+(d*x+c)^2)+3/d*b^3/e^2*\ln(d*x+c \\
& )*\arctan(d*x+c)^2-I/d*b^3/e^2*\arctan(d*x+c)^3+3/2*I/d*a*b^2/e^2*\ln(1+(d*x+c \\
& )^2)*ln(d*x+c+I)-3/2*I/d*b^3/e^2*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/ \\
& ((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*\arctan(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*csgn \\
& n(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*ar \\
& ctan(d*x+c)^2-3/4*I/d*b^3/e^2*\arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+ \\
& (d*x+c)^2))^3-3/4*I/d*b^3/e^2*\arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+ \\
& (d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3+3/4*I/d*b^3/e^2*\arctan(d* \\
& x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^3-3*I/d*a*b^2/e^2*\ln( \\
& d*x+c)*ln(1-I*(d*x+c))-3/2*I/d*a*b^2/e^2*\ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))+3* \\
& I/d*a*b^2/e^2*\ln(d*x+c)*ln(1+I*(d*x+c))+3/2*I/d*b^3/e^2*Pi*csgn(I*((1+I*(d* \\
& x+c))^2/(1+(d*x+c)^2)-1)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3*\arctan(d*x+c \\
& )^2-3/2*I/d*a*b^2/e^2*\ln(1+(d*x+c)^2)*ln(d*x+c-I)+3/2*I/d*a*b^2/e^2*\ln(d*x+c \\
& -I)*ln(-1/2*I*(d*x+c+I))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3}{2} \left( d \left( \frac{\log(d^2 x^2 + 2 c d x + c^2 + 1)}{d^2 e^2} - \frac{2 \log(dx + c)}{d^2 e^2} \right) + \frac{2 \arctan(dx + c)}{d^2 e^2 x + c d e^2} \right) a^2 b - \frac{a^3}{d^2 e^2 x + c d e^2} - \frac{\frac{15}{2} b^3 \arctan(dx + c)^3}{d^2 e^2 x + c d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e)^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -3/2*(d*(\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*\log(dx + c)/(d^2*e \\
& ^2)) + 2*\arctan(dx + c)/(d^2*e^2*x + c*d*e^2))*a^2*b - a^3/(d^2*e^2*x + c* \\
& d*e^2) - 1/32*(4*b^3*\arctan(dx + c)^3 - 3*b^3*\arctan(dx + c)*\log(d^2*x^2 \\
& + 2*c*d*x + c^2 + 1)^2 - 32*(d^2*e^2*x + c*d*e^2)*integrate(1/32*(28*(b^3*d \\
& ^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*\arctan(dx + c)^3 + 12*(8*a*b^2*d^2*x \\
& ^2 + 8*a*b^2*c^2 + b^3*c + 8*a*b^2 + (16*a*b^2*c + b^3)*d*x)*\arctan(dx + c \\
& )^2 - 12*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*\arctan(dx + c)*\log(d^2*x^2 \\
& + 2*c*d*x + c^2 + 1) - 3*(b^3*d*x + b^3*c - (b^3*d^2*x^2 + 2*b^3*c*d*x + b^ \\
& 3*c^2 + b^3)*\arctan(dx + c))*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/(d^4*e^2* \\
& x^4 + 4*c*d^3*e^2*x^3 + (6*c^2 + 1)*d^2*e^2*x^2 + 2*(2*c^3 + c)*d*e^2*x + ( \\
& c^4 + c^2)*e^2), x))/(d^2*e^2*x + c*d*e^2)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(dx + c)^3 + 3 ab^2 \arctan(dx + c)^2 + 3 a^2 b \arctan(dx + c) + a^3}{d^2 e^2 x^2 + 2 c d e^2 x + c^2 e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e)^2,x, algorithm="fricas")



[Out]  $\text{integral}((b^3 \arctan(dx + c))^3 + 3*a*b^2 \arctan(dx + c)^2 + 3*a^2*b \arctan(dx + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2} dx + \int \frac{b^3 \operatorname{atan}^3(c+dx)}{c^2+2cdx+d^2x^2} dx + \int \frac{3ab^2 \operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2} dx + \int \frac{3a^2b \operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2} dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\operatorname{atan}(d*x+c))^3/(d*e*x+c*e)^2, x)$

[Out]  $(\text{Integral}(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + \text{Integral}(b**3*\operatorname{atan}(c + d*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + \text{Integral}(3*a*b**2*\operatorname{atan}(c + d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + \text{Integral}(3*a**2*b*\operatorname{atan}(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2), x))/e**2$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^3}{(dex + ce)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\arctan(d*x+c))^3/(d*e*x+c*e)^2, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}((b*\arctan(d*x + c) + a)^3/(d*e*x + c*e)^2, x)$

$$3.19 \quad \int \frac{(a+b \tan^{-1}(c+dx))^3}{(ce+dex)^3} dx$$

**Optimal.** Leaf size=180

$$-\frac{3ib^3 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)}{2de^3} + \frac{3b^2 \log\left(2 - \frac{2}{1-i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{de^3} - \frac{3b(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)} - \frac{3ib(a + b \tan^{-1}(c + dx))}{de^3}$$

[Out] (((-3\*I)/2)\*b\*(a + b\*ArcTan[c + d\*x])^2)/(d\*e^3) - (3\*b\*(a + b\*ArcTan[c + d\*x])^2)/(2\*d\*e^3\*(c + d\*x)) - (a + b\*ArcTan[c + d\*x])^3/(2\*d\*e^3) - (a + b\*ArcTan[c + d\*x])^3/(2\*d\*e^3\*(c + d\*x)^2) + (3\*b^2\*(a + b\*ArcTan[c + d\*x])\*Log[2 - 2/(1 - I\*(c + d\*x))])/(d\*e^3) - (((3\*I)/2)\*b^3\*PolyLog[2, -1 + 2/(1 - I\*(c + d\*x))])/(d\*e^3)

**Rubi [A]** time = 0.31859, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {5043, 12, 4852, 4918, 4924, 4868, 2447, 4884}

$$-\frac{3ib^3 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)}{2de^3} + \frac{3b^2 \log\left(2 - \frac{2}{1-i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{de^3} - \frac{3b(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)} - \frac{3ib(a + b \tan^{-1}(c + dx))}{de^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^3/(c\*e + d\*e\*x)^3, x]

[Out] (((-3\*I)/2)\*b\*(a + b\*ArcTan[c + d\*x])^2)/(d\*e^3) - (3\*b\*(a + b\*ArcTan[c + d\*x])^2)/(2\*d\*e^3\*(c + d\*x)) - (a + b\*ArcTan[c + d\*x])^3/(2\*d\*e^3) - (a + b\*ArcTan[c + d\*x])^3/(2\*d\*e^3\*(c + d\*x)^2) + (3\*b^2\*(a + b\*ArcTan[c + d\*x])\*Log[2 - 2/(1 - I\*(c + d\*x))])/(d\*e^3) - (((3\*I)/2)\*b^3\*PolyLog[2, -1 + 2/(1 - I\*(c + d\*x))])/(d\*e^3)

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^p\_.\*((e\_.) + (f\_.)\*(x\_.))^m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_.)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_.)\*(v\_.) /; FreeQ[b, x]]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^p\_.\*((d\_.)\*(x\_.))^m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4918

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^p\_.\*((f\_.)\*(x\_.))^m\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTan[c\*x])^p, x],

$x] - \text{Dist}[e/(d*f^2), \text{Int}[(f*x)^{(m+2)}*(a + b*\text{ArcTan}[c*x])^p/(d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1]$

#### Rule 4924

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p/((x)*(d + (e)*(x)^2)), x\_Symbol] :> -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^{p+1})/(b*d*(p+1)), x] + \text{Dist}[I/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^p/(x*(I + c*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[e, c^2*d] \ \&\& \ \text{GtQ}[p, 0]$

#### Rule 4868

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p/((x)*(d + (e)*(x))), x\_Symbol] :> \text{Simp}[(a + b*\text{ArcTan}[c*x])^p*\text{Log}[2 - 2/(1 + (e*x)/d)]/d, x] - \text{Dist}[(b*c*p)/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^{p-1}*\text{Log}[2 - 2/(1 + (e*x)/d)]/(1 + c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[c^2*d^2 + e^2, 0]$

#### Rule 2447

$\text{Int}[\text{Log}[u]*(Pq)^m, x\_Symbol] :> \text{With}\{C = \text{FullSimplify}[(Pq^m*(1 - u))/D[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x] /; \text{IntegerQ}[m] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{RationalFunctionQ}[u, x] \ \&\& \ \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

#### Rule 4884

$\text{Int}[(a + \text{ArcTan}[c*x]*b)^p/((d + (e)*(x)^2), x\_Symbol] :> \text{Simp}[(a + b*\text{ArcTan}[c*x])^{p+1}/(b*c*d*(p+1)), x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[e, c^2*d] \ \&\& \ \text{NeQ}[p, -1]$

#### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan^{-1}(c + dx))^3}{(ce + dex)^3} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{e^3 x^3} dx, x, c + dx\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{x^3} dx, x, c + dx\right)}{de^3} \\
 &= -\frac{(a + b \tan^{-1}(c + dx))^3}{2de^3(c + dx)^2} + \frac{(3b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^2(1+x^2)} dx, x, c + dx\right)}{2de^3} \\
 &= -\frac{(a + b \tan^{-1}(c + dx))^3}{2de^3(c + dx)^2} + \frac{(3b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^2} dx, x, c + dx\right)}{2de^3} - \frac{(3b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx\right)}{2de^3} \\
 &= -\frac{3b(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3(c + dx)^2} + \frac{(3b^2) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx\right)}{2de^3} \\
 &= -\frac{3ib(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{3b(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3(c + dx)^2} \\
 &= -\frac{3ib(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{3b(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3(c + dx)^2} \\
 &= -\frac{3ib(a + b \tan^{-1}(c + dx))^2}{2de^3} - \frac{3b(a + b \tan^{-1}(c + dx))^2}{2de^3(c + dx)} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3} - \frac{(a + b \tan^{-1}(c + dx))^3}{2de^3(c + dx)^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.225925, size = 225, normalized size = 1.25

$$\frac{3b^3(c + dx) \left( i(c + dx) \left( \tan^{-1}(c + dx)^2 + \text{PolyLog}\left(2, e^{2i \tan^{-1}(c+dx)}\right) \right) + \tan^{-1}(c + dx)^2 - 2(c + dx) \tan^{-1}(c + dx) \log\left(1 + \frac{2i(c + dx) \tan^{-1}(c + dx)}{1 + (c + dx)^2}\right) \right)}{2de^3(c + dx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -(a^3 + b^3*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*a^2*b*(c + d*x + (1 + (c + d*x)^2)*ArcTan[c + d*x]) + 3*a*b^2*(2*(c + d*x)*ArcTan[c + d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 2*(c + d*x)^2*Log[(c + d*x)/Sqrt[1 + (c + d*x)^2]]) + 3*b^3*(c + d*x)*(ArcTan[c + d*x]^2 - 2*(c + d*x)*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + I*(c + d*x)*(ArcTan[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c + d*x])]))/(2*d*e^3*(c + d*x)^2)
```

**Maple [B]** time = 0.138, size = 631, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] -3/2/d*a*b^2/e^3*arctan(d*x+c)^2-3/2/d*a*b^2/e^3*ln(1+(d*x+c)^2)+3/d*a*b^2/e^3*ln(d*x+c)-3/2/d*a^2*b/e^3*arctan(d*x+c)-3/4*I/d*b^3/e^3*dilog(1/2*I*(d*x+c))
```

$$\begin{aligned} & x+c-I)) - 3/8 * I/d*b^3/e^3 * \ln(d*x+c+I)^2 + 3/2 * I/d*b^3/e^3 * \operatorname{dilog}(1+I*(d*x+c)) + 3/ \\ & 8 * I/d*b^3/e^3 * \ln(d*x+c-I)^2 + 3/4 * I/d*b^3/e^3 * \operatorname{dilog}(-1/2 * I*(d*x+c+I)) - 3/2 * I/d \\ & *b^3/e^3 * \operatorname{dilog}(1-I*(d*x+c)) - 3/2/d*a^2*b/e^3/(d*x+c) - 1/2/d*b^3/e^3/(d*x+c)^2 \\ & * \arctan(d*x+c)^3 - 3/2/d*b^3/e^3 * \arctan(d*x+c)^2/(d*x+c) - 3/2/d*b^3/e^3 * \arctan \\ & (d*x+c) * \ln(1+(d*x+c)^2) + 3/d*b^3/e^3 * \ln(d*x+c) * \arctan(d*x+c) - 3/4 * I/d*b^3/e^3 \\ & * \ln(d*x+c+I) * \ln(1/2 * I*(d*x+c-I)) - 3/2/d*a^2*b/e^3/(d*x+c)^2 * \arctan(d*x+c) + 3/ \\ & 4 * I/d*b^3/e^3 * \ln(d*x+c-I) * \ln(-1/2 * I*(d*x+c+I)) + 3/4 * I/d*b^3/e^3 * \ln(1+(d*x+c) \\ & ^2) * \ln(d*x+c+I) + 3/2 * I/d*b^3/e^3 * \ln(d*x+c) * \ln(1+I*(d*x+c)) - 3/4 * I/d*b^3/e^3 * \ln \\ & (1+(d*x+c)^2) * \ln(d*x+c-I) - 3/2 * I/d*b^3/e^3 * \ln(d*x+c) * \ln(1-I*(d*x+c)) - 3/2/d* \\ & a*b^2/e^3/(d*x+c)^2 * \arctan(d*x+c)^2 - 3/d*a*b^2/e^3 * \arctan(d*x+c)/(d*x+c) - 1/2 \\ & /d*b^3/e^3 * \arctan(d*x+c)^3 - 1/2/d*a^3/e^3/(d*x+c)^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -3/2 * (d * (1/(d^3 * e^3 * x + c * d^2 * e^3) + \arctan((d^2 * x + c * d)/d)/(d^2 * e^3)) + a \\ & \operatorname{rctan}(d * x + c)/(d^3 * e^3 * x^2 + 2 * c * d^2 * e^3 * x + c^2 * d * e^3)) * a^2 * b - 3/2 * (2 * d * \\ & (1/(d^3 * e^3 * x + c * d^2 * e^3) + \arctan((d^2 * x + c * d)/d)/(d^2 * e^3)) * \arctan(d * x \\ & + c) - (\arctan(d * x + c)^2 - \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1) + 2 * \log(d * x + \\ & c))/(d * e^3)) * a * b^2 - 3/2 * a * b^2 * \arctan(d * x + c)^2/(d^3 * e^3 * x^2 + 2 * c * d^2 * e^3 \\ & * x + c^2 * d * e^3) - 1/32 * (8 * (d^2 * x^2 + 2 * c * d * x + c^2 + 1) * \arctan(d * x + c)^3 + \\ & 12 * (d * x + c) * \arctan(d * x + c)^2 - 3 * (d * x + c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + \\ & 1)^2 - 32 * (d^3 * e^3 * x^2 + 2 * c * d^2 * e^3 * x + c^2 * d * e^3) * \operatorname{integrate}(1/32 * (16 * (d^ \\ & 2 * x^2 + 2 * c * d * x + c^2 + 1) * \arctan(d * x + c)^3 + 12 * (d^3 * x^3 + 3 * c * d^2 * x^2 + \\ & c^3 + (3 * c^2 + 1) * d * x + c) * \arctan(d * x + c)^2 + 3 * (d^3 * x^3 + 3 * c * d^2 * x^2 + c \\ & ^3 + (3 * c^2 + 1) * d * x + c) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1)^2 + 24 * (d^2 * x^2 \\ & + 2 * c * d * x + c^2) * \arctan(d * x + c) - 12 * (d^3 * x^3 + 3 * c * d^2 * x^2 + 3 * c^2 * d * x + \\ & c^3) * \log(d^2 * x^2 + 2 * c * d * x + c^2 + 1))/(d^5 * e^3 * x^5 + 5 * c * d^4 * e^3 * x^4 + (10 \\ & * c^2 + 1) * d^3 * e^3 * x^3 + (10 * c^3 + 3 * c) * d^2 * e^3 * x^2 + (5 * c^4 + 3 * c^2) * d * e^3 * \\ & x + (c^5 + c^3) * e^3), x)) * b^3/(d^3 * e^3 * x^2 + 2 * c * d^2 * e^3 * x + c^2 * d * e^3) - 1 \\ & /2 * a^3/(d^3 * e^3 * x^2 + 2 * c * d^2 * e^3 * x + c^2 * d * e^3) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3 \arctan(dx+c)^3 + 3ab^2 \arctan(dx+c)^2 + 3a^2b \arctan(dx+c) + a^3}{d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(d\*e\*x+c\*e)^3,x, algorithm="fricas")

[Out] 
$$\operatorname{integral}((b^3 * \arctan(d * x + c)^3 + 3 * a * b^2 * \arctan(d * x + c)^2 + 3 * a^2 * b * \arctan(d * x + c) + a^3)/(d^3 * e^3 * x^3 + 3 * c * d^2 * e^3 * x^2 + 3 * c^2 * d * e^3 * x + c^3 * e^3), x)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx + \int \frac{b^3 \operatorname{atan}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx + \int \frac{3ab^2 \operatorname{atan}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx + \int \frac{3a^2b \operatorname{atan}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*atan(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(3*a*b**2*atan(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3), x) + Integral(3*a**2*b*atan(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3), x))/e**3
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^3}{(dex + ce)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

$$3.20 \quad \int \frac{(a+b \tan^{-1}(c+dx))^3}{(ce+dex)^4} dx$$

**Optimal.** Leaf size=287

$$\frac{ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de^4} - \frac{b^3 \text{PolyLog}\left(3, -1 + \frac{2}{1-i(c+dx)}\right)}{2de^4} - \frac{b^2(a+b \tan^{-1}(c+dx))}{de^4(c+dx)} - b$$

```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```

**Rubi [A]** time = 0.499924, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$ , Rules used = {5043, 12, 4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$\frac{ib^2 \text{PolyLog}\left(2, -1 + \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{de^4} - \frac{b^3 \text{PolyLog}\left(3, -1 + \frac{2}{1-i(c+dx)}\right)}{2de^4} - \frac{b^2(a+b \tan^{-1}(c+dx))}{de^4(c+dx)} - b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4, x]
```

```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```

#### Rule 5043

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 4918

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_.))^(m\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTan[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^ (p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 4884

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

Rule 4924

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((x\_.)\*((d\_.) + (e\_.)\*(x\_)^2)), x\_Symbol] := -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*d\*(p + 1)), x] + Dist[I/d, Int[(a + b\*ArcTan[c\*x])^p/(x\*(I + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && GtQ[p, 0]

Rule 4868

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((x\_.)\*((d\_.) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTan[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)]]/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

Rule 4992

Int[(Log[u]\*((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(I\*(a + b\*ArcTan[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] - Dist[(b\*p\*I)/2, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && EqQ[(1 - u)^2 - (1 - (2\*I)/(I + c\*x))^2, 0]

Rule 6610



Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^3}{(ce + dex)^4} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{e^4 x^4} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^3}{x^4} dx, x, c + dx\right)}{de^4} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{3de^4(c + dx)^3} + \frac{b \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^3(1+x^2)} dx, x, c + dx\right)}{de^4} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{3de^4(c + dx)^3} + \frac{b \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x^3} dx, x, c + dx\right)}{de^4} - \frac{b \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x(1+x^2)} dx, x, c + dx\right)}{de^4} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^3}{3de^4} - \frac{(a + b \tan^{-1}(c + dx))^3}{3de^4(c + dx)^3} - \frac{(ib) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))^2}{x(1+x^2)} dx, x, c + dx\right)}{de^4} \\
&= -\frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^3}{3de^4} - \frac{(a + b \tan^{-1}(c + dx))^3}{3de^4(c + dx)^3} - \frac{b(a + b \tan^{-1}(c + dx))^2}{de^4(c + dx)} \\
&= -\frac{b^2(a + b \tan^{-1}(c + dx))}{de^4(c + dx)} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^3}{3de^4} \\
&= -\frac{b^2(a + b \tan^{-1}(c + dx))}{de^4(c + dx)} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^3}{3de^4} \\
&= -\frac{b^2(a + b \tan^{-1}(c + dx))}{de^4(c + dx)} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^3}{3de^4} \\
&= -\frac{b^2(a + b \tan^{-1}(c + dx))}{de^4(c + dx)} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tan^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{i(a + b \tan^{-1}(c + dx))^3}{3de^4}
\end{aligned}$$

**Mathematica [A]** time = 0.823884, size = 360, normalized size = 1.25

$$24ab^2 \left( i \text{PolyLog}\left(2, e^{2i \tan^{-1}(c+dx)}\right) - \frac{(c+dx)^2 + \tan^{-1}(c+dx)^2}{(c+dx)^3} + \tan^{-1}(c+dx) \left( -\frac{1}{(c+dx)^2} + i \tan^{-1}(c+dx) - 2 \log\left(1 - e^{2i \tan^{-1}(c+dx)}\right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^3/(c\*e + d\*e\*x)^4, x]

[Out] ((-8\*a^3)/(c + d\*x)^3 - (12\*a^2\*b)/(c + d\*x)^2 - (24\*a^2\*b\*ArcTan[c + d\*x])/(c + d\*x)^3 - 24\*a^2\*b\*Log[c + d\*x] + 12\*a^2\*b\*Log[1 + c^2 + 2\*c\*d\*x + d^2\*x^2] + 24\*a\*b^2\*(-(((c + d\*x)^2 + ArcTan[c + d\*x]^2)/(c + d\*x)^3) + ArcTan[c + d\*x]\*(-1 - (c + d\*x)^(-2)) + I\*ArcTan[c + d\*x] - 2\*Log[1 - E^((2\*I)\*ArcTan[c + d\*x])]) + I\*PolyLog[2, E^((2\*I)\*ArcTan[c + d\*x])]) + b^3\*(I\*Pi^3 - (24\*ArcTan[c + d\*x])/(c + d\*x) - 12\*ArcTan[c + d\*x]^2 - (12\*ArcTan[c + d\*x]^2)/(c + d\*x)^2 - (8\*I)\*ArcTan[c + d\*x]^3 - (8\*ArcTan[c + d\*x]^3)/(c + d\*x))

$$\begin{aligned} &^3 - 24 \operatorname{ArcTan}[c + d*x]^2 \operatorname{Log}[1 - E^{(-2*I)*\operatorname{ArcTan}[c + d*x]}] + 24 \operatorname{Log}[(c + \\ & d*x)/\operatorname{Sqrt}[1 + (c + d*x)^2]] - (24*I)*\operatorname{ArcTan}[c + d*x]*\operatorname{PolyLog}[2, E^{(-2*I)* \\ & \operatorname{ArcTan}[c + d*x]}] - 12*\operatorname{PolyLog}[3, E^{(-2*I)*\operatorname{ArcTan}[c + d*x]}])/(24*d*e^4) \end{aligned}$$

**Maple [C]** time = 0.864, size = 7083, normalized size = 24.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x)`

[Out] result too large to display

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3 \arctan(dx + c)^3 + 3ab^2 \arctan(dx + c)^2 + 3a^2b \arctan(dx + c) + a^3}{d^4 e^4 x^4 + 4cd^3 e^4 x^3 + 6c^2 d^2 e^4 x^2 + 4c^3 d e^4 x + c^4 e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")`

[Out] `integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arctan(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{b^3 \operatorname{atan}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{3ab^2 \operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{3a^2b \operatorname{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atan(d*x+c))^3/(d*e*x+c*e)^4,x)`

[Out] `(Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(b**3*atan(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atan(c + d*x)**`

```
2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) +
Integral(3*a**2*b*atan(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c
*d**3*x**3 + d**4*x**4), x))/e**4
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^3}{(dex + ce)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

$$3.21 \quad \int \frac{\tan^{-1}(1+x)}{2+2x} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{4}i\text{PolyLog}(2, -i(x+1)) - \frac{1}{4}i\text{PolyLog}(2, i(x+1))$$

[Out] (I/4)\*PolyLog[2, (-I)\*(1 + x)] - (I/4)\*PolyLog[2, I\*(1 + x)]

**Rubi [A]** time = 0.0375427, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5043, 12, 4848, 2391}

$$\frac{1}{4}i\text{PolyLog}(2, -i(x+1)) - \frac{1}{4}i\text{PolyLog}(2, i(x+1))$$

Antiderivative was successfully verified.

[In] Int[ArcTan[1 + x]/(2 + 2\*x), x]

[Out] (I/4)\*PolyLog[2, (-I)\*(1 + x)] - (I/4)\*PolyLog[2, I\*(1 + x)]

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 4848

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 + I\*c\*x]/x, x], x) /; FreeQ[{a, b, c}, x]

#### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(1+x)}{2+2x} dx &= \text{Subst} \left( \int \frac{\tan^{-1}(x)}{2x} dx, x, 1+x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{\tan^{-1}(x)}{x} dx, x, 1+x \right) \\ &= \frac{1}{4}i \text{Subst} \left( \int \frac{\log(1-ix)}{x} dx, x, 1+x \right) - \frac{1}{4}i \text{Subst} \left( \int \frac{\log(1+ix)}{x} dx, x, 1+x \right) \\ &= \frac{1}{4}i\text{Li}_2(-i(1+x)) - \frac{1}{4}i\text{Li}_2(i(1+x)) \end{aligned}$$

**Mathematica [A]** time = 0.0034877, size = 31, normalized size = 1.

$$\frac{1}{4}i\text{PolyLog}(2, -i(x+1)) - \frac{1}{4}i\text{PolyLog}(2, i(x+1))$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[1 + x]/(2 + 2\*x), x]

[Out] (I/4)\*PolyLog[2, (-I)\*(1 + x)] - (I/4)\*PolyLog[2, I\*(1 + x)]

**Maple [B]** time = 0.046, size = 68, normalized size = 2.2

$$\frac{\ln(x+1)\arctan(x+1)}{2} + \frac{i}{4}\ln(x+1)\ln(1+i(x+1)) - \frac{i}{4}\ln(x+1)\ln(1-i(x+1)) + \frac{i}{4}\text{dilog}(1+i(x+1)) - \frac{i}{4}\text{dilog}(1-i(x+1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(x+1)/(2+2\*x), x)

[Out] 1/2\*ln(x+1)\*arctan(x+1)+1/4\*I\*ln(x+1)\*ln(1+I\*(x+1))-1/4\*I\*ln(x+1)\*ln(1-I\*(x+1))+1/4\*I\*dilog(1+I\*(x+1))-1/4\*I\*dilog(1-I\*(x+1))

**Maxima [B]** time = 1.62231, size = 59, normalized size = 1.9

$$-\frac{1}{4}\arctan(x+1,0)\log(x^2+2x+2) + \frac{1}{2}\arctan(x+1)\log(|x+1|) - \frac{1}{4}i\text{Li}_2(ix+i+1) + \frac{1}{4}i\text{Li}_2(-ix-i+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(1+x)/(2+2\*x), x, algorithm="maxima")

[Out] -1/4\*arctan2(x + 1, 0)\*log(x^2 + 2\*x + 2) + 1/2\*arctan(x + 1)\*log(abs(x + 1)) - 1/4\*I\*dilog(I\*x + I + 1) + 1/4\*I\*dilog(-I\*x - I + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(x+1)}{2(x+1)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(1+x)/(2+2\*x), x, algorithm="fricas")

[Out] integral(1/2\*arctan(x + 1)/(x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\text{atan}(x+1)}{x+1} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(1+x)/(2+2\*x),x)

[Out] Integral(atan(x + 1)/(x + 1), x)/2

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x+1)}{2(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(1+x)/(2+2\*x),x, algorithm="giac")

[Out] integrate(1/2\*arctan(x + 1)/(x + 1), x)

$$3.22 \quad \int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$$

**Optimal.** Leaf size=41

$$\frac{i\text{PolyLog}(2, -i(a+bx))}{2d} - \frac{i\text{PolyLog}(2, i(a+bx))}{2d}$$

[Out] ((I/2)\*PolyLog[2, (-I)\*(a + b\*x)])/d - ((I/2)\*PolyLog[2, I\*(a + b\*x)])/d

**Rubi [A]** time = 0.0452878, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {5043, 12, 4848, 2391}

$$\frac{i\text{PolyLog}(2, -i(a+bx))}{2d} - \frac{i\text{PolyLog}(2, i(a+bx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/((a\*d)/b + d\*x), x]

[Out] ((I/2)\*PolyLog[2, (-I)\*(a + b\*x)])/d - ((I/2)\*PolyLog[2, I\*(a + b\*x)])/d

#### Rule 5043

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4848

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 + I\*c\*x]/x, x], x) /; FreeQ[{a, b, c}, x]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a + bx)}{\frac{ad}{b} + dx} dx &= \frac{\text{Subst}\left(\int \frac{b \tan^{-1}(x)}{dx} dx, x, a + bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x} dx, x, a + bx\right)}{d} \\
&= \frac{i \text{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, a + bx\right)}{2d} - \frac{i \text{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, a + bx\right)}{2d} \\
&= \frac{i \text{Li}_2(-i(a + bx))}{2d} - \frac{i \text{Li}_2(i(a + bx))}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.0074928, size = 34, normalized size = 0.83

$$\frac{i(\text{PolyLog}(2, -i(a + bx)) - \text{PolyLog}(2, i(a + bx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/((a\*d)/b + d\*x), x]

[Out] ((I/2)\*(PolyLog[2, (-I)\*(a + b\*x)] - PolyLog[2, I\*(a + b\*x)]))/d

**Maple [B]** time = 0.046, size = 98, normalized size = 2.4

$$\frac{\ln(bx + a) \arctan(bx + a)}{d} + \frac{\frac{i}{2} \ln(bx + a) \ln(1 + i(bx + a))}{d} - \frac{\frac{i}{2} \ln(bx + a) \ln(1 - i(bx + a))}{d} + \frac{\frac{i}{2} \text{dilog}(1 + i(bx + a))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(a\*d/b+d\*x), x)

[Out] 1/d\*ln(b\*x+a)\*arctan(b\*x+a)+1/2\*I/d\*ln(b\*x+a)\*ln(1+I\*(b\*x+a))-1/2\*I/d\*ln(b\*x+a)\*ln(1-I\*(b\*x+a))+1/2\*I/d\*dilog(1+I\*(b\*x+a))-1/2\*I/d\*dilog(1-I\*(b\*x+a))

**Maxima [B]** time = 1.68232, size = 166, normalized size = 4.05

$$\frac{\arctan(bx + a) \log\left(dx + \frac{ad}{b}\right)}{d} - \frac{\arctan\left(\frac{b^2x+ab}{b}\right) \log\left(dx + \frac{ad}{b}\right)}{d} - \frac{\arctan(bx + a, 0) \log\left(b^2x^2 + 2abx + a^2 + 1\right) - 2 \arctan(bx + a) \log\left(\frac{b^2x+ab}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(a\*d/b+d\*x), x, algorithm="maxima")

[Out] arctan(b\*x + a)\*log(d\*x + a\*d/b)/d - arctan((b^2\*x + a\*b)/b)\*log(d\*x + a\*d/b)/d - 1/2\*(arctan2(b\*x + a, 0)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1) - 2\*arctan(b\*x + a)\*log(abs(b\*x + a)) + I\*dilog(I\*b\*x + I\*a + 1) - I\*dilog(-I\*b\*x - I\*a + 1))/d



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arctan(bx + a)}{bdx + ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(a\*d/b+d\*x),x, algorithm="fricas")

[Out] integral(b\*arctan(b\*x + a)/(b\*d\*x + a\*d), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{\text{atan}(a+bx)}{a+bx} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(a\*d/b+d\*x),x)

[Out] b\*Integral(atan(a + b\*x)/(a + b\*x), x)/d

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{dx + \frac{ad}{b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(a\*d/b+d\*x),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(d\*x + a\*d/b), x)

### 3.23 $\int (a + bx)^2 \sqrt{\tan^{-1}(a + bx)} dx$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left((a + bx)^2 \sqrt{\tan^{-1}(a + bx)}, x\right)$$

[Out] Unintegrable[(a + b\*x)^2\*Sqrt[ArcTan[a + b\*x]], x]

**Rubi [A]** time = 0.0165432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (a + bx)^2 \sqrt{\tan^{-1}(a + bx)} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*x)^2\*Sqrt[ArcTan[a + b\*x]], x]

[Out] Defer[Int] [(a + b\*x)^2\*Sqrt[ArcTan[a + b\*x]], x]

Rubi steps

$$\int (a + bx)^2 \sqrt{\tan^{-1}(a + bx)} dx = \int (a + bx)^2 \sqrt{\tan^{-1}(a + bx)} dx$$

**Mathematica [A]** time = 5.40776, size = 0, normalized size = 0.

$$\int (a + bx)^2 \sqrt{\tan^{-1}(a + bx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*x)^2\*Sqrt[ArcTan[a + b\*x]], x]

[Out] Integrate[(a + b\*x)^2\*Sqrt[ArcTan[a + b\*x]], x]

**Maple [A]** time = 0.472, size = 0, normalized size = 0.

$$\int (bx + a)^2 \sqrt{\arctan(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*arctan(b\*x+a)^(1/2), x)

[Out] int((b\*x+a)^2\*arctan(b\*x+a)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (a + bx)^2 \sqrt{\operatorname{atan}(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*atan(b\*x+a)\*\*(1/2),x)

[Out] Integral((a + b\*x)\*\*2\*sqrt(atan(a + b\*x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 \sqrt{\operatorname{arctan}(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)^(1/2),x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*sqrt(arctan(b\*x + a)), x)

### 3.24 $\int (e + fx)^3 (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=233

$$\frac{(e + fx)^4 (a + b \tan^{-1}(c + dx))}{4f} - \frac{bfx(- (1 - 6c^2)f^2 - 12cdef + 6d^2e^2)}{4d^3} - \frac{b(-6(1 - c^2)d^2e^2f^2 + 4c(3 - c^2)def^3 + (c^4 - 4cd^2e^2)f^2)}{4d^4}$$

[Out]  $-(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) - (b*f^2*(d*e - c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d*x)^2])/(2*d^4)$

**Rubi [A]** time = 0.381339, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e + fx)^4 (a + b \tan^{-1}(c + dx))}{4f} - \frac{bfx(- (1 - 6c^2)f^2 - 12cdef + 6d^2e^2)}{4d^3} - \frac{b(-6(1 - c^2)d^2e^2f^2 + 4c(3 - c^2)def^3 + (c^4 - 4cd^2e^2)f^2)}{4d^4}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^3\*(a + b\*ArcTan[c + d\*x]),x]

[Out]  $-(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) - (b*f^2*(d*e - c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d*x)^2])/(2*d^4)$

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4862

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)/((a\_.) + (c\_.)\*(x\_.)^2), x\_Symbol] :> Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

#### Rule 635

Int[((d\_.) + (e\_.)\*(x\_.))/((a\_.) + (c\_.)\*(x\_.)^2), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}

} , x] && !NiceSqrtQ[-(a\*c)]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned}
 \int (e + fx)^3 (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^3 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{(e + fx)^4 (a + b \tan^{-1}(c + dx))}{4f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^4}{1+x^2} dx, x, c + dx\right)}{4f} \\
 &= \frac{(e + fx)^4 (a + b \tan^{-1}(c + dx))}{4f} - \frac{b \text{Subst}\left(\int \left(\frac{f^2(6d^2e^2 - 12cdef - (1-6c^2)f^2)}{d^4} + \frac{4f^3(dx)}{d}\right) dx, x, c + dx\right)}{4f} \\
 &= -\frac{bf(6d^2e^2 - 12cdef - (1-6c^2)f^2)x}{4d^3} - \frac{bf^2(de - cf)(c + dx)^2}{2d^4} - \frac{bf^3(c + dx)^3}{12d^4} \\
 &= -\frac{bf(6d^2e^2 - 12cdef - (1-6c^2)f^2)x}{4d^3} - \frac{bf^2(de - cf)(c + dx)^2}{2d^4} - \frac{bf^3(c + dx)^3}{12d^4} \\
 &= -\frac{bf(6d^2e^2 - 12cdef - (1-6c^2)f^2)x}{4d^3} - \frac{bf^2(de - cf)(c + dx)^2}{2d^4} - \frac{bf^3(c + dx)^3}{12d^4}
 \end{aligned}$$

**Mathematica [C]** time = 0.252758, size = 157, normalized size = 0.67

$$\frac{(e + fx)^4 (a + b \tan^{-1}(c + dx)) - \frac{b(6df^2x((6c^2-1)f^2-12cdef+6d^2e^2)+12f^3(c+dx)^2(de-cf)-3i(de-(c-i)f)^4 \log(-c-dx+i)+3i(de-(c+i)f)^4 \log(c+dx+i))}{6d^4}}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*(a + b\*ArcTan[c + d\*x]), x]

[Out] ((e + f\*x)^4\*(a + b\*ArcTan[c + d\*x]) - (b\*(6\*d\*f^2\*(6\*d^2\*e^2 - 12\*c\*d\*e\*f + (-1 + 6\*c^2)\*f^2)\*x + 12\*f^3\*(d\*e - c\*f)\*(c + d\*x)^2 + 2\*f^4\*(c + d\*x)^3 - (3\*I)\*(d\*e - (-I + c)\*f)^4\*Log[I - c - d\*x] + (3\*I)\*(d\*e - (I + c)\*f)^4\*Log[I + c + d\*x]))/(6\*d^4))/(4\*f)

**Maple [B]** time = 0.042, size = 494, normalized size = 2.1

$$-\frac{bf^3 \arctan(dx + c)}{4d^4} - \frac{b \ln(1 + (dx + c)^2) e^3}{2d} + \frac{bf^3 \arctan(dx + c) x^4}{4} + \arctan(dx + c) x b e^3 + \frac{bf^3 x}{4d^3} + \frac{3afx^2 e^2}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*(a+b*arctan(d*x+c)),x)`

[Out] 
$$-1/4/d^4*b*f^3*arctan(d*x+c)-1/2/d*b*ln(1+(d*x+c)^2)*e^3+1/4*b*f^3*arctan(d*x+c)*x^4+arctan(d*x+c)*x*b*e^3+1/4*b/d^3*f^3*x+3/2*a*f*x^2*e^2+a*f^2*x^3*e-13/12/d^4*b*f^3*c^3+1/4/d^4*b*f^3*c-1/12/d*b*f^3*x^3+1/4*a/f*e^4+b*f^2*arctan(d*x+c)*e*x^3-1/2/d*b*f^2*e*x^2-3/4*b/d^3*f^3*c^2*x-3/2*b/d*f*e^2*x+3/2/d^2*b*f*arctan(d*x+c)*e^2+1/2/d^4*b*f^3*ln(1+(d*x+c)^2)*c^3+1/4/d^2*b*f^3*x^2*c+1/2/d^3*b*f^2*ln(1+(d*x+c)^2)*e+3/2*b*f*arctan(d*x+c)*e^2*x^2-1/4/d^4*b*f^3*arctan(d*x+c)*c^4+1/d*arctan(d*x+c)*b*c*e^3-1/2/d^4*b*f^3*ln(1+(d*x+c)^2)*c+5/2/d^3*b*f^2*c^2*e-3/2/d^2*b*f*c*e^2+a*x*e^3+1/4*a*f^3*x^4+3/2/d^4*b*f^3*arctan(d*x+c)*c^2-3/2/d^2*b*f*arctan(d*x+c)*e^2*c^2-3/d^3*b*f^2*arctan(d*x+c)*c*e-3/2/d^3*b*f^2*ln(1+(d*x+c)^2)*c^2*e+3/2/d^2*b*f*ln(1+(d*x+c)^2)*c*e^2+2*b/d^2*f^2*c*e*x+1/d^3*b*f^2*arctan(d*x+c)*e*c^3$$

**Maxima [A]** time = 1.54294, size = 467, normalized size = 2.

$$\frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2 + \frac{3}{2}\left(x^2 \arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c \log(d^2x^2 + 2cdx + c^2 + 1)}{d^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arctan(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e^3/d$$

**Fricas [A]** time = 1.62432, size = 667, normalized size = 2.86

$$3ad^4f^3x^4 + (12ad^4ef^2 - bd^3f^3)x^3 + 3(6ad^4e^2f - 2bd^3ef^2 + bcd^2f^3)x^2 + 3(4ad^4e^3 - 6bd^3e^2f + 8bcd^2ef^2 - (3bc^2 - b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")`

[Out] 
$$1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 - b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*f - 2*b*d^3*e*f^2 + b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 - 6*b*d^3*e^2*f + 8*b*c*d^2*e*f^2 - (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x + 4*b*c*d^3*e^3 - 6*(b*c^2 - b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2 + b)*f^3)*arctan(d*x + c) - 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b)*d*e*f^2 - (b*c^3 - b*c)*f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4$$

**Sympy [A]** time = 10.2317, size = 627, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*(a+b\*atan(d\*x+c)),x)

[Out] Piecewise((a\*e\*\*3\*x + 3\*a\*e\*\*2\*f\*x\*\*2/2 + a\*e\*f\*\*2\*x\*\*3 + a\*f\*\*3\*x\*\*4/4 - b\*c\*\*4\*f\*\*3\*atan(c + d\*x)/(4\*d\*\*4) + b\*c\*\*3\*e\*f\*\*2\*atan(c + d\*x)/d\*\*3 + b\*c\*\*3\*f\*\*3\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d\*\*4) - 3\*b\*c\*\*2\*e\*\*2\*f\*atan(c + d\*x)/(2\*d\*\*2) - 3\*b\*c\*\*2\*e\*f\*\*2\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d\*\*3) - 3\*b\*c\*\*2\*f\*\*3\*x/(4\*d\*\*3) + 3\*b\*c\*\*2\*f\*\*3\*atan(c + d\*x)/(2\*d\*\*4) + b\*c\*e\*\*3\*atan(c + d\*x)/d + 3\*b\*c\*e\*\*2\*f\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d\*\*2) + 2\*b\*c\*e\*f\*\*2\*x/d\*\*2 + b\*c\*f\*\*3\*x\*\*2/(4\*d\*\*2) - 3\*b\*c\*e\*f\*\*2\*atan(c + d\*x)/d\*\*3 - b\*c\*f\*\*3\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d\*\*4) + b\*e\*\*3\*x\*atan(c + d\*x) + 3\*b\*e\*\*2\*f\*x\*\*2\*atan(c + d\*x)/2 + b\*e\*f\*\*2\*x\*\*3\*atan(c + d\*x) + b\*f\*\*3\*x\*\*4\*atan(c + d\*x)/4 - b\*e\*\*3\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d) - 3\*b\*e\*\*2\*f\*x/(2\*d) - b\*e\*f\*\*2\*x\*\*2/(2\*d) - b\*f\*\*3\*x\*\*3/(12\*d) + 3\*b\*e\*\*2\*f\*atan(c + d\*x)/(2\*d\*\*2) + b\*e\*f\*\*2\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d\*\*3) + b\*f\*\*3\*x/(4\*d\*\*3) - b\*f\*\*3\*atan(c + d\*x)/(4\*d\*\*4), Ne(d, 0)), ((a + b\*atan(c))\*(e\*\*3\*x + 3\*e\*\*2\*f\*x\*\*2/2 + e\*f\*\*2\*x\*\*3 + f\*\*3\*x\*\*4/4), True))

**Giac [B]** time = 3.44363, size = 961, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arctan(d\*x+c)),x, algorithm="giac")

[Out] 1/24\*(6\*b\*d^4\*f^3\*x^4\*arctan(d\*x + c) + 6\*a\*d^4\*f^3\*x^4 + 24\*b\*d^4\*f^2\*x^3\*arctan(d\*x + c)\*e + 24\*a\*d^4\*f^2\*x^3\*e - 2\*b\*d^3\*f^3\*x^3 + 36\*b\*d^4\*f\*x^2\*a\*arctan(d\*x + c)\*e^2 - 3\*pi\*b\*c^4\*f^3\*sgn(d\*x + c) + 12\*pi\*b\*c^3\*d\*f^2\*e\*sgn(d\*x + c) + 3\*pi\*b\*c^4\*f^3 + 6\*b\*c\*d^2\*f^3\*x^2 + 6\*b\*c^4\*f^3\*arctan(1/(d\*x + c)) + 36\*a\*d^4\*f\*x^2\*e^2 - 12\*pi\*b\*c^3\*d\*f^2\*e - 12\*b\*d^3\*f^2\*x^2\*e - 24\*b\*c^3\*d\*f^2\*arctan(1/(d\*x + c))\*e - 18\*pi\*b\*c^2\*d^2\*f\*e^2\*sgn(d\*x + c) - 18\*b\*c^2\*d\*f^3\*x + 24\*b\*d^4\*x\*arctan(d\*x + c)\*e^3 + 18\*pi\*b\*c^2\*d^2\*f\*e^2 + 36\*b\*c^2\*d^2\*f\*arctan(1/(d\*x + c))\*e^2 + 48\*b\*c\*d^2\*f^2\*x\*e + 12\*b\*c^3\*f^3\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) - 36\*b\*c^2\*d\*f^2\*e\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) + 18\*pi\*b\*c^2\*f^3\*sgn(d\*x + c) - 24\*pi\*b\*c\*d^3\*e^3\*sgn(d\*x + c) - 36\*pi\*b\*c\*d\*f^2\*e\*sgn(d\*x + c) - 18\*pi\*b\*c^2\*f^3 - 36\*b\*c^2\*f^3\*arctan(1/(d\*x + c)) + 24\*a\*d^4\*x\*e^3 + 24\*b\*c\*d^3\*arctan(d\*x + c)\*e^3 - 36\*b\*d^3\*f\*x\*e^2 + 36\*pi\*b\*c\*d\*f^2\*e + 72\*b\*c\*d\*f^2\*arctan(1/(d\*x + c))\*e + 36\*b\*c\*d^2\*f\*e^2\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) + 18\*pi\*b\*d^2\*f\*e^2\*sgn(d\*x + c) + 6\*b\*d\*f^3\*x - 18\*pi\*b\*d^2\*f\*e^2 - 36\*b\*d^2\*f\*arctan(1/(d\*x + c))\*e^2 - 12\*b\*c\*f^3\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) - 12\*b\*d^3\*e^3\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) + 12\*b\*d\*f^2\*e\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) - 3\*pi\*b\*f^3\*sgn(d\*x + c) + 3\*pi\*b\*f^3 + 6\*b\*f^3\*arctan(1/(d\*x + c)))/d^4

### 3.25 $\int (e + fx)^2 (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=155

$$\frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))}{3f} - \frac{b(- (1 - 3c^2) f^2 - 6cdef + 3d^2 e^2) \log((c + dx)^2 + 1)}{6d^3} - \frac{b(de - cf)(- (3 - c^2) f^2 - 2cd)}{3d^3 f}$$

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

**Rubi [A]** time = 0.189819, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))}{3f} - \frac{b(- (1 - 3c^2) f^2 - 6cdef + 3d^2 e^2) \log((c + dx)^2 + 1)}{6d^3} - \frac{b(de - cf)(- (3 - c^2) f^2 - 2cd)}{3d^3 f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

#### Rule 5047

```
Int[((a_) + ArcTan[(c_) + (d_)*(x_)])*(b_)^(p_)*((e_) + (f_)*(x_))^(m_), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]
```

#### Rule 4862

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_))^(q_), x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

#### Rule 702

```
Int[((d_) + (e_)*(x_))^(m_)/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

#### Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

#### Rule 203



Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned} \int (e + fx)^2 (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^3}{1+x^2} dx, x, c + dx\right)}{3f} \\ &= \frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \left(\frac{3f^2(de-cf)}{d^3} + \frac{f^3x}{d^3} + \frac{(de-cf)(d^2e^2-2cd)}{d^3}\right) dx, x, c + dx\right)}{3f} \\ &= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3(a+b \tan^{-1}(c+dx))}{3f} - \frac{b \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c + dx\right)}{3f} \\ &= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3(a+b \tan^{-1}(c+dx))}{3f} - \frac{b(3d^2e^2-2cd)}{3d^3} \\ &= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} - \frac{b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2) \tan^{-1}\left(\frac{dx+c}{d}\right)}{3d^3f} \end{aligned}$$

**Mathematica [C]** time = 0.139037, size = 118, normalized size = 0.76

$$\frac{(e + fx)^3 (a + b \tan^{-1}(c + dx)) - \frac{b(6df^2x(de-cf) - i(de-(c-i)f)^3 \log(-c-dx+i) + i(de-(c+i)f)^3 \log(c+dx+i) + f^3(c+dx)^2)}{2d^3}}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcTan[c + d\*x]), x]

[Out] ((e + f\*x)^3\*(a + b\*ArcTan[c + d\*x]) - (b\*(6\*d\*f^2\*(d\*e - c\*f)\*x + f^3\*(c + d\*x)^2 - I\*(d\*e - (-I + c)\*f)^3\*Log[I - c - d\*x] + I\*(d\*e - (I + c)\*f)^3\*Log[I + c + d\*x]))/(2\*d^3)/(3\*f)

**Maple [A]** time = 0.041, size = 283, normalized size = 1.8

$$-\frac{\arctan(dx+c)bfec^2}{d^2} + \frac{bf^2 \ln(1+(dx+c)^2)}{6d^3} + \frac{b \arctan(dx+c)ce^2}{d} + \frac{5bf^2c^2}{6d^3} + bf \arctan(dx+c)ex^2 + \frac{af^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arctan(d\*x+c)), x)



```

c*x/d + x**2 + d**(-2))/(2*d**3) + b*c*e**2*atan(c + d*x)/d + b*c*e*f*log(c
**2/d**2 + 2*c*x/d + x**2 + d**(-2))/d**2 + 2*b*c*f**2*x/(3*d**2) - b*c*f**
2*atan(c + d*x)/d**3 + b*e**2*x*atan(c + d*x) + b*e*f*x**2*atan(c + d*x) +
b*f**2*x**3*atan(c + d*x)/3 - b*e**2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-
2))/(2*d) - b*e*f*x/d - b*f**2*x**2/(6*d) + b*e*f*atan(c + d*x)/d**2 + b*f*
*2*log(c**2/d**2 + 2*c*x/d + x**2 + d**(-2))/(6*d**3), Ne(d, 0)), ((a + b*a
tan(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), True))

```

**Giac [B]** time = 1.51601, size = 562, normalized size = 3.63

$$2bd^3f^2x^3 \arctan(dx+c) + 2ad^3f^2x^3 + 6bd^3fx^2 \arctan(dx+c)e + 6ad^3fx^2e + \pi bc^3f^2 \operatorname{sgn}(dx+c) - 3\pi bc^2df \operatorname{sgn}(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d^3*f^2*x^3*arctan(d*x + c) + 2*a*d^3*f^2*x^3 + 6*b*d^3*f*x^2*arct
an(d*x + c)*e + 6*a*d^3*f*x^2*e + pi*b*c^3*f^2*sgn(d*x + c) - 3*pi*b*c^2*d*
f*e*sgn(d*x + c) - pi*b*c^3*f^2 - b*d^2*f^2*x^2 - 2*b*c^3*f^2*arctan(1/(d*x
+ c)) + 6*b*d^3*x*arctan(d*x + c)*e^2 + 3*pi*b*c^2*d*f*e + 6*b*c^2*d*f*arc
tan(1/(d*x + c))*e - 6*pi*b*c*d^2*e^2*sgn(d*x + c) + 4*b*c*d*f^2*x + 6*a*d^
3*x*e^2 + 6*b*c*d^2*arctan(d*x + c)*e^2 - 6*b*d^2*f*x*e - 3*b*c^2*f^2*log(d
^2*x^2 + 2*c*d*x + c^2 + 1) + 6*b*c*d*f*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
- 3*pi*b*c*f^2*sgn(d*x + c) + 3*pi*b*d*f*e*sgn(d*x + c) + 3*pi*b*c*f^2 + 6*
b*c*f^2*arctan(1/(d*x + c)) - 3*pi*b*d*f*e - 6*b*d*f*arctan(1/(d*x + c))*e
- 3*b*d^2*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + b*f^2*log(d^2*x^2 + 2*c*d*
x + c^2 + 1))/d^3
```

### 3.26 $\int (e + fx) \left( a + b \tan^{-1}(c + dx) \right) dx$

**Optimal.** Leaf size=97

$$\frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} - \frac{b(de - cf) \log((c + dx)^2 + 1)}{2d^2} - \frac{b(-cf + de + f)(de - (c + 1)f) \tan^{-1}(c + dx)}{2d^2 f} - \frac{bfx}{2d}$$

[Out]  $-(b*f*x)/(2*d) - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1 + (c + d*x)^2])/(2*d^2)$

**Rubi [A]** time = 0.112068, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} - \frac{b(de - cf) \log((c + dx)^2 + 1)}{2d^2} - \frac{b(-cf + de + f)(de - (c + 1)f) \tan^{-1}(c + dx)}{2d^2 f} - \frac{bfx}{2d}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)\*(a + b\*ArcTan[c + d\*x]),x]

[Out]  $-(b*f*x)/(2*d) - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1 + (c + d*x)^2])/(2*d^2)$

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4862

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_.))^ (q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

Int[((d\_.) + (e\_.)\*(x\_.))^ (m\_.)/((a\_.) + (c\_.)\*(x\_.)^2), x\_Symbol] :> Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

#### Rule 635

Int[((d\_.) + (e\_.)\*(x\_.))/((a\_.) + (c\_.)\*(x\_.)^2), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 203

Int[((a\_.) + (b\_.)\*(x\_.)^2)^ (-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned} \int (e + fx) (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right) (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2}{1+x^2} dx, x, c + dx\right)}{2f} \\ &= \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \left(\frac{f^2}{d^2} + \frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{d^2(1+x^2)}\right) dx, x, c + dx\right)}{2f} \\ &= -\frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{1+x^2} dx, x, c + dx\right)}{2d^2 f} \\ &= -\frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} - \frac{(b(de - cf)) \text{Subst}\left(\int \frac{x}{1+x^2} dx, x, c + dx\right)}{d^2} \\ &= -\frac{bfx}{2d} - \frac{b(de + f - cf)(de - (1 + c)f) \tan^{-1}(c + dx)}{2d^2 f} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))}{2f} \end{aligned}$$

**Mathematica [C]** time = 0.0548483, size = 163, normalized size = 1.68

$$aex + \frac{1}{2}afx^2 - \frac{be \left( \log(c^2 + 2cdx + d^2x^2 + 1) - 2c \tan^{-1}(c + dx) \right)}{2d} + \frac{bf \left( \frac{1}{2}d \left( \frac{c+dx}{d} - \frac{c}{d} \right)^2 \tan^{-1}(c + dx) - \frac{1}{2}d \left( -\frac{i(-c+i)^2}{1+i} \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)\*(a + b\*ArcTan[c + d\*x]),x]

[Out] a\*e\*x + (a\*f\*x^2)/2 + b\*e\*x\*ArcTan[c + d\*x] + (b\*f\*((d\*(-(c/d) + (c + d\*x)/d)^2\*ArcTan[c + d\*x])/2 - (d\*(x/d - ((I/2)\*(I - c)^2\*Log[I - c - d\*x])/d^2 + ((I/2)\*(I + c)^2\*Log[I + c + d\*x])/d^2))/2)/d - (b\*e\*(-2\*c\*ArcTan[c + d\*x] + Log[1 + c^2 + 2\*c\*d\*x + d^2\*x^2]))/(2\*d)

**Maple [A]** time = 0.039, size = 146, normalized size = 1.5

$$\frac{ax^2f}{2} - \frac{ac^2f}{2d^2} + aex + \frac{ace}{d} + \frac{bf \arctan(dx + c)x^2}{2} - \frac{\arctan(dx + c)bc^2f}{2d^2} + \arctan(dx + c)xbe + \frac{b \arctan(dx + c)c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*(a+b\*arctan(d\*x+c)),x)

[Out] 1/2\*a\*x^2\*f-1/2/d^2\*a\*f\*c^2+a\*e\*x+1/d\*a\*c\*e+1/2\*b\*f\*arctan(d\*x+c)\*x^2-1/2/d^2\*b\*f\*arctan(d\*x+c)\*c^2+arctan(d\*x+c)\*x\*b\*e+1/d\*arctan(d\*x+c)\*b\*c\*e-1/2\*b\*

$f*x/d-1/2/d^2*b*c*f+1/2/d^2*b*\ln(1+(d*x+c)^2)*c*f-1/2/d*b*\ln(1+(d*x+c)^2)*e+1/2/d^2*b*f*\arctan(d*x+c)$

**Maxima [A]** time = 1.48311, size = 157, normalized size = 1.62

$$\frac{1}{2}afx^2 + \frac{1}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right) - c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bf + aex + \frac{(2(dx+c))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arctan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*a\*f\*x^2 + 1/2\*(x^2\*arctan(d\*x + c) - d\*(x/d^2 + (c^2 - 1)\*arctan((d^2\*x + c\*d)/d)/d^3 - c\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)/d^3))\*b\*f + a\*e\*x + 1/2\*(2\*(d\*x + c)\*arctan(d\*x + c) - log((d\*x + c)^2 + 1))\*b\*e/d

**Fricas [A]** time = 1.42201, size = 232, normalized size = 2.39

$$\frac{ad^2fx^2 + (2ad^2e - bdf)x + (bd^2fx^2 + 2bd^2ex + 2bcde - (bc^2 - b)f)\arctan(dx+c) - (bde - bcf)\log(d^2x^2 + 2cdx + c^2 + 1)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arctan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(a\*d^2\*f\*x^2 + (2\*a\*d^2\*e - b\*d\*f)\*x + (b\*d^2\*f\*x^2 + 2\*b\*d^2\*e\*x + 2\*b\*c\*d\*e - (b\*c^2 - b)\*f)\*arctan(d\*x + c) - (b\*d\*e - b\*c\*f)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1))/d^2

**Sympy [A]** time = 2.4684, size = 177, normalized size = 1.82

$$\left\{ \begin{array}{l} aex + \frac{afx^2}{2} - \frac{bc^2f\operatorname{atan}(c+dx)}{2d^2} + \frac{bce\operatorname{atan}(c+dx)}{d} + \frac{bcf\log\left(\frac{c^2}{d^2} + \frac{2cx}{d} + x^2 + \frac{1}{d^2}\right)}{2d^2} + bex\operatorname{atan}(c+dx) + \frac{bf x^2 \operatorname{atan}(c+dx)}{2} - \frac{be\log\left(\frac{c^2}{d^2} + \frac{2cx}{d} + x^2 + \frac{1}{d^2}\right)}{2d} \\ (a + b\operatorname{atan}(c))\left(ex + \frac{fx^2}{2}\right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*atan(d\*x+c)),x)

[Out] Piecewise((a\*e\*x + a\*f\*x\*\*2/2 - b\*c\*\*2\*f\*atan(c + d\*x)/(2\*d\*\*2) + b\*c\*e\*atan(c + d\*x)/d + b\*c\*f\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d\*\*2) + b\*e\*x\*atan(c + d\*x) + b\*f\*x\*\*2\*atan(c + d\*x)/2 - b\*e\*log(c\*\*2/d\*\*2 + 2\*c\*x/d + x\*\*2 + d\*\*(-2))/(2\*d) - b\*f\*x/(2\*d) + b\*f\*atan(c + d\*x)/(2\*d\*\*2), Ne(d, 0)), ((a + b\*atan(c))\*(e\*x + f\*x\*\*2/2), True))

**Giac [B]** time = 1.20523, size = 269, normalized size = 2.77

$$2bd^2fx^2\arctan(dx+c) + 2ad^2fx^2 + 4bd^2x\arctan(dx+c)e - \pi bc^2f\operatorname{sgn}(dx+c) - 4\pi bcde\operatorname{sgn}(dx+c) + \pi bc^2f + 2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*d^2*f*x^2*arctan(d*x + c) + 2*a*d^2*f*x^2 + 4*b*d^2*x*arctan(d*x +
c)*e - pi*b*c^2*f*sgn(d*x + c) - 4*pi*b*c*d*e*sgn(d*x + c) + pi*b*c^2*f +
2*b*c^2*f*arctan(1/(d*x + c)) + 4*a*d^2*x*e + 4*b*c*d*arctan(d*x + c)*e - 2
*b*d*f*x + 2*b*c*f*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*b*d*e*log(d^2*x^2 +
2*c*d*x + c^2 + 1) + pi*b*f*sgn(d*x + c) - pi*b*f - 2*b*f*arctan(1/(d*x +
c)))/d^2
```

### 3.27 $\int (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=38

$$ax - \frac{b \log((c + dx)^2 + 1)}{2d} + \frac{b(c + dx) \tan^{-1}(c + dx)}{d}$$

[Out] a\*x + (b\*(c + d\*x)\*ArcTan[c + d\*x])/d - (b\*Log[1 + (c + d\*x)^2])/(2\*d)

**Rubi [A]** time = 0.0184687, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {5039, 4846, 260}

$$ax - \frac{b \log((c + dx)^2 + 1)}{2d} + \frac{b(c + dx) \tan^{-1}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTan[c + d\*x], x]

[Out] a\*x + (b\*(c + d\*x)\*ArcTan[c + d\*x])/d - (b\*Log[1 + (c + d\*x)^2])/(2\*d)

#### Rule 5039

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 4846

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_.) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned} \int (a + b \tan^{-1}(c + dx)) dx &= ax + b \int \tan^{-1}(c + dx) dx \\ &= ax + \frac{b \text{Subst}\left(\int \tan^{-1}(x) dx, x, c + dx\right)}{d} \\ &= ax + \frac{b(c + dx) \tan^{-1}(c + dx)}{d} - \frac{b \text{Subst}\left(\int \frac{x}{1+x^2} dx, x, c + dx\right)}{d} \\ &= ax + \frac{b(c + dx) \tan^{-1}(c + dx)}{d} - \frac{b \log(1 + (c + dx)^2)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0132084, size = 49, normalized size = 1.29

$$ax - \frac{b(\log(c^2 + 2cdx + d^2x^2 + 1) - 2c \tan^{-1}(c + dx))}{2d} + bx \tan^{-1}(c + dx)$$



Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTan[c + d\*x], x]

[Out]  $a*x + b*x*ArcTan[c + d*x] - (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)$

**Maple [A]** time = 0.033, size = 42, normalized size = 1.1

$$ax + b \arctan(dx + c)x + \frac{b \arctan(dx + c)c}{d} - \frac{b \ln(1 + (dx + c)^2)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arctan(d\*x+c), x)

[Out]  $a*x+b*arctan(d*x+c)*x+b/d*arctan(d*x+c)*c-1/2*b*\ln(1+(d*x+c)^2)/d$

**Maxima [A]** time = 0.973068, size = 49, normalized size = 1.29

$$ax + \frac{(2(dx + c) \arctan(dx + c) - \log((dx + c)^2 + 1))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctan(d\*x+c), x, algorithm="maxima")

[Out]  $a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - \log((d*x + c)^2 + 1))*b/d$

**Fricas [A]** time = 1.51522, size = 119, normalized size = 3.13

$$\frac{2 adx + 2 (bdx + bc) \arctan(dx + c) - b \log(d^2x^2 + 2cdx + c^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctan(d\*x+c), x, algorithm="fricas")

[Out]  $1/2*(2*a*d*x + 2*(b*d*x + b*c)*arctan(d*x + c) - b*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d$

**Sympy [A]** time = 0.565511, size = 51, normalized size = 1.34

$$ax + b \left\{ \begin{array}{ll} \frac{c \operatorname{atan}(c+dx)}{d} + x \operatorname{atan}(c+dx) - \frac{\log(c^2+2cdx+d^2x^2+1)}{2d} & \text{for } d \neq 0 \\ x \operatorname{atan}(c) & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atan(d\*x+c),x)

[Out] a\*x + b\*Piecewise((c\*atan(c + d\*x)/d + x\*atan(c + d\*x) - log(c\*\*2 + 2\*c\*d\*x + d\*\*2\*x\*\*2 + 1)/(2\*d), Ne(d, 0)), (x\*atan(c), True))

**Giac [A]** time = 1.08183, size = 49, normalized size = 1.29

$$ax + \frac{(2(dx + c) \arctan(dx + c) - \log((dx + c)^2 + 1))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctan(d\*x+c),x, algorithm="giac")

[Out] a\*x + 1/2\*(2\*(d\*x + c)\*arctan(d\*x + c) - log((d\*x + c)^2 + 1))\*b/d

$$3.28 \quad \int \frac{a+b \tan^{-1}(c+dx)}{e+fx} dx$$

**Optimal.** Leaf size=162

$$-\frac{ibPolyLog\left(2, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f} + \frac{ibPolyLog\left(2, 1 - \frac{2}{1-i(c+dx)}\right)}{2f} + \frac{(a+b \tan^{-1}(c+dx)) \log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}$$

[Out] -(((a + b\*ArcTan[c + d\*x])\*Log[2/(1 - I\*(c + d\*x))])/f) + ((a + b\*ArcTan[c + d\*x])\*Log[(2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f) + ((I/2)\*b\*PolyLog[2, 1 - 2/(1 - I\*(c + d\*x))])/f - ((I/2)\*b\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f

**Rubi [A]** time = 0.150132, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{ibPolyLog\left(2, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f} + \frac{ibPolyLog\left(2, 1 - \frac{2}{1-i(c+dx)}\right)}{2f} + \frac{(a+b \tan^{-1}(c+dx)) \log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])/(e + f\*x), x]

[Out] -(((a + b\*ArcTan[c + d\*x])\*Log[2/(1 - I\*(c + d\*x))])/f) + ((a + b\*ArcTan[c + d\*x])\*Log[(2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f) + ((I/2)\*b\*PolyLog[2, 1 - 2/(1 - I\*(c + d\*x))])/f - ((I/2)\*b\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.)^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4856

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])\*Log[2/(1 - I\*c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))]/(1 + c^2\*x^2), x], x] + Simp[((a + b\*ArcTan[c\*x])\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_.) + (e\_.)\*(x\_))]/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a + b \tan^{-1}(c + dx)}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{a + b \tan^{-1}(x)}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \tan^{-1}(c + dx)) \log\left(\frac{2}{1 - i(c + dx)}\right)}{f} + \frac{(a + b \tan^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + if - cf)(1 - i(c + dx))}\right)}{f} + \dots$$

$$= -\frac{(a + b \tan^{-1}(c + dx)) \log\left(\frac{2}{1 - i(c + dx)}\right)}{f} + \frac{(a + b \tan^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + if - cf)(1 - i(c + dx))}\right)}{f} - \dots$$

$$= -\frac{(a + b \tan^{-1}(c + dx)) \log\left(\frac{2}{1 - i(c + dx)}\right)}{f} + \frac{(a + b \tan^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + if - cf)(1 - i(c + dx))}\right)}{f} + \dots$$

**Mathematica [A]** time = 0.104057, size = 160, normalized size = 0.99

$$\frac{-ib \text{PolyLog}\left(2, \frac{f(c + dx - i)}{-de + (c - i)f}\right) + ib \text{PolyLog}\left(2, \frac{f(c + dx + i)}{-de + (c + i)f}\right) + 2a \log(d(e + fx)) + ib \log(1 - i(c + dx)) \log\left(\frac{d(e + fx)}{de - (c + i)f}\right) - ib \log\left(\frac{d(e + fx)}{de - (c + i)f}\right)}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x), x]
```

```
[Out] (2*a*Log[d*(e + f*x)] + I*b*Log[(d*(e + f*x))/(d*e - (I + c)*f])*Log[1 - I*(c + d*x)] - I*b*Log[(d*(e + f*x))/(d*e + I*f - c*f])*Log[1 + I*(c + d*x)] - I*b*PolyLog[2, (f*(-I + c + d*x))/(-(d*e) + (-I + c)*f)] + I*b*PolyLog[2, (f*(I + c + d*x))/(-(d*e) + (I + c)*f)]/(2*f)
```

**Maple [A]** time = 0.072, size = 224, normalized size = 1.4

$$\frac{a \ln(f(dx + c) - cf + de)}{f} + \frac{b \ln(f(dx + c) - cf + de) \arctan(dx + c)}{f} + \frac{i}{2} \frac{b \ln(f(dx + c) - cf + de)}{f} \ln\left(\frac{if - f(dx + c)}{de + if - cf}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e), x)
```

```
[Out] a*ln(f*(d*x+c)-c*f+d*e)/f+b*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)+1/2*I*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b*ln(f*(d*x+c)-c*f+d*e)/f*ln((I*f+f*(d*x+c))/(I*f+c*f-d*e))+1/2*I*b/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2b \int \frac{\arctan(dx+c)}{2(fx+e)} dx + \frac{a \log(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e),x, algorithm="maxima")

[Out] 2\*b\*integrate(1/2\*arctan(d\*x + c)/(f\*x + e), x) + a\*log(f\*x + e)/f

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arctan(dx+c) + a}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e),x, algorithm="fricas")

[Out] integral((b\*arctan(d\*x + c) + a)/(f\*x + e), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atan}(c + dx)}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))/(f\*x+e),x)

[Out] Integral((a + b\*atan(c + d\*x))/(e + f\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(dx+c) + a}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e),x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)/(f\*x + e), x)

### 3.29 $\int \frac{a+b \tan^{-1}(c+dx)}{(e+fx)^2} dx$

**Optimal.** Leaf size=151

$$-\frac{a+b \tan^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(c^2+2cdx+d^2x^2+1)}{2((c^2+1)f^2-2cdef+d^2e^2)} + \frac{bd \log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2} + \frac{bd(de-cf) \tan^{-1}(c+dx)}{f((c^2+1)f^2-2cdef+d^2e^2)}$$

[Out] (b\*d\*(d\*e - c\*f)\*ArcTan[c + d\*x])/(f\*(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2)) - (a + b\*ArcTan[c + d\*x])/(f\*(e + f\*x)) + (b\*d\*Log[e + f\*x])/(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2) - (b\*d\*Log[1 + c^2 + 2\*c\*d\*x + d^2\*x^2])/(2\*(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2))

**Rubi [A]** time = 0.121052, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {5045, 1982, 705, 31, 634, 618, 204, 628}

$$-\frac{a+b \tan^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(c^2+2cdx+d^2x^2+1)}{2((c^2+1)f^2-2cdef+d^2e^2)} + \frac{bd \log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2} + \frac{bd(de-cf) \tan^{-1}(c+dx)}{f((c^2+1)f^2-2cdef+d^2e^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])/(e + f\*x)^2, x]

[Out] (b\*d\*(d\*e - c\*f)\*ArcTan[c + d\*x])/(f\*(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2)) - (a + b\*ArcTan[c + d\*x])/(f\*(e + f\*x)) + (b\*d\*Log[e + f\*x])/(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2) - (b\*d\*Log[1 + c^2 + 2\*c\*d\*x + d^2\*x^2])/(2\*(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2))

#### Rule 5045

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^ (m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^(p - 1))/(1 + (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

#### Rule 1982

Int[(u\_)^(m\_.)\*(v\_)^(p\_.), x\_Symbol] := Int[ExpandToSum[u, x]^m\*ExpandToSum[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && ! (LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

#### Rule 705

Int[1/(((d\_.) + (e\_.)\*(x\_.))\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 - b\*d\*e + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 - b\*d\*e + a\*e^2), Int[(c\*d - b\*e - c\*e\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tan^{-1}(c + dx)}{(e + fx)^2} dx &= -\frac{a + b \tan^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1+(c+dx)^2)} dx}{f} \\
&= -\frac{a + b \tan^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1+c^2+2cdx+d^2x^2)} dx}{f} \\
&= -\frac{a + b \tan^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2} dx}{f(d^2e^2 - 2cdef + (1+c^2)f^2)} + \frac{(bdf) \int \frac{1}{e+fx} dx}{d^2e^2 - 2cdef + (1+c^2)f^2} \\
&= -\frac{a + b \tan^{-1}(c + dx)}{f(e + fx)} + \frac{bd \log(e + fx)}{d^2e^2 - 2cdef + (1+c^2)f^2} - \frac{(bd) \int \frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2} dx}{2(d^2e^2 - 2cdef + (1+c^2)f^2)} + \frac{(bdf) \int \frac{1}{e+fx} dx}{d^2e^2 - 2cdef + (1+c^2)f^2} \\
&= -\frac{a + b \tan^{-1}(c + dx)}{f(e + fx)} + \frac{bd \log(e + fx)}{d^2e^2 - 2cdef + (1+c^2)f^2} - \frac{bd \log(1 + c^2 + 2cdx + d^2x^2)}{2(d^2e^2 - 2cdef + (1+c^2)f^2)} - \frac{(bdf) \int \frac{1}{e+fx} dx}{d^2e^2 - 2cdef + (1+c^2)f^2} \\
&= \frac{bd(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1+c^2)f^2)} - \frac{a + b \tan^{-1}(c + dx)}{f(e + fx)} + \frac{bd \log(e + fx)}{d^2e^2 - 2cdef + (1+c^2)f^2} - \frac{bd \log(1 + c^2 + 2cdx + d^2x^2)}{2(d^2e^2 - 2cdef + (1+c^2)f^2)} - \frac{(bdf) \int \frac{1}{e+fx} dx}{d^2e^2 - 2cdef + (1+c^2)f^2}
\end{aligned}$$

**Mathematica [C]** time = 0.156554, size = 121, normalized size = 0.8

$$\frac{-\frac{a+b \tan^{-1}(c+dx)}{e+fx} + \frac{bd(i(-de+(c+i)f) \log(-c-dx+i)+i(-cf+de+if) \log(c+dx+i)+2f \log(d(e+fx)))}{2((c^2+1)f^2-2cdef+d^2e^2)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^2, x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)) + (b*d*(I*(-(d*e) + (I + c)*f)*Log[I - c - d*x] + I*(d*e + I*f - c*f)*Log[I + c + d*x] + 2*f*Log[d*(e + f*x)])) /
```

$$(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))/f$$

**Maple [A]** time = 0.045, size = 205, normalized size = 1.4

$$\frac{ad}{(dfx + de)f} - \frac{bd \arctan(dx + c)}{(dfx + de)f} - \frac{bd \ln(1 + (dx + c)^2)}{2c^2f^2 - 4cdef + 2d^2e^2 + 2f^2} - \frac{bd \arctan(dx + c)c}{c^2f^2 - 2cdef + d^2e^2 + f^2} + \frac{bd^2 \arctan(dx + c)}{f(c^2f^2 - 2cdef + d^2e^2 + f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))/(f\*x+e)^2,x)

[Out] -d\*a/(d\*f\*x+d\*e)/f-d\*b/(d\*f\*x+d\*e)/f\*arctan(d\*x+c)-1/2\*d\*b/(c^2\*f^2-2\*c\*d\*e\*f+d^2\*e^2+f^2)\*ln(1+(d\*x+c)^2)-d\*b/(c^2\*f^2-2\*c\*d\*e\*f+d^2\*e^2+f^2)\*arctan(d\*x+c)\*c+d^2\*b/f/(c^2\*f^2-2\*c\*d\*e\*f+d^2\*e^2+f^2)\*arctan(d\*x+c)\*e+d\*b/(c^2\*f^2-2\*c\*d\*e\*f+d^2\*e^2+f^2)\*ln(f\*(d\*x+c)-c\*f+d\*e)

**Maxima [A]** time = 1.53327, size = 239, normalized size = 1.58

$$\frac{1}{2} \left( d \left( \frac{2(d^2e - cdf) \arctan\left(\frac{d^2x+cd}{d}\right) - \log(d^2x^2 + 2cdx + c^2 + 1)}{(d^2e^2f - 2cdef^2 + (c^2 + 1)f^3)d} - \frac{\log(d^2x^2 + 2cdx + c^2 + 1)}{d^2e^2 - 2cdef + (c^2 + 1)f^2} + \frac{2 \log(fx + e)}{d^2e^2 - 2cdef + (c^2 + 1)f^2} \right) - \frac{2 \arctan(dx + c)}{f^2x + ef} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e)^2,x, algorithm="maxima")

[Out] 1/2\*(d\*(2\*(d^2\*e - c\*d\*f)\*arctan((d^2\*x + c\*d)/d)/((d^2\*e^2\*f - 2\*c\*d\*e\*f^2 + (c^2 + 1)\*f^3)\*d) - log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)/(d^2\*e^2 - 2\*c\*d\*e\*f + (c^2 + 1)\*f^2) + 2\*log(f\*x + e)/(d^2\*e^2 - 2\*c\*d\*e\*f + (c^2 + 1)\*f^2) - 2\*arctan(d\*x + c)/(f^2\*x + e\*f))\*b - a/(f^2\*x + e\*f)

**Fricas [A]** time = 3.03752, size = 435, normalized size = 2.88

$$\frac{2ad^2e^2 - 4acdef + 2(ac^2 + a)f^2 - 2(bcdef - (bc^2 + b)f^2 + (bd^2ef - bcd^2f^2)x) \arctan(dx + c) + (bdf^2x + bdef) \log(dx + c)}{2(d^2e^3f - 2cde^2f^2 + (c^2 + 1)ef^3 + (d^2e^2f^2 - 2cdef^3 + (c^2 + 1)f^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e)^2,x, algorithm="fricas")

[Out] -1/2\*(2\*a\*d^2\*e^2 - 4\*a\*c\*d\*e\*f + 2\*(a\*c^2 + a)\*f^2 - 2\*(b\*c\*d\*e\*f - (b\*c^2 + b)\*f^2 + (b\*d^2\*e\*f - b\*c\*d\*f^2)\*x)\*arctan(d\*x + c) + (b\*d\*f^2\*x + b\*d\*e\*f)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1) - 2\*(b\*d\*f^2\*x + b\*d\*e\*f)\*log(f\*x + e)/(d^2\*e^3\*f - 2\*c\*d\*e^2\*f^2 + (c^2 + 1)\*e\*f^3 + (d^2\*e^2\*f^2 - 2\*c\*d\*e\*f^3 + (c^2 + 1)\*f^4)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))/(f\*x+e)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.09362, size = 393, normalized size = 2.6

$$-\frac{1}{2} \left( df^2 \frac{\log\left(d^2 + \frac{2cdf}{fx+e} + \frac{c^2f^2}{(fx+e)^2} - \frac{2d^2e}{fx+e} - \frac{2cdf e}{(fx+e)^2} + \frac{d^2e^2}{(fx+e)^2} + \frac{f^2}{(fx+e)^2}\right)}{c^2f^4 - 2cdf^3e + d^2f^2e^2 + f^4} + \frac{2(cdf - d^2e) \arctan\left(-\frac{cdf + \frac{c^2f^2}{fx+e} - d^2e - \frac{2cdf e}{fx+e} + \frac{d^2e}{fx+e}}{df}\right)}{(c^2f^3 - 2cdf^2e + d^2fe^2 + f^3)df^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e)^2,x, algorithm="giac")

[Out] 
$$-1/2*(d*f^2*(\log(d^2 + 2*c*d*f/(f*x + e) + c^2*f^2/(f*x + e)^2 - 2*d^2*e/(f*x + e) - 2*c*d*f*e/(f*x + e)^2 + d^2*e^2/(f*x + e)^2 + f^2/(f*x + e)^2)/(c^2*f^4 - 2*c*d*f^3*e + d^2*f^2*e^2 + f^4) + 2*(c*d*f - d^2*e)*\arctan(-(c*d*f + c^2*f^2/(f*x + e) - d^2*e - 2*c*d*f*e/(f*x + e) + d^2*e^2/(f*x + e) + f^2/(f*x + e))/(d*f))/((c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + f^3)*d*f^2)) + 2*\arctan(d*x + c)/((f*x + e)*f)*b - a/((f*x + e)*f)$$

### 3.30 $\int \frac{a+b \tan^{-1}(c+dx)}{(e+fx)^3} dx$

**Optimal.** Leaf size=227

$$\frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} - \frac{bd^2(de - cf) \log(c^2 + 2cdx + d^2x^2 + 1)}{2((c^2 + 1)f^2 - 2cdef + d^2e^2)^2} - \frac{bd}{2(e + fx)((c^2 + 1)f^2 - 2cdef + d^2e^2)} + \frac{bd^2(de - c)}{((c^2 + 1)f^2 - 2cdef + d^2e^2)}$$

[Out]  $-(b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e - c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2)$

**Rubi [A]** time = 0.303089, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {5045, 1982, 709, 800, 634, 618, 204, 628}

$$\frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} - \frac{bd^2(de - cf) \log(c^2 + 2cdx + d^2x^2 + 1)}{2((c^2 + 1)f^2 - 2cdef + d^2e^2)^2} - \frac{bd}{2(e + fx)((c^2 + 1)f^2 - 2cdef + d^2e^2)} + \frac{bd^2(de - c)}{((c^2 + 1)f^2 - 2cdef + d^2e^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*ArcTan[c + d*x])/(e + f*x)^3, x]$

[Out]  $-(b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e - c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2)$

#### Rule 5045

$\text{Int}[(a + b*ArcTan[c + d*x])/(e + f*x)^3, x] \rightarrow \text{Simp}[(e + f*x)^{(m+1)}*(a + b*ArcTan[c + d*x])^p/(f*(m+1)), x] - \text{Dist}[(b*d*p)/(f*(m+1)), \text{Int}[(e + f*x)^{(m+1)}*(a + b*ArcTan[c + d*x])^{(p-1)}]/(1 + (c + d*x)^2), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

#### Rule 1982

$\text{Int}[(u + v*x)^m*(v)^p, x] \rightarrow \text{Int}[\text{ExpandToSum}[u, x]^m*\text{ExpandToSum}[v, x]^p, x] /;$  FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && ! (LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

#### Rule 709

$\text{Int}[(d + e*x)^m/(a + b*x + c*x^2), x] \rightarrow \text{Simp}[(e*(d + e*x)^{(m+1)})/((m+1)*(c*d^2 - b*d*e + a*e^2)), x] + \text{Dist}[1/(c*d^2 - b*d*e + a*e^2), \text{Int}[(d + e*x)^{(m+1)}*\text{Simp}[c*d - b*e - c*e*x, x]/(a + b*x + c*x^2), x], x] /;$  FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && LtQ[m, -1]

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tan^{-1}(c + dx)}{(e + fx)^3} dx &= -\frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1+(c+dx)^2)} dx}{2f} \\
&= -\frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1+c^2+2cdx+d^2x^2)} dx}{2f} \\
&= -\frac{bd}{2(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} - \frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{d(de-2cf)-d^2fx}{(e+fx)(1+c^2+2cdx+d^2x^2)} dx}{2f(d^2e^2 - 2cdef + (1 + c^2) f^2)} \\
&= -\frac{bd}{2(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} - \frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \left( \frac{2df^2(de-cf)}{(d^2e^2-2cdef+(1+c^2)f^2)} \right) dx}{2f(d^2e^2 - 2cdef + (1 + c^2) f^2)} \\
&= -\frac{bd}{2(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} - \frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{bd^2(de - cf) \log(e + fx)}{(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} \\
&= -\frac{bd}{2(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} - \frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{bd^2(de - cf) \log(e + fx)}{(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} \\
&= -\frac{bd}{2(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} - \frac{a + b \tan^{-1}(c + dx)}{2f(e + fx)^2} + \frac{bd^2(de - cf) \log(e + fx)}{(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} \\
&= -\frac{bd}{2(d^2e^2 - 2cdef + (1 + c^2) f^2)(e + fx)} + \frac{bd^2(de - f - cf)(de + f - cf) \tan^{-1}(c + dx)}{2f(d^2e^2 - 2cdef + f^2 + c^2 f^2)^2} - \frac{a}{2f}
\end{aligned}$$

**Mathematica [C]** time = 0.555227, size = 175, normalized size = 0.77

$$\frac{-\frac{a+b \tan^{-1}(c+dx)}{(e+fx)^2} + \frac{1}{2}bd^2 \left( -\frac{2f}{d(e+fx)((c^2+1)f^2-2cdef+d^2e^2)} - \frac{4f(cf-de) \log(d(e+fx))}{((c^2+1)f^2-2cdef+d^2e^2)^2} - \frac{i \log(-c-dx+i)}{(de-(c-i)f)^2} + \frac{i \log(c+dx+i)}{(de-(c+i)f)^2} \right)}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c + d\*x])/(e + f\*x)^3,x]

[Out] (-((a + b\*ArcTan[c + d\*x])/(e + f\*x)^2) + (b\*d^2\*((-2\*f)/(d\*(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2)\*(e + f\*x)) - (I\*Log[I - c - d\*x])/(d\*e - (-I + c)\*f)^2 + (I\*Log[I + c + d\*x])/(d\*e - (I + c)\*f)^2 - (4\*f\*(-(d\*e) + c\*f)\*Log[d\*(e + f\*x)])/(d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2))/2)/(2\*f)

**Maple [A]** time = 0.046, size = 438, normalized size = 1.9

$$-\frac{d^2a}{2(df x + de)^2 f} - \frac{d^2b \arctan(dx + c)}{2(df x + de)^2 f} + \frac{d^2bf \arctan(dx + c)c^2}{2(c^2f^2 - 2cdef + d^2e^2 + f^2)^2} - \frac{bd^3 \arctan(dx + c)ce}{(c^2f^2 - 2cdef + d^2e^2 + f^2)^2} + \frac{d^4b \arctan(dx + c)}{2(c^2f^2 - 2cdef + d^2e^2 + f^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))/(f\*x+e)^3,x)

[Out] -1/2\*d^2\*a/(d\*f\*x+d\*e)^2/f-1/2\*d^2\*b/(d\*f\*x+d\*e)^2/f\*arctan(d\*x+c)+1/2\*d^2\*b\*f/(c^2\*f^2-2\*c\*d\*e\*f+d^2\*e^2+f^2)^2\*arctan(d\*x+c)\*c^2-d^3\*b/(c^2\*f^2-2\*c\*d\*e\*f+d^2\*e^2+f^2)^2

$$d*ef+d^2*e^2+f^2)^2*\arctan(dx+c)*c*e+1/2*d^4*b/f/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)^2*\arctan(dx+c)*e^2+1/2*d^2*b*f/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)^2*\ln(1+(dx+c)^2)*c-1/2*d^3*b/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)^2*\ln(1+(dx+c)^2)*e-1/2*d^2*b*f/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)^2*\arctan(dx+c)-1/2*d^2*b/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)/(d*f*x+d*e)-d^2*b*f/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)^2*\ln(f*(dx+c)-c*f+d*e)*c+d^3*b/(c^2*f^2-2*c*d*ef+d^2*e^2+f^2)^2*\ln(f*(dx+c)-c*f+d*e)*e$$

**Maxima [A]** time = 1.56197, size = 552, normalized size = 2.43

$$-\frac{1}{2} \left( d \left( \frac{(d^2e - cdf) \log(d^2x^2 + 2cdx + c^2 + 1)}{d^4e^4 - 4cd^3e^3f + 2(3c^2 + 1)d^2e^2f^2 - 4(c^3 + c)def^3 + (c^4 + 2c^2 + 1)f^4} - \frac{2(d^2e - cdf)}{d^4e^4 - 4cd^3e^3f + 2(3c^2 + 1)d^2e^2f^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(dx+c))/(f\*x+e)^3,x, algorithm="maxima")

[Out]  $-1/2*(d*((d^2*e - c*d*f)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - 2*(d^2*e - c*d*f)*\log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4*e^2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*\arctan((d^2*x + c*d)/d)/((d^4*e^4*f - 4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c^4 + 2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) + \arctan(dx + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)$

**Fricas [B]** time = 11.9383, size = 1415, normalized size = 6.23

$$\frac{ad^4e^4 - (4ac - b)d^3e^3f + 2(3ac^2 - bc + a)d^2e^2f^2 - (4ac^3 - bc^2 + 4ac - b)def^3 + (ac^4 + 2ac^2 + a)f^4 + (bd^3e^2f^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(dx+c))/(f\*x+e)^3,x, algorithm="fricas")

[Out]  $-1/2*(a*d^4*e^4 - (4*a*c - b)*d^3*e^3*f + 2*(3*a*c^2 - b*c + a)*d^2*e^2*f^2 - (4*a*c^3 - b*c^2 + 4*a*c - b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 + (b*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x - (2*b*c*d^3*e^3*f - (5*b*c^2 + 3*b)*d^2*e^2*f^2 + 4*(b*c^3 + b*c)*d*e*f^3 - (b*c^4 + 2*b*c^2 + b)*f^4 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*f^4)*x^2 + 2*(b*d^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*\arctan(dx + c) + (b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*\log(f*x + e))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 + 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f^5 - 4*(c^3 + c)*d*e*f^6 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4 + 2*c^2 + 1)*e*f^6)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))/(f\*x+e)\*\*3,x)

[Out] Timed out

**Giac [B]** time = 6.7621, size = 1723, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))/(f\*x+e)^3,x, algorithm="giac")

[Out] 
$$-1/2*(\pi*b*c^2*d^2*f^4*x^2*\operatorname{sgn}(d*x + c) - 2*\pi*b*c*d^3*f^3*x^2*e*\operatorname{sgn}(d*x + c) - b*c^2*d^2*f^4*x^2*\arctan(d*x + c) + 2*b*c*d^3*f^3*x^2*\arctan(d*x + c)*e + \pi*b*d^4*f^2*x^2*e^2*\operatorname{sgn}(d*x + c) + 2*\pi*b*c^2*d^2*f^3*x*e*\operatorname{sgn}(d*x + c) - b*d^4*f^2*x^2*\arctan(d*x + c)*e^2 - 2*b*c^2*d^2*f^3*x*\arctan(d*x + c)*e - b*c*d^2*f^4*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + b*d^3*f^3*x^2*e*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*b*c*d^2*f^4*x^2*\log(\operatorname{abs}(f*x + e)) - 2*b*d^3*f^3*x^2*e*\log(\operatorname{abs}(f*x + e)) - \pi*b*d^2*f^4*x^2*\operatorname{sgn}(d*x + c) - 4*\pi*b*c*d^3*f^2*x*e^2*\operatorname{sgn}(d*x + c) + b*c^4*f^4*\arctan(d*x + c) + b*d^2*f^4*x^2*\arctan(d*x + c) + 4*b*c*d^3*f^2*x*\arctan(d*x + c)*e^2 - 4*b*c^3*d*f^3*\arctan(d*x + c)*e - 2*b*c*d^2*f^3*x*e*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*b*c*d^2*f^3*x*e*\log(\operatorname{abs}(f*x + e)) + 2*\pi*b*d^4*f*x*e^3*\operatorname{sgn}(d*x + c) + \pi*b*c^2*d^2*f^2*e^2*\operatorname{sgn}(d*x + c) - 2*\pi*b*d^2*f^3*x*e*\operatorname{sgn}(d*x + c) + a*c^4*f^4 + b*c^2*d*f^4*x - 2*b*d^4*f*x*\arctan(d*x + c)*e^3 + 5*b*c^2*d^2*f^2*\arctan(d*x + c)*e^2 - 4*a*c^3*d*f^3*e - 2*b*c*d^2*f^3*x*e + 2*b*d^2*f^3*x*\arctan(d*x + c)*e + 2*b*d^3*f^2*x*e^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 4*b*d^3*f^2*x*e^2*\log(\operatorname{abs}(f*x + e)) - 2*\pi*b*c*d^3*f*e^3*\operatorname{sgn}(d*x + c) + 2*b*c^2*f^4*\arctan(d*x + c) - 2*b*c*d^3*f*\arctan(d*x + c)*e^3 + 6*a*c^2*d^2*f^2*e^2 + b*d^3*f^2*x*e^2 + b*c^2*d*f^3*e - 4*b*c*d*f^3*\arctan(d*x + c)*e - b*c*d^2*f^2*e^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*b*c*d^2*f^2*e^2*\log(\operatorname{abs}(f*x + e)) + \pi*b*d^4*e^4*\operatorname{sgn}(d*x + c) - \pi*b*d^2*f^2*e^2*\operatorname{sgn}(d*x + c) + 2*a*c^2*f^4 + b*d*f^4*x - 4*a*c*d^3*f*e^3 - 2*b*c*d^2*f^2*e^2 + 3*b*d^2*f^2*\arctan(d*x + c)*e^2 - 4*a*c*d*f^3*e + b*d^3*f*e^3*\log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*b*d^3*f*e^3*\log(\operatorname{abs}(f*x + e)) + b*f^4*\arctan(d*x + c) + a*d^4*e^4 + b*d^3*f*e^3 + 2*a*d^2*f^2*e^2 + b*d*f^3*e + a*f^4)/(c^4*f^7*x^2 - 4*c^3*d*f^6*x^2*e + 6*c^2*d^2*f^5*x^2*e^2 + 2*c^4*f^6*x*e + 2*c^2*f^7*x^2 - 4*c*d^3*f^4*x^2*e^3 - 8*c^3*d*f^5*x*e^2 - 4*c*d*f^6*x^2*e + d^4*f^3*x^2*e^4 + 12*c^2*d^2*f^4*x*e^3 + c^4*f^5*e^2 + 2*d^2*f^5*x^2*e^2 + 4*c^2*f^6*x*e + f^7*x^2 - 8*c*d^3*f^3*x*e^4 - 4*c^3*d*f^4*e^3 - 8*c*d*f^5*x*e^2 + 2*d^4*f^2*x*e^5 + 6*c^2*d^2*f^3*e^4 + 4*d^2*f^4*x*e^3 + 2*c^2*f^5*e^2 + 2*f^6*x*e - 4*c*d^3*f^2*e^5 - 4*c*d*f^4*e^3 + d^4*f*e^6 + 2*d^2*f^3*e^4 + f^5*e^2)$$

### 3.31 $\int (e + fx)^2 (a + b \tan^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=382

$$\frac{ib^2(- (1 - 3c^2)f^2 - 6cdef + 3d^2e^2) \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{3d^3} + \frac{i(- (1 - 3c^2)f^2 - 6cdef + 3d^2e^2)(a + b \tan^{-1}(c + dx))}{3d^3}$$

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/ (3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
- (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/ (3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

**Rubi [A]** time = 0.574053, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.65$ , Rules used = {5047, 4864, 4846, 260, 4852, 321, 203, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2(- (1 - 3c^2)f^2 - 6cdef + 3d^2e^2) \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{3d^3} + \frac{i(- (1 - 3c^2)f^2 - 6cdef + 3d^2e^2)(a + b \tan^{-1}(c + dx))}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/ (3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
- (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/ (3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

#### Rule 5047

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^ (m
_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

#### Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.) + (e_.)*(x_.))^ (q_.), x_Sy
mbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

#### Rule 4846

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 321

Int[((c\_.)\*(x\_.))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 4984

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_.))^(m\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcTan[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && IGtQ[m, 0]

#### Rule 4884

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

#### Rule 4920

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[(a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]]/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rule 2402



```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rubi steps

$$\begin{aligned} \int (e + fx)^2 (a + b \tan^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \left(\frac{de - cf}{d} + \frac{fx}{d}\right)^2 (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst}\left(\int \left(\frac{3f^2(de - cf)(a + b \tan^{-1}(x))}{d^3} + \frac{f^3 x}{d^3}\right) dx, x, c + dx\right)}{3f} \\ &= \frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst}\left(\int \frac{((de - cf)(d^2 e^2 - 2cdef - 3f^2 + c^2 f^2) + f^3 x)}{d^3} dx, x, c + dx\right)}{3f} \\ &= -\frac{2abf(de - cf)x}{d^2} - \frac{bf^2(c + dx)^2 (a + b \tan^{-1}(c + dx))}{3d^3} + \frac{(e + fx)^3 (a + b \tan^{-1}(c + dx))^2}{3f} \\ &= \frac{b^2 f^2 x}{3d^2} - \frac{2abf(de - cf)x}{d^2} - \frac{2b^2 f(de - cf)(c + dx) \tan^{-1}(c + dx)}{d^3} - \frac{bf^2(c + dx) \tan^{-1}(c + dx)}{d^3} \\ &= \frac{b^2 f^2 x}{3d^2} - \frac{2abf(de - cf)x}{d^2} - \frac{b^2 f^2 \tan^{-1}(c + dx)}{3d^3} - \frac{2b^2 f(de - cf)(c + dx) \tan^{-1}(c + dx)}{d^3} \\ &= \frac{b^2 f^2 x}{3d^2} - \frac{2abf(de - cf)x}{d^2} - \frac{b^2 f^2 \tan^{-1}(c + dx)}{3d^3} - \frac{2b^2 f(de - cf)(c + dx) \tan^{-1}(c + dx)}{d^3} \\ &= \frac{b^2 f^2 x}{3d^2} - \frac{2abf(de - cf)x}{d^2} - \frac{b^2 f^2 \tan^{-1}(c + dx)}{3d^3} - \frac{2b^2 f(de - cf)(c + dx) \tan^{-1}(c + dx)}{d^3} \end{aligned}$$

**Mathematica [B]** time = 3.65537, size = 801, normalized size = 2.1

$$\frac{1}{3} a^2 f^2 x^3 + a^2 e f x^2 + a^2 e^2 x + \frac{ab(-dfx(6de - 4cf + dfx) + 2(f^2 c^3 - 3defc^2 + 3(d^2 e^2 - f^2)c + 3def + d^3 x(3e^2 + 3f^2))}{3d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(-(d*f*x*(6*d*e - 4*c*f +
d*f*x)) + 2*(3*d*e*f - 3*c^2*d*e*f + c^3*f^2 + 3*c*(d^2*e^2 - f^2) + d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2))*ArcTan[c + d*x] + (-3*d^2*e^2 + 6*c*d*e*f + (1
```

$$\begin{aligned}
& - 3c^2 f^2 \operatorname{Log}[1 + (c + dx)^2]) / (3d^3) + (b^2 e^2 (\operatorname{ArcTan}[c + dx] * \\
& (-I + c + dx) \operatorname{ArcTan}[c + dx] + 2 \operatorname{Log}[1 + E^{((2I) \operatorname{ArcTan}[c + dx])}]) - I * \\
& \operatorname{PolyLog}[2, -E^{((2I) \operatorname{ArcTan}[c + dx])}]) / d + (b^2 e f ((1 + (2I)c - c^2 + \\
& d^2 x^2) \operatorname{ArcTan}[c + dx]^2 - 2 \operatorname{ArcTan}[c + dx] * (c + dx + 2c \operatorname{Log}[1 + E^{((2I) \\
& \operatorname{ArcTan}[c + dx])}]) + \operatorname{Log}[1 + (c + dx)^2] + (2I)c \operatorname{PolyLog}[2, -E^{((2I) \\
& \operatorname{ArcTan}[c + dx])}]) / d^2 + (b^2 f^2 (1 + (c + dx)^2)^{3/2} ((c + dx) / \operatorname{Sqrt}[1 + (c + dx)^2] \\
& + (6c(c + dx) \operatorname{ArcTan}[c + dx]) / \operatorname{Sqrt}[1 + (c + dx)^2] \\
& + (3(c + dx) \operatorname{ArcTan}[c + dx]^2) / \operatorname{Sqrt}[1 + (c + dx)^2] + (3c^2 (c + dx) \\
& * \operatorname{ArcTan}[c + dx]^2) / \operatorname{Sqrt}[1 + (c + dx)^2] + I \operatorname{ArcTan}[c + dx]^2 \operatorname{Cos}[3 \operatorname{ArcTan}[c + dx]] \\
& - (3I)c^2 \operatorname{ArcTan}[c + dx]^2 \operatorname{Cos}[3 \operatorname{ArcTan}[c + dx]] - 2 \operatorname{ArcTan}[c + dx] \operatorname{Cos}[3 \operatorname{ArcTan}[c + dx]] * \\
& \operatorname{Log}[1 + E^{((2I) \operatorname{ArcTan}[c + dx])}] + 6c^2 \operatorname{ArcTan}[c + dx] \operatorname{Cos}[3 \operatorname{ArcTan}[c + dx]] * \\
& \operatorname{Log}[1 + E^{((2I) \operatorname{ArcTan}[c + dx])}] + 6c \operatorname{Cos}[3 \operatorname{ArcTan}[c + dx]] * \operatorname{Log}[1 / \operatorname{Sqrt}[1 + (c + dx)^2]] \\
& + ((3I - 12c - (9I)c^2) \operatorname{ArcTan}[c + dx]^2 + 2 \operatorname{ArcTan}[c + dx] * (-2 + (-3 + 9c^2) \operatorname{Log}[1 + \\
& E^{((2I) \operatorname{ArcTan}[c + dx])}]) + 18c \operatorname{Log}[1 / \operatorname{Sqrt}[1 + (c + dx)^2]]) / \operatorname{Sqrt}[1 + \\
& (c + dx)^2] - ((4I)(-1 + 3c^2) \operatorname{PolyLog}[2, -E^{((2I) \operatorname{ArcTan}[c + dx])}]) / \\
& (1 + (c + dx)^2)^{3/2} + \operatorname{Sin}[3 \operatorname{ArcTan}[c + dx]] + 6c \operatorname{ArcTan}[c + dx] \operatorname{Sin}[3 \operatorname{ArcTan}[c + dx]] \\
& - \operatorname{ArcTan}[c + dx]^2 \operatorname{Sin}[3 \operatorname{ArcTan}[c + dx]] + 3c^2 \operatorname{ArcTan}[c + dx]^2 \operatorname{Sin}[3 \operatorname{ArcTan}[c + dx]]) / (12d^3)
\end{aligned}$$

**Maple [B]** time = 0.133, size = 1622, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((f*x+e)^2*(a+b*\arctan(dx+c))^2,x)$

[Out]  $I/d^2 b^2 \ln(dx+c-I) \ln(1+(dx+c)^2) c e f + I/d^2 b^2 \ln(dx+c+I) \ln(1/2 I * (dx+c-I)) c e f - I/d^2 b^2 \ln(dx+c+I) \ln(1+(dx+c)^2) c e f - I/d^2 b^2 \ln(dx+c-I) \ln(-1/2 I * (dx+c+I)) c e f - 1/2 I/d^3 b^2 \operatorname{dilog}(1/2 I * (dx+c-I)) c^2 f^2 + 1/4 I/d^3 b^2 \ln(dx+c-I)^2 c^2 f^2 - 1/4 I/d^3 b^2 \ln(dx+c+I)^2 c^2 f^2 + 1/2 I/d^3 b^2 \operatorname{dilog}(-1/2 I * (dx+c+I)) c^2 f^2 + 1/2 I/d^3 b^2 \ln(dx+c+I) \ln(1+(dx+c)^2) e^{2+1/2 I/d^3 b^2 \ln(dx+c-I) \ln(-1/2 I * (dx+c+I))} e^{-1/2 I/d^3 b^2 \ln(dx+c-I) \ln(1+(dx+c)^2)} e^{-1/2 I/d^3 b^2 \ln(dx+c+I) \ln(1/2 I * (dx+c-I))} e^{-1/6 I/d^3 b^2 \ln(dx+c-I) \ln(-1/2 I * (dx+c+I))} f^2 + 1/6 I/d^3 b^2 \ln(dx+c-I) \ln(1+(dx+c)^2) f^2 + 1/6 I/d^3 b^2 \ln(dx+c+I) \ln(1/2 I * (dx+c-I)) f^2 - 1/6 I/d^3 b^2 \ln(dx+c+I) \ln(1+(dx+c)^2) f^2 - 2/d^3 a b f^2 \arctan(dx+c) c + 2/d^2 a b f \arctan(dx+c) e + 2 a b f \arctan(dx+c) e x^2 - 2/d^2 a b f c e + 5/3/d^3 a b f^2 c^2 + 1/2 I/d^3 b^2 \operatorname{dilog}(-1/2 I * (dx+c+I)) e^{-1/2 I/d^3 b^2 \operatorname{dilog}(1/2 I * (dx+c-I))} e^{-1/12 I/d^3 b^2 \ln(dx+c-I)^2} f^2 - 1/4 I/d^3 b^2 \ln(dx+c+I)^2 e^{2+1/4 I/d^3 b^2 \ln(dx+c-I)^2} e^{-1/6 I/d^3 b^2 \operatorname{dilog}(-1/2 I * (dx+c+I))} f^2 - 1/d^2 a b \ln(1+(dx+c)^2) + 1/12 I/d^3 b^2 \ln(dx+c+I)^2 f^2 + 1/3/d^3 b^2 f^2 \arctan(dx+c) \ln(1+(dx+c)^2) - 1/d^2 e^{2 b^2 \arctan(dx+c)} \ln(1+(dx+c)^2) - 1/3/d^2 a b f^2 x^2 + 1/d^2 b^2 f \arctan(dx+c)^2 e + 1/d^2 \arctan(dx+c)^2 b^2 c e^2 + b^2 f \arctan(dx+c)^2 e x^2 + 1/6 I/d^3 b^2 \operatorname{dilog}(1/2 I * (dx+c-I)) f^2 + 1/3/d^3 a b f^2 \ln(1+(dx+c)^2) + 1/d^2 b^2 f \ln(1+(dx+c)^2) e + 1/3/d^3 b^2 f^2 \arctan(dx+c)^2 c^3 - 1/3/d^3 b^2 f^2 \arctan(dx+c) x^2 + 5/3/d^3 b^2 f^2 \arctan(dx+c) c^2 - 1/d^3 b^2 f^2 \arctan(dx+c)^2 c + 2/3 a b f^2 \arctan(dx+c) x^3 - 1/d^3 b^2 f^2 \ln(1+(dx+c)^2) c - 1/d^3 a b f^2 \ln(1+(dx+c)^2) c^2 + 2/d^2 \arctan(dx+c) a b c e^{-1/d^3 b^2 f^2 \arctan(dx+c)} \ln(1+(dx+c)^2) c^2 - 2/d^2 b^2 f \arctan(dx+c) e c + 4/3/d^2 b^2 f^2 \arctan(dx+c) x c + 2/3/d^3 a b f^2 \arctan(dx+c) c^3 - 1/d^2 b^2 f \arctan(dx+c)^2 c^2 e^{-2/d^3 b^2 f \arctan(dx+c)} e x + 1/3 a^2 f^2 x^3 + a^2 x e^2 + 4/3 a b/d^2 f^2 c x^2 - 2 a b/d^2 f e x + 1/3 a^2/f e^3 + 1/3/d^3 b^2 f^2 c + 1/3 b^2 f^2 \arctan(dx+c)^2 x^3 + \arctan(dx+c)^2 x b^2 e^2 + a^2 f x^2 e + 2/d^2 a b f \ln(1+(dx+c)^2) c e - 1/2 I/d^3 b^2 \ln(dx+c+I) \ln(1/2 I * (dx+c-I)) c^2 f^2 + 1/2 I/d^2 b^2 \ln(dx+c+I)^2 c e f - 1/2 I/d$

$$\begin{aligned} &^2*b^2*\ln(d*x+c-I)^2*c*e*f-I/d^2*b^2*dilog(-1/2*I*(d*x+c+I))*c*e*f+1/2*I/d^3*b^2*\ln(d*x+c-I)*\ln(-1/2*I*(d*x+c+I))*c^2*f^2-1/2*I/d^3*b^2*\ln(d*x+c-I)*\ln(1+(d*x+c)^2)*c^2*f^2+1/2*I/d^3*b^2*\ln(d*x+c+I)*\ln(1+(d*x+c)^2)*c^2*f^2+2/d^2*b^2*f*\arctan(d*x+c)*\ln(1+(d*x+c)^2)*c*e+I/d^2*b^2*dilog(1/2*I*(d*x+c-I))*c*e*f-2/d^2*a*b*f*\arctan(d*x+c)*c^2*e+2*\arctan(d*x+c)*x*a*b*e^2+1/3*b^2*f^2*x/d^2-1/3*b^2*f^2*\arctan(d*x+c)/d^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arctan(d\*x+c))^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned} &3/4*b^2*c^2*e^2*\arctan(d*x + c)^2*\arctan((d^2*x + c*d)/d)/d - 1/4*(3*\arctan(d*x + c)*\arctan((d^2*x + c*d)/d)^2/d - \arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*e^2 + 1/3*a^2*f^2*x^3 + 36*b^2*d^2*f^2*\integrate(1/48*x^4*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*f^2*\integrate(1/48*x^4*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*d^2*e*f*\integrate(1/48*x^3*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c*d*f^2*\integrate(1/48*x^3*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*d^2*f^2*\integrate(1/48*x^4*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*d^2*e*f*\integrate(1/48*x^3*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*f^2*\integrate(1/48*x^3*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*d^2*e^2*\integrate(1/48*x^2*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144*b^2*c*d*e*f*\integrate(1/48*x^2*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*c^2*f^2*\integrate(1/48*x^2*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*f*\integrate(1/48*x^3*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*f^2*\integrate(1/48*x^3*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*e^2*\integrate(1/48*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e*f*\integrate(1/48*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*f^2*\integrate(1/48*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c*d*e^2*\integrate(1/48*x*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c^2*e*f*\integrate(1/48*x*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e^2*\integrate(1/48*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e*f*\integrate(1/48*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*e^2*\integrate(1/48*x*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c^2*e*f*\integrate(1/48*x*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e^2*\integrate(1/48*x*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*e^2*\integrate(1/48*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^2*e*f*x^2 + 3/4*b^2*e^2*\arctan(d*x + c)^2*\arctan((d^2*x + c*d)/d)/d - 8*b^2*d*f^2*\integrate(1/48*x^3*\arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^2*d*e*f*\integrate(1/48*x^2*\arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^2*d*e^2*\integrate(1/48*x*\arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1/4*(3*\arctan(d*x + c)*\arctan((d^2*x + c*d)/d)^2/d - \arctan((d^2*x + c*d)/d)^3/d)*b^2*e^2 + 2*(x^2*\arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*\arctan((d^2*x + c*d)/d)/d^3 - c*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*\arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*\arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2*x + 36*b^2*f^2*\integrate \end{aligned}$$

$e(1/48*x^2*\arctan(dx + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*f^2*$   
 $\text{integrate}(1/48*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +$   
 $c^2 + 1), x) + 72*b^2*e*f*\text{integrate}(1/48*x*\arctan(dx + c)^2/(d^2*x^2 + 2*c$   
 $*d*x + c^2 + 1), x) + 6*b^2*e*f*\text{integrate}(1/48*x*\log(d^2*x^2 + 2*c*d*x + c^$   
 $2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*e^2*\text{integrate}(1/48*\log(d$   
 $^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(dx +$   
 $c)*\arctan(dx + c) - \log((dx + c)^2 + 1))*a*b*e^2/d + 1/12*(b^2*f^2*x^3 +$   
 $3*b^2*e*f*x^2 + 3*b^2*e^2*x)*\arctan(dx + c)^2 - 1/48*(b^2*f^2*x^3 + 3*b^2$   
 $*e*f*x^2 + 3*b^2*e^2*x)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$\text{integral}(a^2f^2x^2 + 2a^2efx + a^2e^2 + (b^2f^2x^2 + 2b^2efx + b^2e^2)\arctan(dx + c)^2 + 2(abf^2x^2 + 2abefx + abe^2)\arctan(dx + c), x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*(a+b*arctan(dx+c))^2,x, algorithm="fricas")`

[Out] `integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x + b^2*e^2)*arctan(dx + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arctan(dx + c), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(c + dx))^2 (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**2*(a+b*atan(dx+c))**2,x)`

[Out] `Integral((a + b*atan(c + dx))**2*(e + f*x)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 (b \arctan(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*(a+b*arctan(dx+c))^2,x, algorithm="giac")`

[Out] `integrate((f*x + e)^2*(b*arctan(dx + c) + a)^2, x)`

### 3.32 $\int (e + fx) \left( a + b \tan^{-1}(c + dx) \right)^2 dx$

**Optimal.** Leaf size=222

$$\frac{ib^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{d^2} + \frac{i(de - cf)(a + b \tan^{-1}(c + dx))^2}{d^2} - \frac{(-cf + de + f)(de - (c + 1)f)(a + b \tan^{-1}(c + dx))^2}{2d^2f}$$

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f) + (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^2
```

**Rubi [A]** time = 0.373148, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5047, 4864, 4846, 260, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{d^2} + \frac{i(de - cf)(a + b \tan^{-1}(c + dx))^2}{d^2} - \frac{(-cf + de + f)(de - (c + 1)f)(a + b \tan^{-1}(c + dx))^2}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^2, x]
```

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f) + (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^2
```

#### Rule 5047

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]
```

#### Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 + c^2*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

#### Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^ (p_.), x_Symbol] := Simp[x*(a + b*ArcTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2)], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 4984

Int[(((a\_) + ArcTan[(c\_)\*(x\_)]\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_)^(m\_)))/((d\_) + (e\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcTan[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && IGtQ[m, 0]

#### Rule 4884

Int[((a\_) + ArcTan[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

#### Rule 4920

Int[(((a\_) + ArcTan[(c\_)\*(x\_)]\*(b\_))^(p\_)\*(x\_))/((d\_) + (e\_)\*(x\_)^2), x\_Symbol] := -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_) + ArcTan[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 + c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rule 2402

Int[Log[(c\_)/((d\_) + (e\_)\*(x\_))]/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2315

Int[Log[(c\_)\*(x\_)/((d\_) + (e\_)\*(x\_))], x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rubi steps

$$\begin{aligned}
\int (e + fx)(a + b \tan^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)(a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst}\left(\int \left(\frac{f^2(a+b \tan^{-1}(x))}{d^2} + \frac{(de-f-cf)(de+fx)}{d^2}\right) dx, x, c + dx\right)}{f} \\
&= \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))}{1+x^2} dx, x, c + dx\right)}{d^2 f} \\
&= -\frac{abfx}{d} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst}\left(\int \left(\frac{(de+f-cf)(de-(1+c)f)(a+b \tan^{-1}(x))}{1+x^2}\right) dx, x, c + dx\right)}{d^2} \\
&= -\frac{abfx}{d} - \frac{b^2 f(c + dx) \tan^{-1}(c + dx)}{d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^2}{2f} + \frac{(b^2 f(c + dx) \tan^{-1}(c + dx))^2}{d^2} \\
&= -\frac{abfx}{d} - \frac{b^2 f(c + dx) \tan^{-1}(c + dx)}{d^2} + \frac{i(de - cf)(a + b \tan^{-1}(c + dx))^2}{d^2} - \frac{(b^2 f(c + dx) \tan^{-1}(c + dx))^2}{d^2} \\
&= -\frac{abfx}{d} - \frac{b^2 f(c + dx) \tan^{-1}(c + dx)}{d^2} + \frac{i(de - cf)(a + b \tan^{-1}(c + dx))^2}{d^2} - \frac{(b^2 f(c + dx) \tan^{-1}(c + dx))^2}{d^2} \\
&= -\frac{abfx}{d} - \frac{b^2 f(c + dx) \tan^{-1}(c + dx)}{d^2} + \frac{i(de - cf)(a + b \tan^{-1}(c + dx))^2}{d^2} - \frac{(b^2 f(c + dx) \tan^{-1}(c + dx))^2}{d^2} \\
&= -\frac{abfx}{d} - \frac{b^2 f(c + dx) \tan^{-1}(c + dx)}{d^2} + \frac{i(de - cf)(a + b \tan^{-1}(c + dx))^2}{d^2} - \frac{(b^2 f(c + dx) \tan^{-1}(c + dx))^2}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.379037, size = 264, normalized size = 1.19

$$-2ib^2(de - cf)\text{PolyLog}\left(2, -e^{2i \tan^{-1}(c+dx)}\right) - a^2c^2f + 2a^2cde + 2a^2d^2ex + a^2d^2fx^2 - 2b \tan^{-1}(c + dx)\left(a(c^2f - 2cde - \dots)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] (2\*a^2\*c\*d\*e - 2\*a\*b\*c\*f - a^2\*c^2\*f + 2\*a^2\*d^2\*e\*x - 2\*a\*b\*d\*f\*x + a^2\*d^2\*f\*x^2 + b^2\*(-I + c + d\*x)\*(2\*d\*e + I\*f - c\*f + d\*f\*x)\*ArcTan[c + d\*x]^2 - 2\*b\*ArcTan[c + d\*x]\*(b\*f\*(c + d\*x) + a\*(-2\*c\*d\*e + c^2\*f - 2\*d^2\*e\*x - f\*(1 + d^2\*x^2))) - 2\*b\*(d\*e - c\*f)\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) + 4\*a\*b\*d\*e\*Log[1/Sqrt[1 + (c + d\*x)^2]] - 2\*b^2\*f\*Log[1/Sqrt[1 + (c + d\*x)^2]] - 4\*a\*b\*c\*f\*Log[1/Sqrt[1 + (c + d\*x)^2]] - (2\*I)\*b^2\*(d\*e - c\*f)\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])])/(2\*d^2)

**Maple [B]** time = 0.119, size = 748, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*(a+b\*arctan(d\*x+c))^2,x)

```
[Out] -1/d^2*b^2*arctan(d*x+c)*f*c-1/2/d^2*b^2*arctan(d*x+c)^2*f*c^2+1/d^2*a*b*f*
arctan(d*x+c)+a*b*f*arctan(d*x+c)*x^2+2*arctan(d*x+c)*x*a*b*e-1/2*I/d*b^2*d
ilog(1/2*I*(d*x+c-I))*e+1/4*I/d*b^2*ln(d*x+c-I)^2*e-1/4*I/d*b^2*ln(d*x+c+I)
^2*e+1/2*I/d*b^2*dilog(-1/2*I*(d*x+c+I))*e-1/d*a*b*ln(1+(d*x+c)^2)*e+1/d*ar
ctan(d*x+c)^2*b^2*c*e-1/d*b^2*arctan(d*x+c)*ln(1+(d*x+c)^2)*e-1/d^2*a*b*f*c
-1/d*b^2*arctan(d*x+c)*f*x+a^2*e*x+1/2*a^2*x^2*f-1/2*I/d^2*b^2*ln(d*x+c-I)*
ln(-1/2*I*(d*x+c+I))*c*f-1/2*I/d^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c+I)*c*f+1/2*
I/d^2*b^2*ln(1+(d*x+c)^2)*ln(d*x+c-I)*c*f+1/2*I/d^2*b^2*ln(d*x+c+I)*ln(1/2*
I*(d*x+c-I))*c*f-1/2/d^2*a^2*f*c^2+1/d*a^2*c*e+arctan(d*x+c)^2*x*b^2*e+1/2*
b^2*arctan(d*x+c)^2*f*x^2+1/2/d^2*b^2*arctan(d*x+c)^2*f-1/2*I/d*b^2*ln(d*x+
c+I)*ln(1/2*I*(d*x+c-I))*e+1/2*I/d*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*e+1
/2*I/d*b^2*ln(1+(d*x+c)^2)*ln(d*x+c+I)*e-1/4*I/d^2*b^2*ln(d*x+c-I)^2*c*f+1/
2*I/d^2*b^2*dilog(1/2*I*(d*x+c-I))*c*f+2/d*arctan(d*x+c)*a*b*c*e+1/d^2*b^2*
arctan(d*x+c)*ln(1+(d*x+c)^2)*c*f-1/d^2*a*b*arctan(d*x+c)*c^2*f-1/2*I/d*b^2
*ln(1+(d*x+c)^2)*ln(d*x+c-I)*e-1/2*I/d^2*b^2*dilog(-1/2*I*(d*x+c+I))*c*f+1/
4*I/d^2*b^2*ln(d*x+c+I)^2*c*f+1/d^2*a*b*ln(1+(d*x+c)^2)*c*f-a*b*f*x/d+1/2*b
^2*f*ln(1+(d*x+c)^2)/d^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan(d
*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*
e + 12*b^2*d^2*f*integrate(1/16*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + b^2*d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*integrate(1/16*x^2*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*f*integrate(1
/16*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*d^2*f*i
ntegrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*
e*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12
*b^2*c^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 4*b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*e*integr
ate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + b^2*c^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*c^2*e*integrate(1/
16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1
/2*a^2*f*x^2 + 3/4*b^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^
2*d*f*integrate(1/16*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
- 8*b^2*d*e*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x +
c*d)/d)^3/d)*b^2*e + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^
2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*
x + 12*b^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + b^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + b^2*e*integrate(1/16*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x + c)*arctan(d*x + c) -
```



$$\log((d*x + c)^2 + 1)*a*b*e/d + 1/8*(b^2*f*x^2 + 2*b^2*e*x)*\arctan(d*x + c)^2 - 1/32*(b^2*f*x^2 + 2*b^2*e*x)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(a^2fx + a^2e + (b^2fx + b^2e)\arctan(dx + c)^2 + 2(abfx + abe)\arctan(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arctan(d\*x+c))^2,x, algorithm="fricas")

[Out] integral(a^2\*f\*x + a^2\*e + (b^2\*f\*x + b^2\*e)\*arctan(d\*x + c)^2 + 2\*(a\*b\*f\*x + a\*b\*e)\*arctan(d\*x + c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(c + dx))^2 (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*atan(d\*x+c))\*\*2,x)

[Out] Integral((a + b\*atan(c + d\*x))\*\*2\*(e + f\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)(b \arctan(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arctan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((f\*x + e)\*(b\*arctan(d\*x + c) + a)^2, x)

### 3.33 $\int (a + b \tan^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=102

$$\frac{ib^2 \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{d} + \frac{(c+dx)(a+b \tan^{-1}(c+dx))^2}{d} + \frac{i(a+b \tan^{-1}(c+dx))^2}{d} + \frac{2b \log\left(\frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{d}$$

[Out] (I\*(a + b\*ArcTan[c + d\*x])^2)/d + ((c + d\*x)\*(a + b\*ArcTan[c + d\*x])^2)/d + (2\*b\*(a + b\*ArcTan[c + d\*x])\*Log[2/(1 + I\*(c + d\*x))])/d + (I\*b^2\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/d

**Rubi [A]** time = 0.106977, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5039, 4846, 4920, 4854, 2402, 2315}

$$\frac{ib^2 \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{d} + \frac{(c+dx)(a+b \tan^{-1}(c+dx))^2}{d} + \frac{i(a+b \tan^{-1}(c+dx))^2}{d} + \frac{2b \log\left(\frac{2}{1+i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^2, x]

[Out] (I\*(a + b\*ArcTan[c + d\*x])^2)/d + ((c + d\*x)\*(a + b\*ArcTan[c + d\*x])^2)/d + (2\*b\*(a + b\*ArcTan[c + d\*x])\*Log[2/(1 + I\*(c + d\*x))])/d + (I\*b^2\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/d

#### Rule 5039

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^p], x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 4846

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^p], x\_Symbol] :> Simp[x\*(a + b\*ArcTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 4920

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^p)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] :> -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^p/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{

$c, d, e, f, g, x$  && EqQ[ $c, 2*d$ ] && EqQ[ $e^2*f + d^2*g, 0$ ]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned} \int (a + b \tan^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d} \\ &= \frac{(c + dx)(a + b \tan^{-1}(c + dx))^2}{d} - \frac{(2b) \text{Subst}\left(\int \frac{x(a + b \tan^{-1}(x))}{1+x^2} dx, x, c + dx\right)}{d} \\ &= \frac{i(a + b \tan^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^2}{d} + \frac{(2b) \text{Subst}\left(\int \frac{a + b \tan^{-1}(x)}{i-x} dx\right)}{d} \\ &= \frac{i(a + b \tan^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^2}{d} + \frac{2b(a + b \tan^{-1}(c + dx))}{d} \\ &= \frac{i(a + b \tan^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^2}{d} + \frac{2b(a + b \tan^{-1}(c + dx))}{d} \\ &= \frac{i(a + b \tan^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^2}{d} + \frac{2b(a + b \tan^{-1}(c + dx))}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0730931, size = 109, normalized size = 1.07

$$\frac{-ib^2 \text{PolyLog}\left(2, -e^{2i \tan^{-1}(c+dx)}\right) + a\left(ac + adx + 2b \log\left(\frac{1}{\sqrt{(c+dx)^2+1}}\right)\right) + 2b \tan^{-1}(c + dx)\left(ac + adx + b \log\left(1 + e^{2i \tan^{-1}(c+dx)}\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^2, x]

[Out] (b^2\*(-I + c + d\*x)\*ArcTan[c + d\*x]^2 + 2\*b\*ArcTan[c + d\*x]\*(a\*c + a\*d\*x + b\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) + a\*(a\*c + a\*d\*x + 2\*b\*Log[1/Sqrt[1 + (c + d\*x)^2]]) - I\*b^2\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])])/d

**Maple [A]** time = 0.105, size = 180, normalized size = 1.8

$$(\arctan(dx + c))^2 x b^2 - \frac{i(\arctan(dx + c))^2 b^2}{d} + \frac{(\arctan(dx + c))^2 b^2 c}{d} + 2 \arctan(dx + c) x a b + 2 \frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^2, x)

[Out] arctan(d\*x+c)^2\*x\*b^2-I/d\*arctan(d\*x+c)^2\*b^2+1/d\*arctan(d\*x+c)^2\*b^2\*c+2\*a rctan(d\*x+c)\*x\*a\*b+2/d\*arctan(d\*x+c)\*ln((1+I\*(d\*x+c))^2/(1+(d\*x+c)^2)+1)\*b^2+2/d\*arctan(d\*x+c)\*a\*b\*c-I/d\*polylog(2, -(1+I\*(d\*x+c))^2/(1+(d\*x+c)^2))\*b^2

$+a^2*x-1/d*a*b*\ln(1+(d*x+c)^2)+a^2*c/d$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2,x, algorithm="maxima")

[Out]  $1/16*(12*c^2*\arctan(d*x + c)^2*\arctan((d^2*x + c*d)/d)/d - 4*(3*\arctan(d*x + c)*\arctan((d^2*x + c*d)/d)^2/d - \arctan((d^2*x + c*d)/d)^3/d)*c^2 + 4*x*a$   
 $rctan(d*x + c)^2 + 192*d^2*\int(1/16*x^2*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*d^2*\int(1/16*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*c*d*\int(1/16*x*\arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*d^2*\int(1/16*x^2*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 32*c*d*\int(1/16*x*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*c*d*\int(1/16*x*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*c^2*\int(1/16*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - x*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*\arctan(d*x + c)^2*\arctan((d^2*x + c*d)/d)/d - 12*\arctan(d*x + c)*\arctan((d^2*x + c*d)/d)^2/d + 4*\arctan((d^2*x + c*d)/d)^3/d - 128*d*\int(1/16*x*\arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*\int(1/16*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x))*b^2 + a^2*x + (2*(d*x + c)*\arctan(d*x + c) - \log((d*x + c)^2 + 1))$   
 $*a*b/d$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int (b^2 \arctan(dx + c)^2 + 2ab \arctan(dx + c) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2,x, algorithm="fricas")

[Out] integral(b^2\*arctan(d\*x + c)^2 + 2\*a\*b\*arctan(d\*x + c) + a^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))\*\*2,x)

[Out] Integral((a + b\*atan(c + d\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2, x)
```

$$3.34 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{e+fx} dx$$

**Optimal.** Leaf size=261

$$\frac{ib(a+b \tan^{-1}(c+dx)) \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} + \frac{ib \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{f} + \frac{b^2}{f}$$

[Out] -(((a + b\*ArcTan[c + d\*x])^2\*Log[2/(1 - I\*(c + d\*x))])/f) + ((a + b\*ArcTan[c + d\*x])^2\*Log[(2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f + (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - 2/(1 - I\*(c + d\*x))])/f - (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f - (b^2\*PolyLog[3, 1 - 2/(1 - I\*(c + d\*x))])/(2\*f) + (b^2\*PolyLog[3, 1 - (2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/(2\*f))

**Rubi [A]** time = 0.164644, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {5047, 4858}

$$\frac{ib(a+b \tan^{-1}(c+dx)) \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} + \frac{ib \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{f} + \frac{b^2}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^2/(e + f\*x), x]

[Out] -(((a + b\*ArcTan[c + d\*x])^2\*Log[2/(1 - I\*(c + d\*x))])/f) + ((a + b\*ArcTan[c + d\*x])^2\*Log[(2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f + (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - 2/(1 - I\*(c + d\*x))])/f - (I\*b\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/f - (b^2\*PolyLog[3, 1 - 2/(1 - I\*(c + d\*x))])/(2\*f) + (b^2\*PolyLog[3, 1 - (2\*d\*(e + f\*x))/((d\*e + I\*f - c\*f)\*(1 - I\*(c + d\*x)))]/(2\*f))

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.)^p\_.\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4858

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.)^2/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])^2\*Log[2/(1 - I\*c\*x)])/e, x] + (Simp[((a + b\*ArcTan[c\*x])^2\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x] + Simp[(I\*b\*(a + b\*ArcTan[c\*x])\*PolyLog[2, 1 - 2/(1 - I\*c\*x)])/e, x] - Simp[(I\*b\*(a + b\*ArcTan[c\*x])\*PolyLog[2, 1 - (2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x] - Simp[(b^2\*PolyLog[3, 1 - 2/(1 - I\*c\*x)]]/(2\*e), x] + Simp[(b^2\*PolyLog[3, 1 - (2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))]]/(2\*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rubi steps



$$\begin{aligned} &)^2+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I* \\ &f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi-1/2*I*b^2/f*arctan(d*x+c \\ &)^2*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/( \\ &1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+ \\ &c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*Pi+b^2*c/(I*f+c*f-d \\ &*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I \\ &*f-c*f))+I*a*b/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-I*b^2/f*Pi*arctan(d*x \\ &+c)^2-I*a*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))+2*a*b*ln(f*(d*x+c)-c*f+d \\ &*e)/f*arctan(d*x+c)+I*b^2/f*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d* \\ &x+c)^2))+I*b^2/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c \\ &))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f)) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(fx + e)}{f} + \int \frac{12b^2 \arctan(dx + c)^2 + b^2 \log(d^2x^2 + 2cdx + c^2 + 1)^2 + 32ab \arctan(dx + c)}{16(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(f\*x+e),x, algorithm="maxima")

[Out] a^2\*log(f\*x + e)/f + integrate(1/16\*(12\*b^2\*arctan(d\*x + c)^2 + b^2\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)^2 + 32\*a\*b\*arctan(d\*x + c))/(f\*x + e), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(dx + c)^2 + 2ab \arctan(dx + c) + a^2}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(f\*x+e),x, algorithm="fricas")

[Out] integral((b^2\*arctan(d\*x + c)^2 + 2\*a\*b\*arctan(d\*x + c) + a^2)/(f\*x + e), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(c + dx))^2}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))\*2/(f\*x+e),x)

[Out] Integral((a + b\*atan(c + d\*x))\*2/(e + f\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^2}{fx + e} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(f*x + e), x)
```

$$3.35 \quad \int \frac{(a+b \tan^{-1}(c+dx))^2}{(e+fx)^2} dx$$

**Optimal.** Leaf size=568

$$\frac{ib^2 d \text{PolyLog}\left(2, 1 - \frac{2}{1-i(c+dx)}\right)}{(c^2+1)f^2 - 2cdef + d^2e^2} - \frac{ib^2 d \text{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{(c^2+1)f^2 - 2cdef + d^2e^2} + \frac{ib^2 d \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{(c^2+1)f^2 - 2cdef + d^2e^2} + \frac{2abd \log(\dots)}{(de-cf)}$$

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + I
*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
(I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

**Rubi [A]** time = 1.35103, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 25, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.25$ , Rules used = {5045, 1982, 705, 31, 634, 618, 204, 628, 6741, 5057, 706, 635, 203, 260, 6688, 12, 6725, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854}

$$\frac{ib^2 d \text{PolyLog}\left(2, 1 - \frac{2}{1-i(c+dx)}\right)}{(c^2+1)f^2 - 2cdef + d^2e^2} - \frac{ib^2 d \text{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(de+(-c+i)f)}\right)}{(c^2+1)f^2 - 2cdef + d^2e^2} + \frac{ib^2 d \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{(c^2+1)f^2 - 2cdef + d^2e^2} + \frac{2abd \log(\dots)}{(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2, x]
```

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + (
I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
(I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

**Rule 5045**

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)])*(b_.)^(p_.)*((e_.) + (f_.)*(x_.))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
+ d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

&& IGtQ[p, 0] && ILtQ[m, -1]

### Rule 1982

Int[(u\_)^(m\_)\*(v\_)^(p\_), x\_Symbol] := Int[ExpandToSum[u, x]^m\*ExpandToSum[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

### Rule 705

Int[1/(((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 - b\*d\*e + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 - b\*d\*e + a\*e^2), Int[(c\*d - b\*e - c\*e\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(p\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

### Rule 5057

Int[((a\_) + ArcTan[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_)\*((A\_) + (B\_)\*(x\_) + (C\_)\*(x\_)^2)^(q\_), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(C/d^2 + (C\*x^2)/d^2)^q\*(a + b\*ArcTan[x])^p, x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B\*(1 + c^2) - 2\*A\*c\*d, 0] && EqQ[2\*c\*C - B\*d, 0]

Rule 706

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 4856

```
Int[((a_) + ArcTan[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
)))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[
c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_)*(x_)/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)*((f_) + (g_)*(x_)^(m_))/((
d_) + (e_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symbo
l] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)*(x_)/((d_) + (e_)*(x_)^2),
x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[(((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol]
:= -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^2}{(e + fx)^2} dx &= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \tan^{-1}(c + dx)}{(e + fx)(1 + (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \tan^{-1}(c + dx)}{(e + fx)(1 + c^2 + 2cdx + d^2x^2)} dx}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left( \int \frac{a + b \tan^{-1}(x)}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left( \int \frac{d(a + b \tan^{-1}(x))}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left( \int \frac{a + b \tan^{-1}(x)}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left( \int \left( \frac{a}{(de - cf + fx)(1 + x^2)} + \frac{b \tan^{-1}(x)}{(de - cf + fx)(1 + x^2)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2abd) \text{Subst} \left( \int \frac{1}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} + \frac{(2b^2d) \text{Subst} \left( \int \frac{\tan^{-1}(x)}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b^2d) \text{Subst} \left( \int \left( \frac{f^2 \tan^{-1}(x)}{(d^2e^2 - 2cdef + (1 + c^2)f^2)(de - cf + fx)} + \frac{(de - cf - fx)}{(d^2e^2 - 2cdef + (1 + c^2)f^2)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{2abd \log(e + fx)}{f^2 + (de - cf)^2} + \frac{(2b^2d) \text{Subst} \left( \int \frac{(de - cf - fx) \tan^{-1}(x)}{1 + x^2} dx, x, c + dx \right)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{2abd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} - \frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{2abd \log(e + fx)}{f^2 + (de - cf)^2} - \frac{2b^2d \tan^{-1}(c + dx)}{d^2e^2 - 2cdef + (1 + c^2)f^2} \\
&= \frac{2abd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} - \frac{(a + b \tan^{-1}(c + dx))^2}{f(e + fx)} + \frac{2abd \log(e + fx)}{f^2 + (de - cf)^2} - \frac{2b^2d \tan^{-1}(c + dx)}{d^2e^2 - 2cdef + (1 + c^2)f^2} \\
&= \frac{2abd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{ib^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{b^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{2abd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{ib^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{b^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{2abd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{ib^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{b^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{2abd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{ib^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{b^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)}
\end{aligned}$$

**Mathematica [A]** time = 6.31543, size = 419, normalized size = 0.74

$$b^2 d(e+fx) \left( \frac{i \operatorname{PolyLog}\left(2, \exp\left(2i \left(\tan^{-1}\left(\frac{de-cf}{f}\right) + \tan^{-1}(c+dx)\right)\right)\right) - 2 \left(\tan^{-1}\left(\frac{de-cf}{f}\right) + \tan^{-1}(c+dx)\right) \log\left(1 - \exp\left(2i \left(\tan^{-1}\left(\frac{de-cf}{f}\right) + \tan^{-1}(c+dx)\right)\right)\right) - i \tan^{-1}(c+dx) \left(\pi - 2\right)}{f^2 \left(\frac{(de-cf)^2}{f^2} + 1\right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^2/(e + f\*x)^2, x]

[Out] 
$$\begin{aligned} & \left( -\frac{a^2}{f} + (2ab(-((-(cde) + f + c^2f - d^2ex + cdfx)) \operatorname{ArcTan}[c + dx] + d(e + fx) \operatorname{Log}[(d(e + fx))/\sqrt{1 + (c + dx)^2}])) / (d^2e^2 - 2cdex + (1 + c^2)f^2) + (b^2d(e + fx) * (-((E^{(I \operatorname{ArcTan}[(d^2e - c^2f)/f] * \operatorname{ArcTan}[c + dx]^2) / (f \sqrt{1 + (d^2e - c^2f)/f^2})) + ((c + dx) \operatorname{ArcTan}[c + dx]^2) / (d(e + fx)) - ((d^2e - c^2f) * (-I) * (\pi - 2 \operatorname{ArcTan}[(d^2e - c^2f)/f] * \operatorname{ArcTan}[c + dx] - \pi \operatorname{Log}[1 + E^{((-2I) \operatorname{ArcTan}[c + dx])}] - 2(\operatorname{ArcTan}[(d^2e - c^2f)/f] + \operatorname{ArcTan}[c + dx]) * \operatorname{Log}[1 - E^{((2I) \operatorname{ArcTan}[(d^2e - c^2f)/f] + \operatorname{ArcTan}[c + dx])}] + \pi \operatorname{Log}[1/\sqrt{1 + (c + dx)^2}] + 2 \operatorname{ArcTan}[(d^2e - c^2f)/f] * \operatorname{Log}[\sin[\operatorname{ArcTan}[(d^2e - c^2f)/f] + \operatorname{ArcTan}[c + dx]]) + I \operatorname{PolyLog}[2, E^{((2I) \operatorname{ArcTan}[(d^2e - c^2f)/f] + \operatorname{ArcTan}[c + dx])}])) / (f^2(1 + (d^2e - c^2f)/f^2))) / (d^2e - c^2f) \right) / (e + fx) \end{aligned}$$

**Maple [A]** time = 0.132, size = 1087, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^2/(f\*x+e)^2, x)

[Out] 
$$\begin{aligned} & -\frac{da^2}{(dfx+de)/f - db^2/(dfx+de)/f \operatorname{arctan}(dx+c)^2 - db^2 \operatorname{arctan}(dx+c) / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(1 + (dx+c)^2) - db^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{arctan}(dx+c)^2 * c + d^2b^2 / f / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{arctan}(dx+c)^2 * e + 2db^2 \operatorname{arctan}(dx+c) / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(f * (dx+c) - cf + de) + Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{dilog}((If - f * (dx+c)) / (de + If - cf)) - Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{dilog}((If + f * (dx+c)) / (If + cf - de)) - 1/4 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(dx+c+I)^2 - 1/2 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{dilog}(1/2 * I * (dx+c-I)) + 1/2 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(dx+c+I) * \ln(1 + (dx+c)^2) + 1/4 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(dx+c-I)^2 + Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(f * (dx+c) - cf + de) * \ln((If - f * (dx+c)) / (de + If - cf)) + 1/2 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(dx+c-I) * \ln(-1/2 * I * (dx+c+I)) - Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(f * (dx+c) - cf + de) * \ln((If + f * (dx+c)) / (If + cf - de)) - 1/2 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(dx+c-I) * \ln(1 + (dx+c)^2) - 1/2 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(dx+c+I) * \ln(1/2 * I * (dx+c-I)) + 1/2 * Idb^2 / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{dilog}(-1/2 * I * (dx+c+I)) - 2da * b / (dfx+de) / f \operatorname{arctan}(dx+c) - da * b / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(1 + (dx+c)^2) - 2da * b / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{arctan}(dx+c) * c + 2d^2a * b / f / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \operatorname{arctan}(dx+c) * e + 2da * b / (c^2f^2 - 2cdex + d^2e^2 + f^2) * \ln(f * (dx+c) - cf + de) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( d \left( \frac{2(d^2e - cdf) \arctan\left(\frac{d^2x+cd}{d}\right) - \log(d^2x^2 + 2cdx + c^2 + 1)}{(d^2e^2f - 2cdef^2 + (c^2 + 1)f^3)d} - \frac{\log(d^2x^2 + 2cdx + c^2 + 1)}{d^2e^2 - 2cdef + (c^2 + 1)f^2} + \frac{2 \log(fx + e)}{d^2e^2 - 2cdef + (c^2 + 1)f^2} \right) - \frac{2 \arctan(dx + c)}{f^2x + ef} \right) ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(f\*x+e)^2,x, algorithm="maxima")

[Out] (d\*(2\*(d^2\*e - c\*d\*f)\*arctan((d^2\*x + c\*d)/d)/((d^2\*e^2\*f - 2\*c\*d\*e\*f^2 + (c^2 + 1)\*f^3)\*d) - log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)/(d^2\*e^2 - 2\*c\*d\*e\*f + (c^2 + 1)\*f^2) + 2\*log(f\*x + e)/(d^2\*e^2 - 2\*c\*d\*e\*f + (c^2 + 1)\*f^2)) - 2\*arctan(d\*x + c)/(f^2\*x + e\*f))\*a\*b - 1/16\*(4\*arctan(d\*x + c)^2 - 16\*(f^2\*x + e\*f)\*integrate(1/16\*(12\*(d^2\*f\*x^2 + 2\*c\*d\*f\*x + (c^2 + 1)\*f)\*arctan(d\*x + c)^2 + (d^2\*f\*x^2 + 2\*c\*d\*f\*x + (c^2 + 1)\*f)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)^2 + 8\*(d\*f\*x + d\*e)\*arctan(d\*x + c) - 4\*(d^2\*f\*x^2 + c\*d\*e + (d^2\*e + c\*d\*f)\*x)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1))/(d^2\*f^3\*x^4 + (c^2 + 1)\*e^2\*f + 2\*(d^2\*e\*f^2 + c\*d\*f^3)\*x^3 + (d^2\*e^2\*f + 4\*c\*d\*e\*f^2 + (c^2 + 1)\*f^3)\*x^2 + 2\*(c\*d\*e^2\*f + (c^2 + 1)\*e\*f^2)\*x), x) - log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)^2)\*b^2/(f^2\*x + e\*f) - a^2/(f^2\*x + e\*f)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(dx + c)^2 + 2ab \arctan(dx + c) + a^2}{f^2x^2 + 2efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^2/(f\*x+e)^2,x, algorithm="fricas")

[Out] integral((b^2\*arctan(d\*x + c)^2 + 2\*a\*b\*arctan(d\*x + c) + a^2)/(f^2\*x^2 + 2\*e\*f\*x + e^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))^2/(f\*x+e)^2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^2}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(d*x + c) + a)^2/(f*x + e)^2, x)
```

### 3.36 $\int (e + fx)^2 \left( a + b \tan^{-1}(c + dx) \right)^3 dx$

**Optimal.** Leaf size=564

$$\frac{ib^2 \left( -(1-3c^2)f^2 - 6cdef + 3d^2e^2 \right) \text{PolyLog} \left( 2, 1 - \frac{2}{1+i(c+dx)} \right) (a + b \tan^{-1}(c + dx))}{d^3} + \frac{b^3 \left( -(1-3c^2)f^2 - 6cdef + 3d^2e^2 \right)}{2d^3}$$

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/d^3 - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/d^3
```

**Rubi [A]** time = 0.937082, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5047, 4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4984, 4994, 6610}

$$\frac{ib^2 \left( -(1-3c^2)f^2 - 6cdef + 3d^2e^2 \right) \text{PolyLog} \left( 2, 1 - \frac{2}{1+i(c+dx)} \right) (a + b \tan^{-1}(c + dx))}{d^3} + \frac{b^3 \left( -(1-3c^2)f^2 - 6cdef + 3d^2e^2 \right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/d^3 - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/d^3
```

**Rule 5047**

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)])*(b_.)^p_.)*((e_.) + (f_.)*(x_.))^m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
```

tQ[p, 0]

#### Rule 4864

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcTan[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

#### Rule 4846

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 4920

Int((((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] :> -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 4852

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 4916

Int((((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((f\_.)\*(x\_))^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] :> Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcTan[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcTan[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

#### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

#### Rule 4884

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_)^(p_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

#### Rule 4984

```
Int[(((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)*((f_) + (g_)*(x_)^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

#### Rule 4994

```
Int[(Log[u]*((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

#### Rule 6610

```
Int[(u)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

#### Rubi steps



$$\begin{aligned}
& 2) * \text{ArcTan}[c + d*x]^2 - (c + d*x) * \text{ArcTan}[c + d*x] * (3 + 2*c * \text{ArcTan}[c + d*x]) \\
& - 6 * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] - 6 * c * \text{ArcTan}[c + d*x] * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] \\
& + (3*I) * (1 + 2*c * \text{ArcTan}[c + d*x]) * \text{PolyLog}[2, -E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] - 3 * c * \text{PolyLog}[3, -E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] \\
& ) / d^2 + (a * b^2 * f^2 * (1 + (c + d*x)^2)^{3/2} * ((c + d*x) / \text{Sqrt}[1 + (c + d*x)^2] + (6 * c * (c + d*x) * \text{ArcTan}[c + d*x]) / \text{Sqrt}[1 + (c + d*x)^2] \\
& + (3 * (c + d*x) * \text{ArcTan}[c + d*x]^2) / \text{Sqrt}[1 + (c + d*x)^2] + I * \text{ArcTan}[c + d*x]^2 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] - (3 * I) * c^2 * \text{ArcTan}[c + d*x]^2 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] \\
& - 2 * \text{ArcTan}[c + d*x] * \text{Cos}[3 * \text{ArcTan}[c + d*x]] * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] + 6 * c^2 * \text{ArcTan}[c + d*x] * \text{Cos}[3 * \text{ArcTan}[c + d*x]] * \text{Log}[1 / \text{Sqrt}[1 + (c + d*x)^2]] \\
& + (\text{ArcTan}[c + d*x] * (-4 + (3 * I - 12 * c - (9 * I) * c^2) * \text{ArcTan}[c + d*x]) + 6 * (-1 + 3 * c^2) * \text{ArcTan}[c + d*x] * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] \\
& + 18 * c * \text{Log}[1 / \text{Sqrt}[1 + (c + d*x)^2]]) / \text{Sqrt}[1 + (c + d*x)^2] - ((4 * I) * (-1 + 3 * c^2) * \text{PolyLog}[2, -E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] / (1 + (c + d*x)^2)^{3/2} \\
& + \text{Sin}[3 * \text{ArcTan}[c + d*x]] + 6 * c * \text{ArcTan}[c + d*x] * \text{Sin}[3 * \text{ArcTan}[c + d*x]] - \text{ArcTan}[c + d*x]^2 * \text{Sin}[3 * \text{ArcTan}[c + d*x]] + 3 * c^2 * \text{ArcTan}[c + d*x]^2 * \text{Sin}[3 * \text{ArcTan}[c + d*x]]) / (4 * d^3) \\
& + (b^3 * f^2 * ((-I) * (3 * c - \text{ArcTan}[c + d*x] + 3 * c^2 * \text{ArcTan}[c + d*x]) * \text{PolyLog}[2, -E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] + ((1 + (c + d*x)^2)^{3/2} * ((3 * (c + d*x) * \text{ArcTan}[c + d*x]) / \text{Sqrt}[1 + (c + d*x)^2] + (9 * c * (c + d*x) * \text{ArcTan}[c + d*x]^2) / \text{Sqrt}[1 + (c + d*x)^2] + (3 * (c + d*x) * \text{ArcTan}[c + d*x]^3) / \text{Sqrt}[1 + (c + d*x)^2] + (3 * c^2 * (c + d*x) * \text{ArcTan}[c + d*x]^3) / \text{Sqrt}[1 + (c + d*x)^2] - (9 * I) * c * \text{ArcTan}[c + d*x]^2 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] + I * \text{ArcTan}[c + d*x]^3 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] - (3 * I) * c^2 * \text{ArcTan}[c + d*x]^3 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] + 18 * c * \text{ArcTan}[c + d*x] * \text{Cos}[3 * \text{ArcTan}[c + d*x]] * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] - 3 * \text{ArcTan}[c + d*x]^2 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] + 9 * c^2 * \text{ArcTan}[c + d*x]^2 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] + 3 * \text{Cos}[3 * \text{ArcTan}[c + d*x]] * \text{Log}[1 / \text{Sqrt}[1 + (c + d*x)^2]] + (3 * (\text{ArcTan}[c + d*x]^2 * (-2 - (9 * I) * c + I * \text{ArcTan}[c + d*x] - 4 * c * \text{ArcTan}[c + d*x] - (3 * I) * c^2 * \text{ArcTan}[c + d*x]) + 3 * \text{ArcTan}[c + d*x] * (6 * c - \text{ArcTan}[c + d*x] + 3 * c^2 * \text{ArcTan}[c + d*x]) * \text{Log}[1 + E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] + 3 * \text{Log}[1 / \text{Sqrt}[1 + (c + d*x)^2]])) / \text{Sqrt}[1 + (c + d*x)^2] + (6 * (-1 + 3 * c^2) * \text{PolyLog}[3, -E^{\left((2*I) * \text{ArcTan}[c + d*x]\right)}] / (1 + (c + d*x)^2)^{3/2} + 3 * \text{ArcTan}[c + d*x] * \text{Sin}[3 * \text{ArcTan}[c + d*x]] + 9 * c * \text{ArcTan}[c + d*x]^2 * \text{Sin}[3 * \text{ArcTan}[c + d*x]] - \text{ArcTan}[c + d*x]^3 * \text{Sin}[3 * \text{ArcTan}[c + d*x]] + 3 * c^2 * \text{ArcTan}[c + d*x]^3 * \text{Sin}[3 * \text{ArcTan}[c + d*x]])) / 12)) / d^3
\end{aligned}$$

**Maple [C]** time = 2.95, size = 6682, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arctan(d\*x+c))^3,x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arctan(d\*x+c))^3,x, algorithm="maxima")

```
[Out] 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e^2 + 1/3*a^3*f^2*x^3 + 7/8*b^3*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*e*f*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*e*f*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*e*f*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^3*e*f*x^2 + 3*a*b^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^3*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d*f^2*integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*e*f*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*e^2*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*ar
```

```

ctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^
3*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/
d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*ar
ctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)
/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^
3*e^2*x + 28*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 3*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*f^2*in
tegrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b
^3*e*f*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
+ 6*b^3*e*f*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*e^2*integrate(1/3
2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*
b*e^2/d + 1/24*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)^
3 - 1/32*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$\int (a^3 f^2 x^2 + 2 a^3 e f x + a^3 e^2 + (b^3 f^2 x^2 + 2 b^3 e f x + b^3 e^2) \arctan(dx + c)^3 + 3 (ab^2 f^2 x^2 + 2 ab^2 e f x + ab^2 e^2) \arctan(dx + c)^2) dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arctan(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(a^3\*f^2\*x^2 + 2\*a^3\*e\*f\*x + a^3\*e^2 + (b^3\*f^2\*x^2 + 2\*b^3\*e\*f\*x + b^3\*e^2)\*arctan(d\*x + c)^3 + 3\*(a\*b^2\*f^2\*x^2 + 2\*a\*b^2\*e\*f\*x + a\*b^2\*e^2)\*arctan(d\*x + c)^2 + 3\*(a^2\*b\*f^2\*x^2 + 2\*a^2\*b\*e\*f\*x + a^2\*b\*e^2)\*arctan(d\*x + c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(c + dx))^3 (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*(a+b\*atan(d\*x+c))\*\*3,x)

[Out] Integral((a + b\*atan(c + d\*x))\*\*3\*(e + f\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 (b \operatorname{arctan}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arctan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*(b\*arctan(d\*x + c) + a)^3, x)



### 3.37 $\int (e + fx) \left( a + b \tan^{-1}(c + dx) \right)^3 dx$

**Optimal.** Leaf size=337

$$\frac{3ib^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{d^2} + \frac{3b^3(de - cf)\text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)}{2d^2} - \frac{3ib^3 f \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{d^2}$$

```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/d^2
```

**Rubi [A]** time = 0.631683, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5047, 4864, 4846, 4920, 4854, 2402, 2315, 4984, 4884, 4994, 6610}

$$\frac{3ib^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{d^2} + \frac{3b^3(de - cf)\text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)}{2d^2} - \frac{3ib^3 f \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^3, x]
```

```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/d^2
```

#### Rule 5047

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_)])*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]
```

#### Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^ (p_.)*((d_.) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

#### Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[(a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(m_.)))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p*I)/2, Int[(a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u]]/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \tan^{-1}(c + dx))^3 dx &= \frac{\text{Subst} \left( \int \left( \frac{de-cf}{d} + \frac{fx}{d} \right) (a + b \tan^{-1}(x))^3 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \left( \frac{f^2 (a + b \tan^{-1}(x))^2}{d^2} + \frac{((de-f-cf)(e+fx))}{d^2} \right) dx, x, c + dx \right)}{2d^2} \\
&= \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= -\frac{3bf(c + dx) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= -\frac{3ibf (a + b \tan^{-1}(c + dx))^2}{2d^2} - \frac{3bf(c + dx) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= -\frac{3ibf (a + b \tan^{-1}(c + dx))^2}{2d^2} - \frac{3bf(c + dx) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{i(de - cf) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= -\frac{3ibf (a + b \tan^{-1}(c + dx))^2}{2d^2} - \frac{3bf(c + dx) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{i(de - cf) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= -\frac{3ibf (a + b \tan^{-1}(c + dx))^2}{2d^2} - \frac{3bf(c + dx) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{i(de - cf) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= -\frac{3ibf (a + b \tan^{-1}(c + dx))^2}{2d^2} - \frac{3bf(c + dx) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{i(de - cf) (a + b \tan^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tan^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b \tan^{-1}(x))^2}{1+x^2} dx, x, c + dx \right)}{2d^2 f}
\end{aligned}$$

**Mathematica [A]** time = 0.678918, size = 592, normalized size = 1.76

$$6ab^2de \left( \tan^{-1}(c + dx) \left( (c + dx - i) \tan^{-1}(c + dx) + 2 \log \left( 1 + e^{2i \tan^{-1}(c + dx)} \right) \right) - i \text{PolyLog} \left( 2, -e^{2i \tan^{-1}(c + dx)} \right) \right) - 6ab^2c$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*(a + b\*ArcTan[c + d\*x])^3,x]

[Out] (a^2\*(2\*a\*d\*e - 3\*b\*f - 2\*a\*c\*f)\*(c + d\*x) + a^3\*f\*(c + d\*x)^2 + 3\*a^2\*b\*f\*ArcTan[c + d\*x] - 3\*a^2\*b\*(c + d\*x)\*(c\*f - d\*(2\*e + f\*x))\*ArcTan[c + d\*x] + 6\*a\*b^2\*f\*(-((c + d\*x)\*ArcTan[c + d\*x]) + ((1 + (c + d\*x)^2)\*ArcTan[c + d\*x]^2)/2 - Log[1/Sqrt[1 + (c + d\*x)^2]]) - 3\*a^2\*b\*(d\*e - c\*f)\*Log[1 + (c + d\*x)^2] + 6\*a\*b^2\*d\*e\*(ArcTan[c + d\*x]\*((-I + c + d\*x)\*ArcTan[c + d\*x] + 2\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) - I\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) - 6\*a\*b^2\*c\*f\*(ArcTan[c + d\*x]\*((-I + c + d\*x)\*ArcTan[c + d\*x] + 2\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) - I\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) + b^3\*f\*(ArcTan[c + d\*x]\*((3\*I)\*ArcTan[c + d\*x] - 3\*(c + d\*x)\*ArcTan[c + d\*x] + (1 + (c + d\*x)^2)\*ArcTan[c + d\*x]^2 - 6\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) + (3\*I)\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) + 2\*b^3\*d\*e\*(ArcTan[c + d\*x]^2\*(-I + c + d\*x)\*ArcTan[c + d\*x] + 3\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) - (3\*I)\*ArcTan[c + d\*x]\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) + (3\*PolyLog[3, -E^((2\*I)\*ArcTan[c + d\*x])])/2 - 2\*b^3\*c\*f\*(ArcTan[c + d\*x]^2\*(-I + c + d\*x)\*ArcTan[c + d\*x] + 3\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) -

$(3*I)*\text{ArcTan}[c + d*x]*\text{PolyLog}[2, -E^{((2*I)*\text{ArcTan}[c + d*x])}] + (3*\text{PolyLog}[3, -E^{((2*I)*\text{ArcTan}[c + d*x])}])/2)/(2*d^2)$

**Maple [C]** time = 1.261, size = 16362, normalized size = 48.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*(a+b*arctan(d*x+c))^3,x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 7/8*b^3*c^2*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e + 7/8*b^3*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 56*b^3*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*integrate(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*f*integrate(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*integrate(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*f*integrate(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*f*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*f*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/2*a^3*f*x^2 + 3*a*b^2*e \end{aligned}$$

```

*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*f*integrate(1/64*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*f*integrate
(1/64*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) - 24*b^3*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 6*b^3*d*e*integrate(1/64*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x + c)*arctan((d^2*x + c*d)
/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*e - 7/32*(6*arctan(d*x + c)^2*
arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d
+ arctan((d^2*x + c*d)/d)^4/d)*b^3*e + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2
+ (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/d^3))*a^2*b*f + a^3*e*x + 56*b^3*f*integrate(1/64*x*arctan(d*x + c)^3/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*f*integrate(1/64*x*arctan(d*x + c)*
log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*
a*b^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 6*b^3*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - l
og((d*x + c)^2 + 1))*a^2*b*e/d + 1/16*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x +
c)^3 - 3/64*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2

```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral( $a^3fx + a^3e + (b^3fx + b^3e) \arctan(dx + c)^3 + 3(ab^2fx + ab^2e) \arctan(dx + c)^2 + 3(a^2bfx + a^2be) \arctan(dx + c) + a^3e$ ), x)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arctan(d*x + c)^3 + 3*(a*b^2*f
*x + a*b^2*e)*arctan(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctan(d*x + c),
x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(c + dx))^3 (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)(b \arctan(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arctan(d*x + c) + a)^3, x)
```

### 3.38 $\int (a + b \tan^{-1}(c + dx))^3 dx$

**Optimal.** Leaf size=143

$$\frac{3ib^2 \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{d} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)}{2d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d}$$

[Out] (I\*(a + b\*ArcTan[c + d\*x])^3)/d + ((c + d\*x)\*(a + b\*ArcTan[c + d\*x])^3)/d + (3\*b\*(a + b\*ArcTan[c + d\*x])^2\*Log[2/(1 + I\*(c + d\*x))])/d + ((3\*I)\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/d + (3\*b^3\*PolyLog[3, 1 - 2/(1 + I\*(c + d\*x))])/(2\*d)

**Rubi [A]** time = 0.214405, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {5039, 4846, 4920, 4854, 4884, 4994, 6610}

$$\frac{3ib^2 \text{PolyLog}\left(2, 1 - \frac{2}{1+i(c+dx)}\right)(a + b \tan^{-1}(c + dx))}{d} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1+i(c+dx)}\right)}{2d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c + d\*x])^3, x]

[Out] (I\*(a + b\*ArcTan[c + d\*x])^3)/d + ((c + d\*x)\*(a + b\*ArcTan[c + d\*x])^3)/d + (3\*b\*(a + b\*ArcTan[c + d\*x])^2\*Log[2/(1 + I\*(c + d\*x))])/d + ((3\*I)\*b^2\*(a + b\*ArcTan[c + d\*x])\*PolyLog[2, 1 - 2/(1 + I\*(c + d\*x))])/d + (3\*b^3\*PolyLog[3, 1 - 2/(1 + I\*(c + d\*x))])/(2\*d)

#### Rule 5039

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 4846

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 4920

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[(a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]]/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rule 4884

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

#### Rule 4994

Int[(Log[u\_]\*((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> -Simp[(I\*(a + b\*ArcTan[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p\*I)/2, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && EqQ[(1 - u)^2 - (1 - (2\*I)/(I - c\*x))^2, 0]

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

#### Rubi steps

$$\begin{aligned}
 \int (a + b \tan^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int (a + b \tan^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
 &= \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d} - \frac{(3b) \text{Subst}\left(\int \frac{x^{(a+b \tan^{-1}(x))^2}}{1+x^2} dx, x, c + dx\right)}{d} \\
 &= \frac{i(a + b \tan^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d} + \frac{(3b) \text{Subst}\left(\int \frac{(a+b \tan^{-1}(x))}{i-x} dx\right)}{d} \\
 &= \frac{i(a + b \tan^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d} + \frac{3b(a + b \tan^{-1}(c + dx))^2}{d} \\
 &= \frac{i(a + b \tan^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d} + \frac{3b(a + b \tan^{-1}(c + dx))^2}{d} \\
 &= \frac{i(a + b \tan^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tan^{-1}(c + dx))^3}{d} + \frac{3b(a + b \tan^{-1}(c + dx))^2}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.114604, size = 212, normalized size = 1.48

$$\frac{6ab^2 \left( \tan^{-1}(c + dx) \left( (c + dx - i) \tan^{-1}(c + dx) + 2 \log \left( 1 + e^{2i \tan^{-1}(c + dx)} \right) \right) - i \text{PolyLog} \left( 2, -e^{2i \tan^{-1}(c + dx)} \right) \right) + 2b^3 \left( -3 \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^3,x]

[Out] (2\*a^3\*(c + d\*x) + 6\*a^2\*b\*(c + d\*x)\*ArcTan[c + d\*x] - 3\*a^2\*b\*Log[1 + (c + d\*x)^2] + 6\*a\*b^2\*(ArcTan[c + d\*x]\*((-I + c + d\*x)\*ArcTan[c + d\*x] + 2\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) - I\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) + 2\*b^3\*(ArcTan[c + d\*x]^2\*((-I + c + d\*x)\*ArcTan[c + d\*x] + 3\*Log[1 + E^((2\*I)\*ArcTan[c + d\*x])]) - (3\*I)\*ArcTan[c + d\*x]\*PolyLog[2, -E^((2\*I)\*ArcTan[c + d\*x])]) + (3\*PolyLog[3, -E^((2\*I)\*ArcTan[c + d\*x])])/(2))/d

---

**Maple [B]** time = 0.155, size = 359, normalized size = 2.5

$$xa^3 + \frac{a^3c}{d} - \frac{3ib^3 \arctan(dx+c)}{d} \operatorname{polylog}\left(2, -\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right) + (\arctan(dx+c))^3 xb^3 + \frac{(\arctan(dx+c))^3 b^3c}{d} + 3 \frac{b^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^3,x)

[Out]  $x*a^3+1/d*a^3*c-3*I/d*b^3*\arctan(d*x+c)*\operatorname{polylog}(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+\arctan(d*x+c)^3*x*b^3+1/d*\arctan(d*x+c)^3*b^3*c+3/d*b^3*\arctan(d*x+c)^2*\ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)-3*I/d*\arctan(d*x+c)^2*a*b^2+3/2/d*b^3*\operatorname{polylog}(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I/d*\operatorname{polylog}(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))*a*b^2+3*\arctan(d*x+c)^2*x*a*b^2+3/d*\arctan(d*x+c)^2*a*b^2*c+6/d*\arctan(d*x+c)*\ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)*a*b^2-I/d*b^3*\arctan(d*x+c)^3+3*\arctan(d*x+c)*x*a^2*b+3/d*\arctan(d*x+c)*a^2*b*c-3/2/d*a^2*b*\ln(1+(d*x+c)^2)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3,x, algorithm="maxima")

[Out]  $7/8*b^3*c^2*\arctan(d*x+c)^3*\arctan((d^2*x+c*d)/d)/d+1/8*b^3*x*\arctan(d*x+c)^3+3*a*b^2*c^2*\arctan(d*x+c)^2*\arctan((d^2*x+c*d)/d)/d-3/32*b^3*x*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)^2-(3*\arctan(d*x+c)*\arctan((d^2*x+c*d)/d)^2/d-\arctan((d^2*x+c*d)/d)^3/d)*a*b^2*c^2-7/32*(6*\arctan(d*x+c)^2*\arctan((d^2*x+c*d)/d)^2/d-4*\arctan(d*x+c)*\arctan((d^2*x+c*d)/d)^3/d+\arctan((d^2*x+c*d)/d)^4/d)*b^3*c^2+7/8*b^3*\arctan(d*x+c)^3*\arctan((d^2*x+c*d)/d)/d+28*b^3*d^2*\int(1/32*x^2*\arctan(d*x+c)^3/(d^2*x^2+2*c*d*x+c^2+1),x)+3*b^3*d^2*\int(1/32*x^2*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+96*a*b^2*d^2*\int(1/32*x^2*\arctan(d*x+c)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+56*b^3*c*d*\int(1/32*x*\arctan(d*x+c)^3/(d^2*x^2+2*c*d*x+c^2+1),x)+12*b^3*d^2*\int(1/32*x^2*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)/(d^2*x^2+2*c*d*x+c^2+1),x)+6*b^3*c*d*\int(1/32*x*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+192*a*b^2*c*d*\int(1/32*x*\arctan(d*x+c)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+12*b^3*c*d*\int(1/32*x*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)/(d^2*x^2+2*c*d*x+c^2+1),x)+3*b^3*c^2*\int(1/32*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+3*a*b^2*\arctan(d*x+c)^2*\arctan((d^2*x+c*d)/d)/d-12*b^3*d*\int(1/32*x*\arctan(d*x+c)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+3*b^3*d*\int(1/32*x*\log(d^2*x^2+2*c*d*x+c^2+1)^2/(d^2*x^2+2*c*d*x+c^2+1),x)-(3*\arctan(d*x+c)*\arctan((d^2*x+c*d)/d)^2/d-\arctan((d^2*x+c*d)/d)^3/d)*a*b^2-7/32*(6*\arctan(d*x+c)^2*\arctan((d^2*x+c*d)/d)^2/d-4*\arctan(d*x+c)*\arctan((d^2*x+c*d)/d)^3/d+\arctan((d^2*x+c*d)/d)^4/d)*b^3+a^3*x+3*b^3*\int(1/32*\arctan(d*x+c)*\log(d^2*x^2+2*c*d*x+c^2+1)^2/(d^2*x^2+2*c*d*x+c^2+1),x)+3/2*(2*(d*x+c)*\arctan(d*x+c)-\log((d*x+c)^2+1))*a^2*b/d$



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^3 \arctan(dx + c)^3 + 3ab^2 \arctan(dx + c)^2 + 3a^2b \arctan(dx + c) + a^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(b^3\*arctan(d\*x + c)^3 + 3\*a\*b^2\*arctan(d\*x + c)^2 + 3\*a^2\*b\*arctan(d\*x + c) + a^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))\*\*3,x)

[Out] Integral((a + b\*atan(c + d\*x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)^3, x)

$$3.39 \quad \int \frac{(a+b \tan^{-1}(c+dx))^3}{e+fx} dx$$

**Optimal.** Leaf size=372

$$\frac{3b^2(a+b \tan^{-1}(c+dx)) \operatorname{PolyLog}\left(3, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f} - \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2f}$$

```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x))])/f +
(((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))
])/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x)
)/((d*e + I*f - c*f)*(1 - I*(c + d*x))])/f - (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))]/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))]/f
```

**Rubi [A]** time = 0.20282, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {5047, 4860}

$$\frac{3b^2(a+b \tan^{-1}(c+dx)) \operatorname{PolyLog}\left(3, 1 - \frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{2f} - \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-i(c+dx)}\right)(a+b \tan^{-1}(c+dx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x), x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x))])/f +
(((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))
])/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x)
)/((d*e + I*f - c*f)*(1 - I*(c + d*x))])/f - (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))]/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))]/f
```

#### Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^p_.*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

#### Rule 4860

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^3*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(3*
I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)]]/(2*e), x] - Simp[(
3*I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
- I*c*x))]]/(2*e), x] - Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1
- I*c*x)]]/(2*e), x] + Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - (2*c
```

$\frac{(d + ex)}{(cd + Ie)(1 - Icx)}}{(2e), x] - \text{Simp}[(3Ib^3 \text{PolyLog}[4, 1 - 2/(1 - Icx)])/(4e), x] + \text{Simp}[(3Ib^3 \text{PolyLog}[4, 1 - (2c(d + ex))/(cd + Ie)(1 - Icx)])/(4e), x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2d^2 + e^2, 0]$

### Rubi steps

$$\int \frac{(a + b \tan^{-1}(c + dx))^3}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{(a + b \tan^{-1}(x))^3}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \tan^{-1}(c + dx))^3 \log\left(\frac{2}{1 - i(c + dx)}\right)}{f} + \frac{(a + b \tan^{-1}(c + dx))^3 \log\left(\frac{2d(e + fx)}{(de + if - cf)(1 - i(c + dx))}\right)}{f}$$

**Mathematica [F]** time = 71.5293, size = 0, normalized size = 0.

$$\int \frac{(a + b \tan^{-1}(c + dx))^3}{e + fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^3/(e + f\*x),x]

[Out] Integrate[(a + b\*ArcTan[c + d\*x])^3/(e + f\*x), x]

**Maple [C]** time = 0.765, size = 4389, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^3/(f\*x+e),x)

[Out]  $\frac{3}{2}Ia^2b \ln(f(d*x+c) - cf + d*e) / f \ln\left(\frac{I*f - f*(d*x+c)}{d*e + I*f - c*f}\right) - \frac{3}{2}Ia^2b \ln(f(d*x+c) - cf + d*e) / f \ln\left(\frac{I*f + f*(d*x+c)}{I*f + c*f - d*e}\right) + 3Ia*b^2 / (I*f + c*f - d*e) * \arctan(d*x+c)^2 * \ln(1 - (I*f + c*f - d*e) * (1 + I*(d*x+c))^2 / (1 + (d*x+c)^2)) / (d*e + I*f - c*f) + 3Ia*b^2 / f * \arctan(d*x+c) * \text{polylog}(2, -(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2)) - 1/2 * I*b^3 / f * \arctan(d*x+c)^3 * \text{Pi} * \text{csgn}(I*(I*f*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + c*f*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) - d*e*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) - I*f + c*f - d*e) / ((1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + 1))^3 - 3/2 * I*b^3 * c / (I*f + c*f - d*e) * \arctan(d*x+c)^2 * \text{polylog}(2, (I*f + c*f - d*e) * (1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) / (d*e + I*f - c*f)) + 3a*b^2 * c / (I*f + c*f - d*e) * \arctan(d*x+c)^2 * \ln(1 - (I*f + c*f - d*e) * (1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) / (d*e + I*f - c*f)) + I*b^3 / f * \arctan(d*x+c)^3 * \text{Pi} * \text{csgn}(I*(I*f*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + c*f*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) - d*e*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) - I*f + c*f - d*e) / ((1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + 1))^3 - 3Ia*b^2 / f * \text{Pi} * \arctan(d*x+c)^2 + a^3 * \ln(f(d*x+c) - cf + d*e) / f - 3/4 * b^3 / (I*f + c*f - d*e) * \text{polylog}(4, (I*f + c*f - d*e) * (1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) / (d*e + I*f - c*f)) + 6I*d*a*b^2 / f * e * \arctan(d*x+c) * \text{polylog}(2, (I*f + c*f - d*e) * (1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) / (d*e + I*f - c*f)) / (2I*f + 2*c*f - 2*d*e) + 3/2 * I*a*b^2 / f * \arctan(d*x+c)^2 * \text{Pi} * \text{csgn}(I / ((1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + 1)) * \text{csgn}(I*(I*f*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + c*f*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) - d*e*(1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) - I*f + c*f - d*e) / ((1 + I*(d*x+c))^2 / (1 + (d*x+c)^2) + 1))$

$$\begin{aligned}
& I*(d*x+c)^2/(1+(d*x+c)^2+1))+3/2*a*b^2*c/(I*f+c*f-d*e)*polylog(3, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*I*a*b^2*c/(I*f+c*f-d*e) \\
& )*arctan(d*x+c)*polylog(2, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*I*a*b^2/f*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-b^3/f*arctan(d*x+c)^3*ln \\
& (I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-3/2*b^3/f*arctan(d*x+c)*polylog(3, \\
& -(1+I*(d*x+c))^2/(1+(d*x+c)^2))+I*b^3/(I*f+c*f-d*e)*arctan(d*x+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+b^3*c/(I*f+c*f-d*e) \\
& )*arctan(d*x+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*b^3*c/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*a*b^2*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^2-3*a*b^2/f*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)+3*a*b^2/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(2, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*a^2*b*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)+3/2*I*a*b^2/(I*f+c*f-d*e)*polylog(3, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3/f*arctan(d*x+c)^2*polylog(2, -(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*I*a^2*b/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-3/2*I*a^2*b/f*dilog((I*f+f*(d*x+c))/(I*f+c*f-d*e))-I*b^3/f*Pi*arctan(d*x+c)^3+3/4*I*b^3*c/(I*f+c*f-d*e)*polylog(4, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*b^3/(I*f+c*f-d*e)*arctan(d*x+c)^2*polylog(2, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+b^3*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^3-3/2*a*b^2/f*polylog(3, -(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3/4*I*b^3/f*polylog(4, -(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*d*a*b^2/f*e/(I*f+c*f-d*e)*arctan(d*x+c)^2*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*a*b^2/f*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-3/2*I*a*b^2/f*arctan(d*x+c)^2*Pi*csgn(I/(1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2+1/2*I*b^3/f*arctan(d*x+c)^3*Pi*csgn(I/(1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3-3*I*d*b^3/f*e*arctan(d*x+c)^2*polylog(2, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)-3/2*I*a*b^2/f*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^3-3*I*d*b^3/f*e*polylog(4, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(4*I*f+4*c*f-4*d*e)-3/2*d*a*b^2/f*e/(I*f+c*f-d*e)*polylog(3, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-d*b^3/f*e/(I*f+c*f-d*e)*arctan(d*x+c)^3*ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*d*b^3/f*e/(I*f+c*f-d*e)*arctan(d*x+c)*polylog(3, (I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I*b^3/f*arctan(d*x+c)^3*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2-1/2*I*b^3/f*arctan(d*x+c)^3*Pi*csgn(I/(1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2
\end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(fx + e)}{f} + \int \frac{28b^3 \arctan(dx + c)^3 + 3b^3 \arctan(dx + c) \log(d^2x^2 + 2cdx + c^2 + 1)^2 + 96ab^2 \arctan(dx + c)}{32(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(f\*x+e),x, algorithm="maxima")

[Out] a^3\*log(f\*x + e)/f + integrate(1/32\*(28\*b^3\*arctan(d\*x + c)^3 + 3\*b^3\*arctan(d\*x + c)\*log(d^2\*x^2 + 2\*c\*d\*x + c^2 + 1)^2 + 96\*a\*b^2\*arctan(d\*x + c)^2 + 96\*a^2\*b\*arctan(d\*x + c))/(f\*x + e), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(dx + c)^3 + 3ab^2 \arctan(dx + c)^2 + 3a^2b \arctan(dx + c) + a^3}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(f\*x+e),x, algorithm="fricas")

[Out] integral((b^3\*arctan(d\*x + c)^3 + 3\*a\*b^2\*arctan(d\*x + c)^2 + 3\*a^2\*b\*arctan(d\*x + c) + a^3)/(f\*x + e), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(c + dx))^3}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))\*\*3/(f\*x+e),x)

[Out] Integral((a + b\*atan(c + d\*x))\*\*3/(e + f\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^3}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(f\*x+e),x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)^3/(f\*x + e), x)

$$3.40 \quad \int \frac{(a+b \tan^{-1}(c+dx))^3}{(e+fx)^2} dx$$

**Optimal.** Leaf size=1233

result too large to display

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
+ (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
- (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))]/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))]/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I
*(c + d*x)))]/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
(1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + I*f - c*f)*(1 - I*(c + d*x)))]/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

**Rubi [A]** time = 2.31078, antiderivative size = 1233, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 22, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.1$ , Rules used = {5045, 6741, 5057, 6688, 12, 6725, 706, 31, 635, 203, 260, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858, 4994, 6610}

$$\frac{id \tan^{-1}(c+dx)^3 b^3}{d^2 e^2 - 2cdfe + (c^2 + 1) f^2} + \frac{d(de - cf) \tan^{-1}(c+dx)^3 b^3}{f(d^2 e^2 - 2cdfe + (c^2 + 1) f^2)} - \frac{3d \tan^{-1}(c+dx)^2 \log\left(\frac{2}{1-i(c+dx)}\right) b^3}{d^2 e^2 - 2cdfe + (c^2 + 1) f^2} + \frac{3d \tan^{-1}(c+dx)^2 \log\left(\frac{2}{1+i(c+dx)}\right) b^3}{d^2 e^2 - 2cdfe + (c^2 + 1) f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2, x]
```

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
+ (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
- (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
```

$$\begin{aligned}
& + (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
\end{aligned}$$
Rule 5045

```

Int[((a_) + ArcTan[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(m_)
_, x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

```

Rule 6741

```

Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

```

Rule 5057

```

Int[((a_) + ArcTan[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(m_)
_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)^(q_), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]

```

Rule 6688

```

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 6725

```

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ

```

[n, 0]

### Rule 706

Int[1/(((d\_) + (e\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 + a\*e^2), Int[(c\*d - c\*e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(n\_), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 4856

Int[((a\_) + ArcTan[(c\_)\*(x\_)])\*(b\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])\*Log[2/(1 - I\*c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))]/(1 + c^2\*x^2), x], x] + Simp[((a + b\*ArcTan[c\*x])\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

### Rule 2402

Int[Log[(c\_)/((d\_) + (e\_)\*(x\_))]/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_)\*(x\_)/((d\_) + (e\_)\*(x\_))], x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 4984



Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(m\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcTan[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && IGtQ[m, 0]

#### Rule 4884

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTan[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2\*d] && NeQ[p, -1]

#### Rule 4920

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 + c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rule 4858

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^2/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])^2\*Log[2/(1 - I\*c\*x)])/e, x] + (Simp[((a + b\*ArcTan[c\*x])^2\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x] + Simp[(I\*b\*(a + b\*ArcTan[c\*x])\*PolyLog[2, 1 - 2/(1 - I\*c\*x)])/e, x] - Simp[(I\*b\*(a + b\*ArcTan[c\*x])\*PolyLog[2, 1 - (2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x] - Simp[(b^2\*PolyLog[3, 1 - 2/(1 - I\*c\*x)])/((2\*e)), x] + Simp[(b^2\*PolyLog[3, 1 - (2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/((2\*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rule 4994

Int[(Log[u\_]\*((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[(I\*(a + b\*ArcTan[c\*x])^p\*PolyLog[2, 1 - u])/((2\*c\*d)), x] + Dist[(b\*p\*I)/2, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/((d + e\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2\*d] && EqQ[(1 - u)^2 - (1 - (2\*I)/(I - c\*x))^2, 0]

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan^{-1}(c + dx))^3}{(e + fx)^2} dx &= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a + b \tan^{-1}(c + dx))^2}{(e + fx)(1 + (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a + b \tan^{-1}(c + dx))^2}{(e + fx)(1 + c^2 + 2cdx + d^2x^2)} dx}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left( \int \frac{(a + b \tan^{-1}(x))^2}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left( \int \frac{d(a + b \tan^{-1}(x))^2}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left( \int \frac{(a + b \tan^{-1}(x))^2}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left( \int \left( \frac{a^2}{(de - cf + fx)(1 + x^2)} + \frac{2ab \tan^{-1}(x)}{(de - cf + fx)(1 + x^2)} + \frac{b^2 \tan^{-1}(x)}{(de - cf + fx)(1 + x^2)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3a^2bd) \text{Subst} \left( \int \frac{1}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} + \frac{(6ab^2d) \text{Subst} \left( \int \frac{\tan^{-1}(x)}{(de - cf + fx)(1 + x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{(6ab^2d) \text{Subst} \left( \int \left( \frac{f^2 \tan^{-1}(x)}{(d^2e^2 - 2cdef + (1 + c^2)f^2)(de - cf + fx)} + \frac{(de - cf - fx) \tan^{-1}(x)}{(d^2e^2 - 2cdef + (1 + c^2)f^2)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{3a^2bd \log(e + fx)}{f^2 + (de - cf)^2} + \frac{(6ab^2d) \text{Subst} \left( \int \frac{(de - cf - fx) \tan^{-1}(x)}{1 + x^2} dx, x, c + dx \right)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{3a^2bd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} - \frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{3a^2bd \log(e + fx)}{f^2 + (de - cf)^2} - \frac{6ab^2d \tan^{-1}(c + dx)}{d^2e^2 - 2cdef + (1 + c^2)f^2} \\
&= \frac{3a^2bd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} - \frac{(a + b \tan^{-1}(c + dx))^3}{f(e + fx)} + \frac{3a^2bd \log(e + fx)}{f^2 + (de - cf)^2} - \frac{6ab^2d \tan^{-1}(c + dx)}{d^2e^2 - 2cdef + (1 + c^2)f^2} \\
&= \frac{3a^2bd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{3iab^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{3ab^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{3a^2bd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{3iab^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{3ab^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{3a^2bd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{3iab^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{3ab^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)} \\
&= \frac{3a^2bd(de - cf) \tan^{-1}(c + dx)}{f(f^2 + (de - cf)^2)} + \frac{3iab^2d \tan^{-1}(c + dx)^2}{d^2e^2 - 2cdef + (1 + c^2)f^2} + \frac{3ab^2d(de - cf) \tan^{-1}(c + dx)}{f(d^2e^2 - 2cdef + (1 + c^2)f^2)}.
\end{aligned}$$

**Mathematica [F]** time = 70.2675, size = 0, normalized size = 0.

$$\int \frac{(a + b \tan^{-1}(c + dx))^3}{(e + fx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTan[c + d\*x])^3/(e + f\*x)^2,x]

[Out] Integrate[(a + b\*ArcTan[c + d\*x])^3/(e + f\*x)^2, x]

**Maple [C]** time = 1.013, size = 4764, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctan(d\*x+c))^3/(f\*x+e)^2,x)

[Out] 
$$-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^{(1/2)})^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(I*f+c*f-d*e)*\arctan(d*x+c)^2*\ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*e)*\arctan(d*x+c)^2*\ln(1-(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)^2-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))+3*I*d^2*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(I*f+c*f-d*e)*\arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1)^2)-3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2+3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2)^2+3/2*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))-3/4*I*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*Pi*csgn(I/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/((1+I*(d*x+c))^2/(1+(d*x+c)^2)+1))^2)-3*I*d*b^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(I*f+c*f-d*e)*\arctan(d*x+c)*polylog(2,(I*f+c*f-d*e)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*d*b^3*\arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\ln(f*(d*x+c)-c*f+d*e)-3/2*d*b^3*\arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\ln(1+(d*x+c)^2)-3*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*\arctan(d*x+c)^2*\ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)+3*d*b^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)$$

$$\begin{aligned}
& ) * \arctan(dx+c)^2 \ln((1+I*(dx+c))/(1+(dx+c)^2)^{(1/2)}) - db^3/(df*x+de)/ \\
& * \arctan(dx+c)^3 - 3/2 * da^2 * b / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(1+(dx+c)^2) \\
& + 3 * da^2 * b / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(f*(dx+c) - c*f + d*e) - db^3 * \arctan(dx+c)^3 \\
& / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * c + 3 * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \ln(2) \\
& - I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^3 + d^2 * b^3 / f * \arctan(dx+c)^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * e \\
& - 3 * da^2 * b / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c) * c - 3 * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * c \\
& - 3 * da^2 * b / (df*x+de) / f * \arctan(dx+c) - 3 * da * b^2 / (df*x+de) / f * \arctan(dx+c)^2 - 3 * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi \\
& - 3/2 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \operatorname{dilog}(1/2 * I * (dx+c-I)) + 3/2 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \operatorname{dilog}(-1/2 * I * (dx+c+I)) \\
& - 3 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \operatorname{dilog}((I*f + f*(dx+c))/(I*f + c*f - d*e)) + 3/4 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(dx+c-I)^2 + 3 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \operatorname{dilog}((I*f - f*(dx+c))/(d*e + I*f - c*f)) \\
& - 3/4 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(dx+c+I)^2 - 3/2 * d^2 * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * e / (I*f + c*f - d*e) * \operatorname{polylog}(3, (I*f + c*f - d*e) * (1+I*(dx+c))^2 / (1+(dx+c)^2) / (d*e + I*f - c*f)) \\
& - 3 * da * b^2 * \arctan(dx+c) / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(1+(dx+c)^2) + 6 * da * b^2 * \arctan(dx+c) / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(f*(dx+c) - c*f + d*e) - da^3 / (df*x+de) / f \\
& + 3/2 * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * (1+I*(dx+c)) / (1+(dx+c)^2)^{(1/2)}) * \operatorname{csgn}(I * (1+I*(dx+c))^2 / (1+(dx+c)^2))^{2+3/4} * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * (1+I*(dx+c))^2 / (1+(dx+c)^2) / ((1+I*(dx+c))^2 / (1+(dx+c)^2) + 1)^2)^{2-3/4} \\
& * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * (1+I*(dx+c))^2 / (1+(dx+c)^2) / ((1+I*(dx+c))^2 / (1+(dx+c)^2) + 1)^2)^{3+3/4} * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * ((1+I*(dx+c))^2 / (1+(dx+c)^2) + 1)^2)^{3-3/4} \\
& * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * (1+I*(dx+c))^2 / (1+(dx+c)^2))^{3-3} * d^2 * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * e / (I*f + c*f - d*e) * \arctan(dx+c)^2 * \ln(1 - (I*f + c*f - d*e) * (1+I*(dx+c))^2 / (1+(dx+c)^2) / (d*e + I*f - c*f)) \\
& + 3 * d * b^3 * f / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) / (I*f + c*f - d*e) * \arctan(dx+c) * \operatorname{polylog}(2, (I*f + c*f - d*e) * (1+I*(dx+c))^2 / (1+(dx+c)^2) / (d*e + I*f - c*f)) + 3 * d^2 * a^2 * b / f / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c) * e \\
& + 3/2 * d * b^3 * f / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * c / (I*f + c*f - d*e) * \operatorname{polylog}(3, (I*f + c*f - d*e) * (1+I*(dx+c))^2 / (1+(dx+c)^2) / (d*e + I*f - c*f)) + 3 * d^2 * a * b^2 / f / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * e \\
& - 3/2 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(dx+c+I) * \ln(1/2 * I * (dx+c-I)) + 3/2 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(dx+c-I) * \ln(-1/2 * I * (dx+c+I)) \\
& + 3/2 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(1+(dx+c)^2) * \ln(dx+c+I) + 3 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(f*(dx+c) - c*f + d*e) * \ln((I*f - f*(dx+c))/(d*e + I*f - c*f)) \\
& - 3/2 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(1+(dx+c)^2) * \ln(dx+c-I) + 3/2 * I * d * b^3 * f / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) / (I*f + c*f - d*e) * \operatorname{polylog}(3, (I*f + c*f - d*e) * (1+I*(dx+c))^2 / (1+(dx+c)^2) / (d*e + I*f - c*f)) \\
& - 3 * I * da * b^2 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \ln(f*(dx+c) - c*f + d*e) * \ln((I*f + f*(dx+c))/(I*f + c*f - d*e)) + 3 * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * (I*f * (1+I*(dx+c))^2 / (1+(dx+c)^2) + c*f * (1+I*(dx+c))^2 / (1+(dx+c)^2) - d*e * (1+I*(dx+c))^2 / (1+(dx+c)^2) - I*f + c*f - d*e) / ((1+I*(dx+c))^2 / (1+(dx+c)^2) + 1)^2)^{2-3/2} \\
& * I * d * b^3 / (c^2 * f^2 - 2 * c * d * e * f + d^2 * e^2 + f^2) * \arctan(dx+c)^2 * \pi * \operatorname{csgn}(I * (I*f * (1+I*(dx+c))^2 / (1+(dx+c)^2) + c*f * (1+I*(dx+c))^2 / (1+(dx+c)^2) - d*e * (1+I*(dx+c))^2 / (1+(dx+c)^2) - I*f + c*f - d*e) / ((1+I*(dx+c))^2 / (1+(dx+c)^2) + 1))^3
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} \left( d \left( \frac{2(d^2e - cdf) \arctan\left(\frac{d^2x+cd}{d}\right)}{(d^2e^2f - 2cdef^2 + (c^2+1)f^3)d} - \frac{\log(d^2x^2 + 2cdx + c^2 + 1)}{d^2e^2 - 2cdef + (c^2+1)f^2} + \frac{2 \log(fx + e)}{d^2e^2 - 2cdef + (c^2+1)f^2} \right) - \frac{2 \arctan(dx + c)}{f^2x + ef} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(f\*x+e)^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 3/2*(d*(2*(d^2*e - c*d*f)*\arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 \\ & + (c^2 + 1)*f^3)*d) - \log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e* \\ & f + (c^2 + 1)*f^2) + 2*\log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2)) \\ & - 2*\arctan(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4*b^3* \\ & \arctan(d*x + c)^3 - 3*b^3*\arctan(d*x + c)*\log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 \\ & - 32*(f^2*x + e*f)*\int(1/32*(28*(b^3*d^2*f*x^2 + 2*b^3*c*d*f*x + (b \\ & ^3*c^2 + b^3)*f)*\arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*f*x^2 + b^3*d*e + (16* \\ & a*b^2*c + b^3)*d*f*x + 8*(a*b^2*c^2 + a*b^2)*f)*\arctan(d*x + c)^2 - 12*(b^3 \\ & *d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*\arctan(d*x + c)*\log(d^2 \\ & *x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*f*x + b^3*d*e - (b^3*d^2*f*x^2 + 2*b^3 \\ & *c*d*f*x + (b^3*c^2 + b^3)*f)*\arctan(d*x + c))*\log(d^2*x^2 + 2*c*d*x + c^2 \\ & + 1)^2)/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2 \\ & *e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2) \\ & *x), x))/(f^2*x + e*f) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b^3 \arctan(dx + c)^3 + 3ab^2 \arctan(dx + c)^2 + 3a^2b \arctan(dx + c) + a^3}{f^2x^2 + 2efx + e^2}, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(f\*x+e)^2,x, algorithm="fricas")

[Out] integral((b^3\*arctan(d\*x + c)^3 + 3\*a\*b^2\*arctan(d\*x + c)^2 + 3\*a^2\*b\*arctan(d\*x + c) + a^3)/(f^2\*x^2 + 2\*e\*f\*x + e^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atan(d\*x+c))^3/(f\*x+e)^2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(dx + c) + a)^3}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctan(d\*x+c))^3/(f\*x+e)^2,x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)^3/(f\*x + e)^2, x)

### 3.41 $\int (e + fx)^m (a + b \tan^{-1}(c + dx)) dx$

**Optimal.** Leaf size=177

$$\frac{ibd(e + fx)^{m+2} \text{Hypergeometric2F1}\left(1, m + 2, m + 3, \frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de + (-c+i)f)} + \frac{ibd(e + fx)^{m+2} \text{Hypergeometric2F1}\left(1, m + 2, m + 3, \frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de - (c+i)f)}$$

[Out]  $((e + f*x)^{(1 + m)*(a + b*ArcTan[c + d*x])})/(f*(1 + m)) - ((I/2)*b*d*(e + f*x)^{(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c*f)]})/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^{(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)]})/(f*(d*e - (I + c)*f)*(1 + m)*(2 + m))$

**Rubi [A]** time = 0.246324, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5047, 4862, 712, 68}

$$\frac{ibd(e + fx)^{m+2} \text{Hypergeometric2F1}\left(1, m + 2, m + 3, \frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de + (-c+i)f)} + \frac{ibd(e + fx)^{m+2} \text{Hypergeometric2F1}\left(1, m + 2, m + 3, \frac{d(e+fx)}{-cf+de+if}\right)}{2f(m+1)(m+2)(de - (c+i)f)}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x]),x]

[Out]  $((e + f*x)^{(1 + m)*(a + b*ArcTan[c + d*x])})/(f*(1 + m)) - ((I/2)*b*d*(e + f*x)^{(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c*f)]})/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^{(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)]})/(f*(d*e - (I + c)*f)*(1 + m)*(2 + m))$

#### Rule 5047

Int[((a\_) + ArcTan[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IntegerQ[p, 0]

#### Rule 4862

Int[((a\_) + ArcTan[(c\_)\*(x\_)])\*(b\_)\*((d\_) + (e\_)\*(x\_))^(q\_), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 712

Int[((d\_) + (e\_)\*(x\_))^(m\_)/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m, 1/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[m]

#### Rule 68

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((b\*c - a\*d)^n\*(a + b\*x)^(m + 1)\*Hypergeometric2F1[-n, m + 1, m + 2, -(d\*(a + b\*x))/(b\*c - a\*d)])/(b^(n + 1)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]

&& NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

### Rubi steps

$$\begin{aligned}
 \int (e + fx)^m (a + b \tan^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \tan^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{(e + fx)^{1+m} (a + b \tan^{-1}(c + dx))}{f(1+m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1+x^2} dx, x, c + dx\right)}{f(1+m)} \\
 &= \frac{(e + fx)^{1+m} (a + b \tan^{-1}(c + dx))}{f(1+m)} - \frac{b \text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(i-x)} + \frac{i\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(i+x)}\right) dx, x, c + dx\right)}{f(1+m)} \\
 &= \frac{(e + fx)^{1+m} (a + b \tan^{-1}(c + dx))}{f(1+m)} - \frac{(ib) \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{i-x} dx, x, c + dx\right)}{2f(1+m)} \\
 &= \frac{(e + fx)^{1+m} (a + b \tan^{-1}(c + dx))}{f(1+m)} - \frac{ibd(e + fx)^{2+m} {}_2F_1\left(1, 2 + m; 3 + m; \frac{d(e+fx)}{de+if}\right)}{2f(de + (i - c)f)(1 + m)(2 + m)}
 \end{aligned}$$

**Mathematica [A]** time = 0.287765, size = 162, normalized size = 0.92

$$\frac{(e + fx)^{m+1} \left( 2(a + b \tan^{-1}(c + dx)) + \frac{bd(e+fx)\left((de-(c+i)f)\text{Hypergeometric2F1}\left(1, m+2, m+3, \frac{d(e+fx)}{de-(c-i)f}\right) + (-de+(c-i)f)\text{Hypergeometric2F1}\left(1, m+2, m+3, \frac{d(e+fx)}{de-(c-i)f}\right)\right)}{(m+2)(-icf+ide+f)(de-(c-i)f)} \right)}{2f(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x]), x]

[Out] ((e + f\*x)^(1 + m)\*(2\*(a + b\*ArcTan[c + d\*x]) + (b\*d\*(e + f\*x)\*((d\*e - (I + c)\*f)\*Hypergeometric2F1[1, 2 + m, 3 + m, (d\*(e + f\*x))/(d\*e - (-I + c)\*f)] + (-d\*e) + (-I + c)\*f)\*Hypergeometric2F1[1, 2 + m, 3 + m, (d\*(e + f\*x))/(d\*e - (I + c)\*f)])))/((I\*d\*e + f - I\*c\*f)\*(d\*e - (-I + c)\*f)\*(2 + m)))/(2\*f\*(1 + m))

**Maple [F]** time = 1.284, size = 0, normalized size = 0.

$$\int (fx + e)^m (a + b \arctan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arctan(d\*x+c)), x)

[Out] int((f\*x+e)^m\*(a+b\*arctan(d\*x+c)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \arctan(dx + c) + a)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c)),x, algorithm="fricas")

[Out] integral((b\*arctan(d\*x + c) + a)\*(f\*x + e)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*m\*(a+b\*atan(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(dx + c) + a)(fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c)),x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)\*(f\*x + e)^m, x)



$$3.42 \quad \int (e + fx)^m (a + b \tan^{-1}(c + dx))^2 dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left((e + fx)^m (a + b \tan^{-1}(c + dx))^2, x\right)$$

[Out] Unintegrable[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^2, x]

**Rubi [A]** time = 0.057976, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (e + fx)^m (a + b \tan^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] Defer[Subst][Defer[Int][((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^2, x], x, c + d\*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \tan^{-1}(c + dx))^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \tan^{-1}(x))^2 dx, x, c + dx\right)}{d}$$

**Mathematica [A]** time = 4.44536, size = 0, normalized size = 0.

$$\int (e + fx)^m (a + b \tan^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^2,x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^2, x]

**Maple [A]** time = 1.105, size = 0, normalized size = 0.

$$\int (fx + e)^m (a + b \arctan(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^2,x)

[Out] int((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \arctan(dx+c)^2 + 2ab \arctan(dx+c) + a^2\right)(fx+e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^2,x, algorithm="fricas")

[Out] integral((b^2\*arctan(d\*x + c)^2 + 2\*a\*b\*arctan(d\*x + c) + a^2)\*(f\*x + e)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*m\*(a+b\*atan(d\*x+c))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(dx+c) + a)^2 (fx+e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)^2\*(f\*x + e)^m, x)

$$3.43 \quad \int (e + fx)^m (a + b \tan^{-1}(c + dx))^3 dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left((e + fx)^m (a + b \tan^{-1}(c + dx))^3, x\right)$$

[Out] Unintegrable[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^3, x]

**Rubi [A]** time = 0.0564118, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (e + fx)^m (a + b \tan^{-1}(c + dx))^3 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^3,x]

[Out] Defer[Subst][Defer[Int][((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^3, x], x, c + d\*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \tan^{-1}(c + dx))^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \tan^{-1}(x))^3 dx, x, c + dx\right)}{d}$$

**Mathematica [A]** time = 0.405131, size = 0, normalized size = 0.

$$\int (e + fx)^m (a + b \tan^{-1}(c + dx))^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^3,x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcTan[c + d\*x])^3, x]

**Maple [A]** time = 1.097, size = 0, normalized size = 0.

$$\int (fx + e)^m (a + b \arctan(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^3,x)

[Out] int((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^3,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \arctan(dx+c)^3 + 3ab^2 \arctan(dx+c)^2 + 3a^2b \arctan(dx+c) + a^3\right)(fx+e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^3,x, algorithm="fricas")

[Out] integral((b^3\*arctan(d\*x + c)^3 + 3\*a\*b^2\*arctan(d\*x + c)^2 + 3\*a^2\*b\*arctan(d\*x + c) + a^3)\*(f\*x + e)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*m\*(a+b\*atan(d\*x+c))\*\*3,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(dx+c) + a)^3 (fx+e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arctan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((b\*arctan(d\*x + c) + a)^3\*(f\*x + e)^m, x)

### 3.44 $\int x^3 \tan^{-1}(a + bx) dx$

**Optimal.** Leaf size=106

$$\frac{(1-6a^2)x}{4b^3} - \frac{a(1-a^2)\log((a+bx)^2+1)}{2b^4} - \frac{(a^4-6a^2+1)\tan^{-1}(a+bx)}{4b^4} - \frac{(a+bx)^3}{12b^4} + \frac{a(a+bx)^2}{2b^4} + \frac{1}{4}x^4 \tan^{-1}(a+bx)$$

[Out]  $((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) - ((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)$

**Rubi [A]** time = 0.110634, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1-6a^2)x}{4b^3} - \frac{a(1-a^2)\log((a+bx)^2+1)}{2b^4} - \frac{(a^4-6a^2+1)\tan^{-1}(a+bx)}{4b^4} - \frac{(a+bx)^3}{12b^4} + \frac{a(a+bx)^2}{2b^4} + \frac{1}{4}x^4 \tan^{-1}(a+bx)$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcTan[a + b\*x], x]

[Out]  $((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) - ((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)$

#### Rule 5047

Int[((a\_) + ArcTan[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4862

Int[((a\_) + ArcTan[(c\_)\*(x\_)])\*(b\_)\*((d\_) + (e\_)\*(x\_))^(q\_), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

Int[((d\_) + (e\_)\*(x\_))^(m\_)/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

#### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned} \int x^3 \tan^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \tan^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{4}x^4 \tan^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^4}{1 + x^2} dx, x, a + bx\right) \\ &= \frac{1}{4}x^4 \tan^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \left(-\frac{1 - 6a^2}{b^4} - \frac{4ax}{b^4} + \frac{x^2}{b^4} + \frac{1 - 6a^2 + a^4 + 4a(1 - a^2)x}{b^4(1 + x^2)}\right) dx, x, a + bx\right) \\ &= \frac{(1 - 6a^2)x}{4b^3} + \frac{a(a + bx)^2}{2b^4} - \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \tan^{-1}(a + bx) - \frac{\text{Subst}\left(\int \frac{1 - 6a^2 + a^4 + 4a(1 - a^2)x}{1 + x^2} dx, x, a + bx\right)}{4b^4} \\ &= \frac{(1 - 6a^2)x}{4b^3} + \frac{a(a + bx)^2}{2b^4} - \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \tan^{-1}(a + bx) - \frac{(a(1 - a^2)) \text{Subst}\left(\int \frac{x}{1 + x^2} dx, x, a + bx\right)}{b^4} \\ &= \frac{(1 - 6a^2)x}{4b^3} + \frac{a(a + bx)^2}{2b^4} - \frac{(a + bx)^3}{12b^4} - \frac{(1 - 6a^2 + a^4) \tan^{-1}(a + bx)}{4b^4} + \frac{1}{4}x^4 \tan^{-1}(a + bx) - \frac{a(1 - a^2)}{4b^4} \end{aligned}$$

**Mathematica [C]** time = 0.0699706, size = 95, normalized size = 0.9

$$\frac{6(1 - 6a^2)bx + 6b^4x^4 \tan^{-1}(a + bx) - 2(a + bx)^3 + 12a(a + bx)^2 + 3i(a - i)^4 \log(-a - bx + i) - 3i(a + i)^4 \log(a + bx + i)}{24b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcTan[a + b\*x], x]

[Out] (6\*(1 - 6\*a^2)\*b\*x + 12\*a\*(a + b\*x)^2 - 2\*(a + b\*x)^3 + 6\*b^4\*x^4\*ArcTan[a + b\*x] + (3\*I)\*(-I + a)^4\*Log[I - a - b\*x] - (3\*I)\*(I + a)^4\*Log[I + a + b\*x])/(24\*b^4)

**Maple [A]** time = 0.039, size = 132, normalized size = 1.3

$$\frac{x^4 \arctan(bx + a)}{4} - \frac{\arctan(bx + a)a^4}{4b^4} - \frac{x^3}{12b} + \frac{ax^2}{4b^2} - \frac{3a^2x}{4b^3} - \frac{13a^3}{12b^4} + \frac{x}{4b^3} + \frac{a}{4b^4} + \frac{\ln(1 + (bx + a)^2)a^3}{2b^4} - \frac{\ln(1 + (bx + a)^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arctan(b\*x+a), x)

[Out] 1/4\*x^4\*arctan(b\*x+a)-1/4/b^4\*arctan(b\*x+a)\*a^4-1/12/b\*x^3+1/4/b^2\*x^2\*a-3/4/b^3\*x\*a^2-13/12/b^4\*a^3+1/4/b^3\*x+1/4/b^4\*a+1/2/b^4\*ln(1+(b\*x+a)^2)\*a^3-1/2/b^4\*ln(1+(b\*x+a)^2)\*a+3/2/b^4\*arctan(b\*x+a)\*a^2-1/4/b^4\*arctan(b\*x+a)

---

**Maxima [A]** time = 1.50389, size = 140, normalized size = 1.32

$$\frac{1}{4} x^4 \arctan (bx + a) - \frac{1}{12} b \left( \frac{b^2 x^3 - 3 abx^2 + 3 (3 a^2 - 1) x}{b^4} + \frac{3 (a^4 - 6 a^2 + 1) \arctan \left( \frac{b^2 x + ab}{b} \right)}{b^5} - \frac{6 (a^3 - a) \log (b^2 x^2 + 2 abx + a^2 + 1)}{b^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arctan(b\*x+a),x, algorithm="maxima")

[Out] 1/4\*x^4\*arctan(b\*x + a) - 1/12\*b\*((b^2\*x^3 - 3\*a\*b\*x^2 + 3\*(3\*a^2 - 1)\*x)/b^4 + 3\*(a^4 - 6\*a^2 + 1)\*arctan((b^2\*x + a\*b)/b)/b^5 - 6\*(a^3 - a)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1)/b^5)

---

**Fricas [A]** time = 1.68514, size = 203, normalized size = 1.92

$$\frac{b^3 x^3 - 3 ab^2 x^2 + 3 (3 a^2 - 1) bx - 3 (b^4 x^4 - a^4 + 6 a^2 - 1) \arctan (bx + a) - 6 (a^3 - a) \log (b^2 x^2 + 2 abx + a^2 + 1)}{12 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arctan(b\*x+a),x, algorithm="fricas")

[Out] -1/12\*(b^3\*x^3 - 3\*a\*b^2\*x^2 + 3\*(3\*a^2 - 1)\*b\*x - 3\*(b^4\*x^4 - a^4 + 6\*a^2 - 1)\*arctan(b\*x + a) - 6\*(a^3 - a)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1))/b^4

---

**Sympy [A]** time = 2.36794, size = 155, normalized size = 1.46

$$\left\{ \begin{array}{l} \frac{a^4 \operatorname{atan}(a+bx)}{4b^4} + \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} - \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{atan}(a+bx)}{2b^4} + \frac{ax^2}{4b^2} - \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{atan}(a+bx)}{4} - \frac{x^3}{12b} + \frac{x}{4b^3} - a \\ \frac{x^4 \operatorname{atan}(a)}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*atan(b\*x+a),x)

[Out] Piecewise((-a\*\*4\*atan(a + b\*x)/(4\*b\*\*4) + a\*\*3\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(2\*b\*\*4) - 3\*a\*\*2\*x/(4\*b\*\*3) + 3\*a\*\*2\*atan(a + b\*x)/(2\*b\*\*4) + a\*x\*\*2/(4\*b\*\*2) - a\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(2\*b\*\*4) + x\*\*4\*atan(a + b\*x)/4 - x\*\*3/(12\*b) + x/(4\*b\*\*3) - atan(a + b\*x)/(4\*b\*\*4), Ne(b, 0)), (x\*\*4\*atan(a)/4, True))

---

**Giac [A]** time = 1.10813, size = 139, normalized size = 1.31

$$\frac{1}{4} x^4 \arctan (bx + a) - \frac{1}{12} b \left( \frac{3 (a^4 - 6 a^2 + 1) \arctan (bx + a)}{b^5} - \frac{6 (a^3 - a) \log (b^2 x^2 + 2 abx + a^2 + 1)}{b^5} + \frac{b^4 x^3 - 3 ab^3 x^2 + 3 (3 a^2 - 1) bx - 3 (b^4 x^4 - a^4 + 6 a^2 - 1) \arctan (bx + a) - 6 (a^3 - a) \log (b^2 x^2 + 2 abx + a^2 + 1)}{12 b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(b*x + a) - 1/12*b*(3*(a^4 - 6*a^2 + 1)*arctan(b*x + a)/b^5 -  
6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5 + (b^4*x^3 - 3*a*b^3*x^2  
+ 9*a^2*b^2*x - 3*b^2*x)/b^6)
```



### 3.45 $\int x^2 \tan^{-1}(a + bx) dx$

**Optimal.** Leaf size=79

$$\frac{(1 - 3a^2) \log((a + bx)^2 + 1)}{6b^3} - \frac{a(3 - a^2) \tan^{-1}(a + bx)}{3b^3} + \frac{ax}{b^2} - \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \tan^{-1}(a + bx)$$

[Out] (a\*x)/b^2 - (a + b\*x)^2/(6\*b^3) - (a\*(3 - a^2)\*ArcTan[a + b\*x])/(3\*b^3) + (x^3\*ArcTan[a + b\*x])/3 + ((1 - 3\*a^2)\*Log[1 + (a + b\*x)^2])/(6\*b^3)

**Rubi [A]** time = 0.0917566, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1 - 3a^2) \log((a + bx)^2 + 1)}{6b^3} - \frac{a(3 - a^2) \tan^{-1}(a + bx)}{3b^3} + \frac{ax}{b^2} - \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \tan^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcTan[a + b\*x],x]

[Out] (a\*x)/b^2 - (a + b\*x)^2/(6\*b^3) - (a\*(3 - a^2)\*ArcTan[a + b\*x])/(3\*b^3) + (x^3\*ArcTan[a + b\*x])/3 + ((1 - 3\*a^2)\*Log[1 + (a + b\*x)^2])/(6\*b^3)

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4862

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

#### Rule 635

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

`Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rubi steps

$$\begin{aligned}
 \int x^2 \tan^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \tan^{-1}(x) dx, x, a + bx\right)}{b} \\
 &= \frac{1}{3} x^3 \tan^{-1}(a + bx) - \frac{1}{3} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^3}{1 + x^2} dx, x, a + bx\right) \\
 &= \frac{1}{3} x^3 \tan^{-1}(a + bx) - \frac{1}{3} \text{Subst}\left(\int \left(-\frac{3a}{b^3} + \frac{x}{b^3} + \frac{a(3 - a^2) - (1 - 3a^2)x}{b^3(1 + x^2)}\right) dx, x, a + bx\right) \\
 &= \frac{ax}{b^2} - \frac{(a + bx)^2}{6b^3} + \frac{1}{3} x^3 \tan^{-1}(a + bx) - \frac{\text{Subst}\left(\int \frac{a(3 - a^2) - (1 - 3a^2)x}{1 + x^2} dx, x, a + bx\right)}{3b^3} \\
 &= \frac{ax}{b^2} - \frac{(a + bx)^2}{6b^3} + \frac{1}{3} x^3 \tan^{-1}(a + bx) + \frac{(1 - 3a^2) \text{Subst}\left(\int \frac{x}{1 + x^2} dx, x, a + bx\right)}{3b^3} - \frac{(a(3 - a^2)) \text{Subst}\left(\int \frac{1}{1 + x^2} dx, x, a + bx\right)}{3b^3} \\
 &= \frac{ax}{b^2} - \frac{(a + bx)^2}{6b^3} - \frac{a(3 - a^2) \tan^{-1}(a + bx)}{3b^3} + \frac{1}{3} x^3 \tan^{-1}(a + bx) + \frac{(1 - 3a^2) \log(1 + (a + bx)^2)}{6b^3}
 \end{aligned}$$

**Mathematica [C]** time = 0.0483124, size = 114, normalized size = 1.44

$$\frac{\frac{1}{3} b \left( \frac{a+bx}{b} - \frac{a}{b} \right)^3 \tan^{-1}(a + bx) - \frac{1}{3} b \left( \frac{(a+bx)^2}{2b^3} - \frac{3ax}{b^2} - \frac{(1-ia)^3 \log(a+bx+i)}{2b^3} - \frac{(1+ia)^3 \log(-a-bx+i)}{2b^3} \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcTan[a + b\*x], x]

[Out] ((b\*(-(a/b) + (a + b\*x)/b)^3\*ArcTan[a + b\*x])/3 - (b\*((-3\*a\*x)/b^2 + (a + b\*x)^2/(2\*b^3) - ((1 + I\*a)^3\*Log[I - a - b\*x])/(2\*b^3) - ((1 - I\*a)^3\*Log[I + a + b\*x])/(2\*b^3)))/3)/b

**Maple [A]** time = 0.036, size = 95, normalized size = 1.2

$$\frac{x^3 \arctan(bx + a)}{3} + \frac{\arctan(bx + a) a^3}{3b^3} - \frac{x^2}{6b} + \frac{2ax}{3b^2} + \frac{5a^2}{6b^3} - \frac{\ln(1 + (bx + a)^2) a^2}{2b^3} + \frac{\ln(1 + (bx + a)^2)}{6b^3} - \frac{\arctan(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arctan(b\*x+a), x)

[Out] 1/3\*x^3\*arctan(b\*x+a)+1/3/b^3\*arctan(b\*x+a)\*a^3-1/6/b\*x^2+2/3\*a\*x/b^2+5/6/b^3\*a^2-1/2/b^3\*ln(1+(b\*x+a)^2)\*a^2+1/6/b^3\*ln(1+(b\*x+a)^2)-1/b^3\*arctan(b\*x+a)\*a

**Maxima [A]** time = 1.50323, size = 115, normalized size = 1.46

$$\frac{1}{3} x^3 \arctan (bx + a) - \frac{1}{6} b \left( \frac{bx^2 - 4ax}{b^3} - \frac{2(a^3 - 3a) \arctan \left( \frac{b^2x + ab}{b} \right)}{b^4} + \frac{(3a^2 - 1) \log (b^2x^2 + 2abx + a^2 + 1)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arctan(b\*x+a),x, algorithm="maxima")

[Out] 1/3\*x^3\*arctan(b\*x + a) - 1/6\*b\*((b\*x^2 - 4\*a\*x)/b^3 - 2\*(a^3 - 3\*a)\*arctan((b^2\*x + a\*b)/b)/b^4 + (3\*a^2 - 1)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1)/b^4)

**Fricas [A]** time = 1.74905, size = 161, normalized size = 2.04

$$\frac{b^2x^2 - 4abx - 2(b^3x^3 + a^3 - 3a) \arctan (bx + a) + (3a^2 - 1) \log (b^2x^2 + 2abx + a^2 + 1)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arctan(b\*x+a),x, algorithm="fricas")

[Out] -1/6\*(b^2\*x^2 - 4\*a\*b\*x - 2\*(b^3\*x^3 + a^3 - 3\*a)\*arctan(b\*x + a) + (3\*a^2 - 1)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1))/b^3

**Sympy [A]** time = 1.4325, size = 117, normalized size = 1.48

$$\begin{cases} \frac{a^3 \operatorname{atan}(a+bx)}{3b^3} - \frac{a^2 \log(a^2+2abx+b^2x^2+1)}{2b^3} + \frac{2ax}{3b^2} - \frac{a \operatorname{atan}(a+bx)}{b^3} + \frac{x^3 \operatorname{atan}(a+bx)}{3} - \frac{x^2}{6b} + \frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{atan}(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*atan(b\*x+a),x)

[Out] Piecewise((a\*\*3\*atan(a + b\*x)/(3\*b\*\*3) - a\*\*2\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(2\*b\*\*3) + 2\*a\*x/(3\*b\*\*2) - a\*atan(a + b\*x)/b\*\*3 + x\*\*3\*atan(a + b\*x)/3 - x\*\*2/(6\*b) + log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(6\*b\*\*3), Ne(b, 0)), (x\*\*3\*atan(a)/3, True))

**Giac [A]** time = 1.09272, size = 111, normalized size = 1.41

$$\frac{1}{3} x^3 \arctan (bx + a) + \frac{1}{6} b \left( \frac{2(a^3 - 3a) \arctan (bx + a)}{b^4} - \frac{(3a^2 - 1) \log (b^2x^2 + 2abx + a^2 + 1)}{b^4} - \frac{b^2x^2 - 4abx}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arctan(b\*x+a),x, algorithm="giac")

[Out] 1/3\*x^3\*arctan(b\*x + a) + 1/6\*b\*(2\*(a^3 - 3\*a)\*arctan(b\*x + a)/b^4 - (3\*a^2 - 1)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1)/b^4 - (b^2\*x^2 - 4\*a\*b\*x)/b^4)

### 3.46 $\int x \tan^{-1}(a + bx) dx$

**Optimal.** Leaf size=60

$$\frac{(1 - a^2) \tan^{-1}(a + bx)}{2b^2} + \frac{a \log((a + bx)^2 + 1)}{2b^2} + \frac{1}{2} x^2 \tan^{-1}(a + bx) - \frac{x}{2b}$$

[Out]  $-x/(2*b) + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 + (a*Log[1 + (a + b*x)^2])/(2*b^2)$

**Rubi [A]** time = 0.0546893, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {5047, 4862, 702, 635, 203, 260}

$$\frac{(1 - a^2) \tan^{-1}(a + bx)}{2b^2} + \frac{a \log((a + bx)^2 + 1)}{2b^2} + \frac{1}{2} x^2 \tan^{-1}(a + bx) - \frac{x}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcTan[a + b\*x], x]

[Out]  $-x/(2*b) + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 + (a*Log[1 + (a + b*x)^2])/(2*b^2)$

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4862

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcTan[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] :> Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

#### Rule 635

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 260**

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x\_Symbol] :> \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

**Rubi steps**

$$\begin{aligned} \int x \tan^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right) \tan^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{2}x^2 \tan^{-1}(a + bx) - \frac{1}{2} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^2}{1 + x^2} dx, x, a + bx\right) \\ &= \frac{1}{2}x^2 \tan^{-1}(a + bx) - \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{1 - a^2 + 2ax}{b^2(1 + x^2)}\right) dx, x, a + bx\right) \\ &= -\frac{x}{2b} + \frac{1}{2}x^2 \tan^{-1}(a + bx) + \frac{\text{Subst}\left(\int \frac{1 - a^2 + 2ax}{1 + x^2} dx, x, a + bx\right)}{2b^2} \\ &= -\frac{x}{2b} + \frac{1}{2}x^2 \tan^{-1}(a + bx) + \frac{a \text{Subst}\left(\int \frac{x}{1 + x^2} dx, x, a + bx\right)}{b^2} + \frac{(1 - a^2) \text{Subst}\left(\int \frac{1}{1 + x^2} dx, x, a + bx\right)}{2b^2} \\ &= -\frac{x}{2b} + \frac{(1 - a^2) \tan^{-1}(a + bx)}{2b^2} + \frac{1}{2}x^2 \tan^{-1}(a + bx) + \frac{a \log(1 + (a + bx)^2)}{2b^2} \end{aligned}$$

**Mathematica [C]** time = 0.0307717, size = 90, normalized size = 1.5

$$\frac{-ia^2 \log(a + bx + i) + 2b^2 x^2 \tan^{-1}(a + bx) + 2a \log(a + bx + i) + i(a - i)^2 \log(-a - bx + i) + i \log(a + bx + i) - 2bx}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcTan[a + b\*x],x]

[Out]  $(-2*b*x + 2*b^2*x^2*ArcTan[a + b*x] + I*(-I + a)^2*Log[I - a - b*x] + I*Log[I + a + b*x] + 2*a*Log[I + a + b*x] - I*a^2*Log[I + a + b*x])/(4*b^2)$

**Maple [A]** time = 0.035, size = 66, normalized size = 1.1

$$\frac{x^2 \arctan(bx + a)}{2} - \frac{\arctan(bx + a) a^2}{2b^2} - \frac{x}{2b} - \frac{a}{2b^2} + \frac{a \ln(1 + (bx + a)^2)}{2b^2} + \frac{\arctan(bx + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arctan(b\*x+a),x)

[Out]  $1/2*x^2*arctan(b*x+a) - 1/2/b^2*arctan(b*x+a)*a^2 - 1/2*x/b - 1/2/b^2*a + 1/2*a*\ln(1+(b*x+a)^2)/b^2 + 1/2/b^2*arctan(b*x+a)$

**Maxima [A]** time = 1.51409, size = 92, normalized size = 1.53

$$\frac{1}{2}x^2 \arctan(bx + a) - \frac{1}{2}b \left( \frac{x}{b^2} + \frac{(a^2 - 1) \arctan\left(\frac{b^2x + ab}{b}\right)}{b^3} - \frac{a \log(b^2x^2 + 2abx + a^2 + 1)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arctan(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}x^2\arctan(bx+a) - \frac{1}{2}b\left(\frac{x}{b^2} + (a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)\right) - a\log(b^2x^2+2abx+a^2+1)/b^3$

**Fricas [A]** time = 1.69324, size = 123, normalized size = 2.05

$$\frac{bx - (b^2x^2 - a^2 + 1)\arctan(bx + a) - a\log(b^2x^2 + 2abx + a^2 + 1)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arctan(b\*x+a),x, algorithm="fricas")

[Out]  $-1/2*(bx - (b^2*x^2 - a^2 + 1)*\arctan(b*x + a) - a*\log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^2$

**Sympy [A]** time = 0.965333, size = 78, normalized size = 1.3

$$\begin{cases} \frac{-\frac{a^2 \operatorname{atan}(a+bx)}{2b^2} + \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^2} + \frac{x^2 \operatorname{atan}(a+bx)}{2} - \frac{x}{2b} + \frac{\operatorname{atan}(a+bx)}{2b^2}}{\frac{x^2 \operatorname{atan}(a)}{2}} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{atan}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*atan(b\*x+a),x)

[Out] Piecewise((-a\*\*2\*atan(a + b\*x)/(2\*b\*\*2) + a\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(2\*b\*\*2) + x\*\*2\*atan(a + b\*x)/2 - x/(2\*b) + atan(a + b\*x)/(2\*b\*\*2), Ne(b, 0)), (x\*\*2\*atan(a)/2, True))

**Giac [A]** time = 1.10012, size = 81, normalized size = 1.35

$$\frac{1}{2}x^2\arctan(bx+a) - \frac{1}{2}b\left(\frac{x}{b^2} + \frac{(a^2-1)\arctan(bx+a)}{b^3} - \frac{a\log(b^2x^2+2abx+a^2+1)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arctan(b\*x+a),x, algorithm="giac")

[Out]  $\frac{1}{2}x^2\arctan(bx+a) - \frac{1}{2}b\left(\frac{x}{b^2} + (a^2-1)\arctan(bx+a)\right) - a\log(b^2x^2+2abx+a^2+1)/b^3$

### 3.47 $\int \tan^{-1}(a + bx) dx$

**Optimal.** Leaf size=33

$$\frac{(a + bx) \tan^{-1}(a + bx)}{b} - \frac{\log((a + bx)^2 + 1)}{2b}$$

[Out] ((a + b\*x)\*ArcTan[a + b\*x])/b - Log[1 + (a + b\*x)^2]/(2\*b)

**Rubi [A]** time = 0.0113888, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5039, 4846, 260}

$$\frac{(a + bx) \tan^{-1}(a + bx)}{b} - \frac{\log((a + bx)^2 + 1)}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x], x]

[Out] ((a + b\*x)\*ArcTan[a + b\*x])/b - Log[1 + (a + b\*x)^2]/(2\*b)

#### Rule 5039

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 4846

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcTan[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTan[c\*x])^(p - 1))/(1 + c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned} \int \tan^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \tan^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \tan^{-1}(a + bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2} dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \tan^{-1}(a + bx)}{b} - \frac{\log(1 + (a + bx)^2)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0138611, size = 39, normalized size = 1.18

$$\frac{\log(a^2 + 2abx + b^2x^2 + 1) - 2(a + bx) \tan^{-1}(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x],x]

[Out]  $-(2*(a + b*x)*\text{ArcTan}[a + b*x] + \text{Log}[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b)$

**Maple [A]** time = 0.035, size = 36, normalized size = 1.1

$$x \arctan(bx + a) + \frac{\arctan(bx + a)a}{b} - \frac{\ln(1 + (bx + a)^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a),x)

[Out]  $x*\arctan(b*x+a)+1/b*\arctan(b*x+a)*a-1/2*\ln(1+(b*x+a)^2)/b$

**Maxima [A]** time = 1.00836, size = 42, normalized size = 1.27

$$\frac{2(bx + a) \arctan(bx + a) - \log((bx + a)^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a),x, algorithm="maxima")

[Out]  $1/2*(2*(b*x + a)*\arctan(b*x + a) - \log((b*x + a)^2 + 1))/b$

**Fricas [A]** time = 1.74759, size = 97, normalized size = 2.94

$$\frac{2(bx + a) \arctan(bx + a) - \log(b^2x^2 + 2abx + a^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a),x, algorithm="fricas")

[Out]  $1/2*(2*(b*x + a)*\arctan(b*x + a) - \log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b$

**Sympy [A]** time = 0.558806, size = 46, normalized size = 1.39

$$\begin{cases} \frac{a \operatorname{atan}(a+bx)}{b} + x \operatorname{atan}(a + bx) - \frac{\log(a^2+2abx+b^2x^2+1)}{2b} & \text{for } b \neq 0 \\ x \operatorname{atan}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a),x)



```
[Out] Piecewise((a*atan(a + b*x)/b + x*atan(a + b*x) - log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b), Ne(b, 0)), (x*atan(a), True))
```

**Giac [A]** time = 1.11763, size = 42, normalized size = 1.27

$$\frac{2(bx + a)\arctan(bx + a) - \log((bx + a)^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

$$3.48 \quad \int \frac{\tan^{-1}(a+bx)}{x} dx$$

**Optimal.** Leaf size=120

$$\frac{1}{2}i\text{PolyLog}\left(2, 1 - \frac{2}{1-i(a+bx)}\right) - \frac{1}{2}i\text{PolyLog}\left(2, 1 - \frac{2bx}{(-a+i)(1-i(a+bx))}\right) + \log\left(\frac{2}{1-i(a+bx)}\right)(-\tan^{-1}(a+bx)) +$$

[Out] -(ArcTan[a + b\*x]\*Log[2/(1 - I\*(a + b\*x))]) + ArcTan[a + b\*x]\*Log[(2\*b\*x)/(I - a)\*(1 - I\*(a + b\*x))] + (I/2)\*PolyLog[2, 1 - 2/(1 - I\*(a + b\*x))] - (I/2)\*PolyLog[2, 1 - (2\*b\*x)/((I - a)\*(1 - I\*(a + b\*x)))]

**Rubi [A]** time = 0.106247, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5047, 4856, 2402, 2315, 2447}

$$\frac{1}{2}i\text{PolyLog}\left(2, 1 - \frac{2}{1-i(a+bx)}\right) - \frac{1}{2}i\text{PolyLog}\left(2, 1 - \frac{2bx}{(-a+i)(1-i(a+bx))}\right) + \log\left(\frac{2}{1-i(a+bx)}\right)(-\tan^{-1}(a+bx)) +$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/x,x]

[Out] -(ArcTan[a + b\*x]\*Log[2/(1 - I\*(a + b\*x))]) + ArcTan[a + b\*x]\*Log[(2\*b\*x)/(I - a)\*(1 - I\*(a + b\*x))] + (I/2)\*PolyLog[2, 1 - 2/(1 - I\*(a + b\*x))] - (I/2)\*PolyLog[2, 1 - (2\*b\*x)/((I - a)\*(1 - I\*(a + b\*x)))]

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4856

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])\*Log[2/(1 - I\*c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/(c\*d + I\*e)\*(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] + Simp[((a + b\*ArcTan[c\*x])\*Log[(2\*c\*(d + e\*x))/(c\*d + I\*e)\*(1 - I\*c\*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_.), x\_Symbol] :> With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&

PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(a+bx)}{x} dx &= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}} dx, x, a+bx\right)}{b} \\ &= -\tan^{-1}(a+bx) \log\left(\frac{2}{1-i(a+bx)}\right) + \tan^{-1}(a+bx) \log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) + \text{Subst}\left(\int \frac{\log(x)}{x} dx, x, a+bx\right) \\ &= -\tan^{-1}(a+bx) \log\left(\frac{2}{1-i(a+bx)}\right) + \tan^{-1}(a+bx) \log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) - \frac{1}{2}i\text{Li}_2\left(1-\frac{2bx}{(i-a)(1-i(a+bx))}\right) \\ &= -\tan^{-1}(a+bx) \log\left(\frac{2}{1-i(a+bx)}\right) + \tan^{-1}(a+bx) \log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) + \frac{1}{2}i\text{Li}_2\left(1-\frac{2bx}{(i-a)(1-i(a+bx))}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0081672, size = 171, normalized size = 1.42

$$\frac{1}{2}i\text{PolyLog}\left(2, \frac{i(1-i(a+bx))}{a+i}\right) - \frac{1}{2}i\text{PolyLog}\left(2, -\frac{i(1+i(a+bx))}{a-i}\right) - \frac{1}{2}i \log(1+i(a+bx)) \log\left(\frac{i\left(\frac{a+bx}{b} - \frac{a}{b}\right)}{-\frac{1}{b} - \frac{ia}{b}}\right) + \frac{1}{2}i \log\left(\frac{a+bx}{b}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/x, x]

[Out] (-I/2)\*Log[1 + I\*(a + b\*x)]\*Log[(I\*(-(a/b) + (a + b\*x)/b))/(-b^(-1) - (I\*a)/b)] + (I/2)\*Log[1 - I\*(a + b\*x)]\*Log[((-I)\*(-(a/b) + (a + b\*x)/b))/(-b^(-1) + (I\*a)/b)] + (I/2)\*PolyLog[2, (I\*(1 - I\*(a + b\*x)))/(I + a)] - (I/2)\*PolyLog[2, ((-I)\*(1 + I\*(a + b\*x)))/(-I + a)]

**Maple [A]** time = 0.047, size = 103, normalized size = 0.9

$$\ln(bx) \arctan(bx+a) + \frac{i}{2} \ln(bx) \ln\left(\frac{i-a-bx}{i-a}\right) - \frac{i}{2} \ln(bx) \ln\left(\frac{i+a+bx}{i+a}\right) + \frac{i}{2} \text{dilog}\left(\frac{i-a-bx}{i-a}\right) - \frac{i}{2} \text{dilog}\left(\frac{i+a+bx}{i+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/x, x)

[Out] ln(b\*x)\*arctan(b\*x+a)+1/2\*I\*ln(b\*x)\*ln((I-a-b\*x)/(I-a))-1/2\*I\*ln(b\*x)\*ln((I+a+b\*x)/(I+a))+1/2\*I\*dilog((I-a-b\*x)/(I-a))-1/2\*I\*dilog((I+a+b\*x)/(I+a))

**Maxima [A]** time = 1.71072, size = 181, normalized size = 1.51

$$-\frac{1}{2} \arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right) \log(b^2x^2 + 2abx + a^2 + 1) + \frac{1}{2} \arctan(bx+a) \log\left(\frac{b^2x^2}{a^2+1}\right) + \arctan(bx+a) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x,x, algorithm="maxima")

[Out]  $-1/2*\arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*\log(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1/2*\arctan(b*x + a)*\log(b^2*x^2/(a^2 + 1)) + \arctan(b*x + a)*\log(x) - \arctan((b^2*x + a*b)/b)*\log(x) - 1/2*I*\operatorname{dilog}((I*b*x + I*a + 1)/(I*a + 1)) + 1/2*I*\operatorname{dilog}((I*b*x + I*a - 1)/(I*a - 1))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\arctan(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x,x, algorithm="fricas")

[Out] integral(arctan(b\*x + a)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x,x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/x, x)

$$3.49 \quad \int \frac{\tan^{-1}(a+bx)}{x^2} dx$$

**Optimal.** Leaf size=62

$$\frac{b \log(x)}{a^2+1} - \frac{b \log((a+bx)^2+1)}{2(a^2+1)} - \frac{ab \tan^{-1}(a+bx)}{a^2+1} - \frac{\tan^{-1}(a+bx)}{x}$$

[Out] -((a\*b\*ArcTan[a + b\*x])/(1 + a^2)) - ArcTan[a + b\*x]/x + (b\*Log[x])/(1 + a^2) - (b\*Log[1 + (a + b\*x)^2])/(2\*(1 + a^2))

**Rubi [A]** time = 0.038679, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5045, 371, 706, 31, 635, 203, 260}

$$\frac{b \log(x)}{a^2+1} - \frac{b \log((a+bx)^2+1)}{2(a^2+1)} - \frac{ab \tan^{-1}(a+bx)}{a^2+1} - \frac{\tan^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/x^2,x]

[Out] -((a\*b\*ArcTan[a + b\*x])/(1 + a^2)) - ArcTan[a + b\*x]/x + (b\*Log[x])/(1 + a^2) - (b\*Log[1 + (a + b\*x)^2])/(2\*(1 + a^2))

#### Rule 5045

Int[((a\_) + ArcTan[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^(p - 1))/(1 + (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

#### Rule 371

Int[((a\_) + (b\_)\*(v\_)^(n\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m\*(a + b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

#### Rule 706

Int[1/(((d\_) + (e\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 + a\*e^2), Int[(c\*d - c\*e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}

}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(a+bx)}{x^2} dx &= -\frac{\tan^{-1}(a+bx)}{x} + b \int \frac{1}{x(1+(a+bx)^2)} dx \\ &= -\frac{\tan^{-1}(a+bx)}{x} + b \operatorname{Subst}\left(\int \frac{1}{(-a+x)(1+x^2)} dx, x, a+bx\right) \\ &= -\frac{\tan^{-1}(a+bx)}{x} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-a+x} dx, x, a+bx\right)}{1+a^2} + \frac{b \operatorname{Subst}\left(\int \frac{-a-x}{1+x^2} dx, x, a+bx\right)}{1+a^2} \\ &= -\frac{\tan^{-1}(a+bx)}{x} + \frac{b \log(x)}{1+a^2} - \frac{b \operatorname{Subst}\left(\int \frac{x}{1+x^2} dx, x, a+bx\right)}{1+a^2} - \frac{(ab) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, a+bx\right)}{1+a^2} \\ &= -\frac{ab \tan^{-1}(a+bx)}{1+a^2} - \frac{\tan^{-1}(a+bx)}{x} + \frac{b \log(x)}{1+a^2} - \frac{b \log(1+(a+bx)^2)}{2(1+a^2)} \end{aligned}$$

**Mathematica [C]** time = 0.0560829, size = 67, normalized size = 1.08

$$-\frac{\tan^{-1}(a+bx)}{x} + \frac{b(i(a+i)\log(-a-bx+i) + (-1-ia)\log(a+bx+i) + 2\log(x))}{2(a^2+1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/x^2, x]

[Out] -(ArcTan[a + b\*x]/x) + (b\*(2\*Log[x] + I\*(I + a)\*Log[I - a - b\*x] + (-1 - I\*a)\*Log[I + a + b\*x]))/(2\*(1 + a^2))

**Maple [A]** time = 0.043, size = 63, normalized size = 1.

$$-\frac{\arctan(bx+a)}{x} - \frac{b \ln(1+(bx+a)^2)}{2a^2+2} - \frac{ab \arctan(bx+a)}{a^2+1} + \frac{b \ln(bx)}{a^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/x^2, x)

[Out] -arctan(b\*x+a)/x-1/2\*b\*ln(1+(b\*x+a)^2)/(a^2+1)-a\*b\*arctan(b\*x+a)/(a^2+1)+b/(a^2+1)\*ln(b\*x)

---

**Maxima [A]** time = 1.54389, size = 104, normalized size = 1.68

$$-\frac{1}{2}b\left(\frac{2a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1} + \frac{\log(b^2x^2+2abx+a^2+1)}{a^2+1} - \frac{2\log(x)}{a^2+1}\right) - \frac{\arctan(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^2,x, algorithm="maxima")

[Out] -1/2\*b\*(2\*a\*arctan((b^2\*x + a\*b)/b)/(a^2 + 1) + log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1)/(a^2 + 1) - 2\*log(x)/(a^2 + 1)) - arctan(b\*x + a)/x

---

**Fricas [A]** time = 1.71557, size = 151, normalized size = 2.44

$$\frac{bx\log(b^2x^2+2abx+a^2+1)-2bx\log(x)+2(abx+a^2+1)\arctan(bx+a)}{2(a^2+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^2,x, algorithm="fricas")

[Out] -1/2\*(b\*x\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1) - 2\*b\*x\*log(x) + 2\*(a\*b\*x + a^2 + 1)\*arctan(b\*x + a))/((a^2 + 1)\*x)

---

**Sympy [B]** time = 10.9512, size = 323, normalized size = 5.21

$$\begin{cases} \frac{ib^2x^2\operatorname{atan}(bx-i)}{2bx^2-4ix} - \frac{4bx\operatorname{atan}(bx-i)}{2bx^2-4ix} - \frac{ibx}{2bx^2-4ix} + \frac{4i\operatorname{atan}(bx-i)}{2bx^2-4ix} - \frac{2}{2bx^2-4ix} & \text{for } a = -i \\ \frac{ib^2x^2\operatorname{atan}(bx+i)}{2bx^2+4ix} - \frac{4bx\operatorname{atan}(bx+i)}{2bx^2+4ix} + \frac{ibx}{2bx^2+4ix} - \frac{4i\operatorname{atan}(bx+i)}{2bx^2+4ix} - \frac{2}{2bx^2+4ix} & \text{for } a = i \\ -\frac{2a^2\operatorname{atan}(a+bx)}{2a^2x+2x} - \frac{2abx\operatorname{atan}(a+bx)}{2a^2x+2x} + \frac{2bx\log(x)}{2a^2x+2x} - \frac{bx\log(a^2+2abx+b^2x^2+1)}{2a^2x+2x} - \frac{2\operatorname{atan}(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/x\*\*2,x)

[Out] Piecewise((-I\*b\*\*2\*x\*\*2\*atan(b\*x - I)/(2\*b\*x\*\*2 - 4\*I\*x) - 4\*b\*x\*atan(b\*x - I)/(2\*b\*x\*\*2 - 4\*I\*x) - I\*b\*x/(2\*b\*x\*\*2 - 4\*I\*x) + 4\*I\*atan(b\*x - I)/(2\*b\*x\*\*2 - 4\*I\*x) - 2/(2\*b\*x\*\*2 - 4\*I\*x), Eq(a, -I)), (I\*b\*\*2\*x\*\*2\*atan(b\*x + I)/(2\*b\*x\*\*2 + 4\*I\*x) - 4\*b\*x\*atan(b\*x + I)/(2\*b\*x\*\*2 + 4\*I\*x) + I\*b\*x/(2\*b\*x\*\*2 + 4\*I\*x) - 4\*I\*atan(b\*x + I)/(2\*b\*x\*\*2 + 4\*I\*x) - 2/(2\*b\*x\*\*2 + 4\*I\*x), Eq(a, I)), (-2\*a\*\*2\*atan(a + b\*x)/(2\*a\*\*2\*x + 2\*x) - 2\*a\*b\*x\*atan(a + b\*x)/(2\*a\*\*2\*x + 2\*x) + 2\*b\*x\*log(x)/(2\*a\*\*2\*x + 2\*x) - b\*x\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(2\*a\*\*2\*x + 2\*x) - 2\*atan(a + b\*x)/(2\*a\*\*2\*x + 2\*x), True))

---

**Giac [A]** time = 1.11186, size = 95, normalized size = 1.53

$$-\frac{1}{2}b\left(\frac{2a\arctan(bx+a)}{a^2+1} + \frac{\log(b^2x^2+2abx+a^2+1)}{a^2+1} - \frac{2\log(|x|)}{a^2+1}\right) - \frac{\arctan(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*(2*a*arctan(b*x + a)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^2 + 1) - 2*log(abs(x))/(a^2 + 1)) - arctan(b*x + a)/x
```



### 3.50 $\int \frac{\tan^{-1}(a+bx)}{x^3} dx$

**Optimal.** Leaf size=96

$$-\frac{ab^2 \log(x)}{(a^2+1)^2} + \frac{ab^2 \log((a+bx)^2+1)}{2(a^2+1)^2} - \frac{(1-a^2)b^2 \tan^{-1}(a+bx)}{2(a^2+1)^2} - \frac{b}{2(a^2+1)x} - \frac{\tan^{-1}(a+bx)}{2x^2}$$

[Out]  $-b/(2*(1+a^2)*x) - ((1-a^2)*b^2*ArcTan[a+b*x])/(2*(1+a^2)^2) - ArcTan[a+b*x]/(2*x^2) - (a*b^2*Log[x])/(1+a^2)^2 + (a*b^2*Log[1+(a+b*x)^2])/(2*(1+a^2)^2)$

**Rubi [A]** time = 0.0833072, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5045, 371, 710, 801, 635, 203, 260}

$$-\frac{ab^2 \log(x)}{(a^2+1)^2} + \frac{ab^2 \log((a+bx)^2+1)}{2(a^2+1)^2} - \frac{(1-a^2)b^2 \tan^{-1}(a+bx)}{2(a^2+1)^2} - \frac{b}{2(a^2+1)x} - \frac{\tan^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/x^3,x]

[Out]  $-b/(2*(1+a^2)*x) - ((1-a^2)*b^2*ArcTan[a+b*x])/(2*(1+a^2)^2) - ArcTan[a+b*x]/(2*x^2) - (a*b^2*Log[x])/(1+a^2)^2 + (a*b^2*Log[1+(a+b*x)^2])/(2*(1+a^2)^2)$

#### Rule 5045

Int[((a\_.) + ArcTan[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^(p - 1))/(1 + (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

#### Rule 371

Int[((a\_.) + (b\_.)\*(v\_)^(n\_.))^ (p\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m\*(a + b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

#### Rule 710

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(d - e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1]

#### Rule 801

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((f\_.) + (g\_.)\*(x\_.))/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{x^3} dx &= -\frac{\tan^{-1}(a+bx)}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2(1+(a+bx)^2)} dx \\
&= -\frac{\tan^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2 \operatorname{Subst}\left(\int \frac{1}{(-a+x)^2(1+x^2)} dx, x, a+bx\right) \\
&= -\frac{b}{2(1+a^2)x} - \frac{\tan^{-1}(a+bx)}{2x^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{-a-x}{(-a+x)(1+x^2)} dx, x, a+bx\right)}{2(1+a^2)} \\
&= -\frac{b}{2(1+a^2)x} - \frac{\tan^{-1}(a+bx)}{2x^2} + \frac{b^2 \operatorname{Subst}\left(\int \left(\frac{2a}{(1+a^2)(a-x)} + \frac{-1+a^2+2ax}{(1+a^2)(1+x^2)}\right) dx, x, a+bx\right)}{2(1+a^2)} \\
&= -\frac{b}{2(1+a^2)x} - \frac{\tan^{-1}(a+bx)}{2x^2} - \frac{ab^2 \log(x)}{(1+a^2)^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2} dx, x, a+bx\right)}{2(1+a^2)^2} \\
&= -\frac{b}{2(1+a^2)x} - \frac{\tan^{-1}(a+bx)}{2x^2} - \frac{ab^2 \log(x)}{(1+a^2)^2} + \frac{(ab^2) \operatorname{Subst}\left(\int \frac{x}{1+x^2} dx, x, a+bx\right)}{(1+a^2)^2} - \frac{((1-a^2)b^2)S}{(1+a^2)^2} \\
&= -\frac{b}{2(1+a^2)x} - \frac{(1-a^2)b^2 \tan^{-1}(a+bx)}{2(1+a^2)^2} - \frac{\tan^{-1}(a+bx)}{2x^2} - \frac{ab^2 \log(x)}{(1+a^2)^2} + \frac{ab^2 \log(1+(a+bx)^2)}{2(1+a^2)^2}
\end{aligned}$$

**Mathematica [C]** time = 0.0958245, size = 92, normalized size = 0.96

$$\frac{-2 \tan^{-1}(a+bx) + \frac{bx(-i(a+i)^2bx \log(-a-bx+i) - 4abx \log(x) + (a-i)((1+ia)bx \log(a+bx+i) - 2(a+i)))}{(a^2+1)^2}}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/x^3, x]

[Out] (-2\*ArcTan[a + b\*x] + (b\*x\*(-4\*a\*b\*x\*Log[x] - I\*(I + a)^2\*b\*x\*Log[I - a - b\*x] + (-I + a)\*(-2\*(I + a) + (1 + I\*a)\*b\*x\*Log[I + a + b\*x]))) / ((1 + a^2)^2) / (4\*x^2)

---

**Maple [A]** time = 0.043, size = 105, normalized size = 1.1

$$-\frac{\arctan(bx+a)}{2x^2} + \frac{b^2 \arctan(bx+a)a^2}{2(a^2+1)^2} + \frac{ab^2 \ln(1+(bx+a)^2)}{2(a^2+1)^2} - \frac{b^2 \arctan(bx+a)}{2(a^2+1)^2} - \frac{b}{(2a^2+2)x} - \frac{ab^2 \ln(bx)}{(a^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/x^3,x)

[Out]  $-1/2*\arctan(b*x+a)/x^2+1/2*b^2/(a^2+1)^2*\arctan(b*x+a)*a^2+1/2*a*b^2*\ln(1+(b*x+a)^2)/(a^2+1)^2-1/2*b^2/(a^2+1)^2*\arctan(b*x+a)-1/2*b/(a^2+1)/x-b^2/(a^2+1)^2*a*\ln(b*x)$

---

**Maxima [A]** time = 1.52569, size = 151, normalized size = 1.57

$$\frac{1}{2} \left( \frac{(a^2-1)b \arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1} + \frac{ab \log(b^2x^2+2abx+a^2+1)}{a^4+2a^2+1} - \frac{2ab \log(x)}{a^4+2a^2+1} - \frac{1}{(a^2+1)x} \right) b - \frac{\arctan(bx+a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^3,x, algorithm="maxima")

[Out]  $1/2*((a^2-1)*b*\arctan((b^2*x+a*b)/b)/(a^4+2*a^2+1)+a*b*\log(b^2*x^2+2*a*b*x+a^2+1)/(a^4+2*a^2+1)-2*a*b*\log(x)/(a^4+2*a^2+1)-1/((a^2+1)*x))*b-1/2*\arctan(b*x+a)/x^2$

---

**Fricas [A]** time = 1.89237, size = 225, normalized size = 2.34

$$\frac{ab^2x^2 \log(b^2x^2+2abx+a^2+1) - 2ab^2x^2 \log(x) - (a^2+1)bx + ((a^2-1)b^2x^2 - a^4 - 2a^2 - 1) \arctan(bx+a)}{2(a^4+2a^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^3,x, algorithm="fricas")

[Out]  $1/2*(a*b^2*x^2*\log(b^2*x^2+2*a*b*x+a^2+1)-2*a*b^2*x^2*\log(x)-(a^2+1)*b*x+((a^2-1)*b^2*x^2-a^4-2*a^2-1)*\arctan(b*x+a))/((a^4+2*a^2+1)*x^2)$

---

**Sympy [B]** time = 16.5722, size = 644, normalized size = 6.71

$$\left\{ \begin{array}{l} -\frac{2b^3x^3 \operatorname{atan}(bx-i)}{16bx^3-32ix^2} + \frac{ib^3x^3}{16bx^3-32ix^2} + \frac{4ib^2x^2 \operatorname{atan}(bx-i)}{16bx^3-32ix^2} - \frac{8bx \operatorname{atan}(bx-i)}{16bx^3-32ix^2} + \frac{2ibx}{16bx^3-32ix^2} + \frac{16i \operatorname{atan}(bx-i)}{16bx^3-32ix^2} - \frac{4}{16bx^3-32ix^2} \\ -\frac{2b^3x^3 \operatorname{atan}(bx+i)}{16bx^3+32ix^2} - \frac{ib^3x^3}{16bx^3+32ix^2} - \frac{4ib^2x^2 \operatorname{atan}(bx+i)}{16bx^3+32ix^2} - \frac{8bx \operatorname{atan}(bx+i)}{16bx^3+32ix^2} - \frac{2ibx}{16bx^3+32ix^2} - \frac{16i \operatorname{atan}(bx+i)}{16bx^3+32ix^2} - \frac{4}{16bx^3+32ix^2} \\ -\frac{a^4 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{2ab^2x^2 \log(x)}{2a^4x^2+4a^2x^2+2x^2} + \frac{ab^2x^2 \log(a^2+2abx+b^2x^2+1)}{2a^4x^2+4a^2x^2+2x^2} - \frac{b^2x^2}{2a^4x^2+4a^2x^2+2x^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/x\*\*3,x)

[Out] Piecewise((-2\*b\*\*3\*x\*\*3\*atan(b\*x - I)/(16\*b\*x\*\*3 - 32\*I\*x\*\*2) + I\*b\*\*3\*x\*\*3/(16\*b\*x\*\*3 - 32\*I\*x\*\*2) + 4\*I\*b\*\*2\*x\*\*2\*atan(b\*x - I)/(16\*b\*x\*\*3 - 32\*I\*x\*\*2) - 8\*b\*x\*atan(b\*x - I)/(16\*b\*x\*\*3 - 32\*I\*x\*\*2) + 2\*I\*b\*x/(16\*b\*x\*\*3 - 32\*I\*x\*\*2) + 16\*I\*atan(b\*x - I)/(16\*b\*x\*\*3 - 32\*I\*x\*\*2) - 4/(16\*b\*x\*\*3 - 32\*I\*x\*\*2), Eq(a, -I)), (-2\*b\*\*3\*x\*\*3\*atan(b\*x + I)/(16\*b\*x\*\*3 + 32\*I\*x\*\*2) - I\*b\*\*3\*x\*\*3/(16\*b\*x\*\*3 + 32\*I\*x\*\*2) - 4\*I\*b\*\*2\*x\*\*2\*atan(b\*x + I)/(16\*b\*x\*\*3 + 32\*I\*x\*\*2) - 8\*b\*x\*atan(b\*x + I)/(16\*b\*x\*\*3 + 32\*I\*x\*\*2) - 2\*I\*b\*x/(16\*b\*x\*\*3 + 32\*I\*x\*\*2) - 16\*I\*atan(b\*x + I)/(16\*b\*x\*\*3 + 32\*I\*x\*\*2) - 4/(16\*b\*x\*\*3 + 32\*I\*x\*\*2), Eq(a, I)), (-a\*\*4\*atan(a + b\*x)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) + a\*\*2\*b\*\*2\*x\*\*2\*atan(a + b\*x)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) - a\*\*2\*b\*x/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) - 2\*a\*\*2\*atan(a + b\*x)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) - 2\*a\*b\*\*2\*x\*\*2\*log(x)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) + a\*b\*\*2\*x\*\*2\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) - b\*\*2\*x\*\*2\*atan(a + b\*x)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) - b\*x/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2) - atan(a + b\*x)/(2\*a\*\*4\*x\*\*2 + 4\*a\*\*2\*x\*\*2 + 2\*x\*\*2), True))

**Giac [A]** time = 1.08494, size = 155, normalized size = 1.61

$$\frac{1}{2} \left( \frac{ab \log(b^2x^2 + 2abx + a^2 + 1)}{a^4 + 2a^2 + 1} - \frac{2ab \log(|x|)}{a^4 + 2a^2 + 1} + \frac{(a^2b^2 - b^2) \arctan(bx + a)}{(a^4 + 2a^2 + 1)b} - \frac{1}{(a^2 + 1)x} \right) b - \frac{\arctan(bx + a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^3,x, algorithm="giac")

[Out] 1/2\*(a\*b\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1)/(a^4 + 2\*a^2 + 1) - 2\*a\*b\*log(abs(x))/(a^4 + 2\*a^2 + 1) + (a^2\*b^2 - b^2)\*arctan(b\*x + a)/((a^4 + 2\*a^2 + 1)\*b) - 1/((a^2 + 1)\*x))\*b - 1/2\*arctan(b\*x + a)/x^2

### 3.51 $\int \frac{\tan^{-1}(a+bx)}{x^4} dx$

**Optimal.** Leaf size=129

$$\frac{2ab^2}{3(a^2+1)^2 x} - \frac{(1-3a^2)b^3 \log(x)}{3(a^2+1)^3} + \frac{(1-3a^2)b^3 \log((a+bx)^2+1)}{6(a^2+1)^3} + \frac{a(3-a^2)b^3 \tan^{-1}(a+bx)}{3(a^2+1)^3} - \frac{b}{6(a^2+1)x^2}$$

[Out] -b/(6\*(1 + a^2)\*x^2) + (2\*a\*b^2)/(3\*(1 + a^2)^2\*x) + (a\*(3 - a^2)\*b^3\*ArcTan[a + b\*x])/(3\*(1 + a^2)^3) - ArcTan[a + b\*x]/(3\*x^3) - ((1 - 3\*a^2)\*b^3\*Log[x])/(3\*(1 + a^2)^3) + ((1 - 3\*a^2)\*b^3\*Log[1 + (a + b\*x)^2])/(6\*(1 + a^2)^3)

**Rubi [A]** time = 0.114371, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5045, 371, 710, 801, 635, 203, 260}

$$\frac{2ab^2}{3(a^2+1)^2 x} - \frac{(1-3a^2)b^3 \log(x)}{3(a^2+1)^3} + \frac{(1-3a^2)b^3 \log((a+bx)^2+1)}{6(a^2+1)^3} + \frac{a(3-a^2)b^3 \tan^{-1}(a+bx)}{3(a^2+1)^3} - \frac{b}{6(a^2+1)x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/x^4, x]

[Out] -b/(6\*(1 + a^2)\*x^2) + (2\*a\*b^2)/(3\*(1 + a^2)^2\*x) + (a\*(3 - a^2)\*b^3\*ArcTan[a + b\*x])/(3\*(1 + a^2)^3) - ArcTan[a + b\*x]/(3\*x^3) - ((1 - 3\*a^2)\*b^3\*Log[x])/(3\*(1 + a^2)^3) + ((1 - 3\*a^2)\*b^3\*Log[1 + (a + b\*x)^2])/(6\*(1 + a^2)^3)

#### Rule 5045

Int[((a\_) + ArcTan[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcTan[c + d\*x])^(p - 1))/(1 + (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

#### Rule 371

Int[((a\_) + (b\_)\*(v\_)^(n\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m\*(a + b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

#### Rule 710

Int[((d\_) + (e\_)\*(x\_))^(m\_)/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(d - e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1]

#### Rule 801

Int((((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_)))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x],

`x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]`

### Rule 635

`Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]`

### Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

### Rule 260

`Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{-1}(a+bx)}{x^4} dx &= -\frac{\tan^{-1}(a+bx)}{3x^3} + \frac{1}{3}b \int \frac{1}{x^3(1+(a+bx)^2)} dx \\
 &= -\frac{\tan^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3 \text{Subst} \left( \int \frac{1}{(-a+x)^3(1+x^2)} dx, x, a+bx \right) \\
 &= -\frac{b}{6(1+a^2)x^2} - \frac{\tan^{-1}(a+bx)}{3x^3} + \frac{b^3 \text{Subst} \left( \int \frac{-a-x}{(-a+x)^2(1+x^2)} dx, x, a+bx \right)}{3(1+a^2)} \\
 &= -\frac{b}{6(1+a^2)x^2} - \frac{\tan^{-1}(a+bx)}{3x^3} + \frac{b^3 \text{Subst} \left( \int \left( -\frac{2a}{(1+a^2)(a-x)^2} + \frac{1-3a^2}{(1+a^2)^2(a-x)} + \frac{a(3-a^2)+(1-3a^2)x}{(1+a^2)^2(1+x^2)} \right) dx, x, a+bx \right)}{3(1+a^2)} \\
 &= -\frac{b}{6(1+a^2)x^2} + \frac{2ab^2}{3(1+a^2)^2 x} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{(1-3a^2)b^3 \log(x)}{3(1+a^2)^3} + \frac{b^3 \text{Subst} \left( \int \frac{a(3-a^2)+(1-3a^2)x}{1+x^2} dx, x, a+bx \right)}{3(1+a^2)^3} \\
 &= -\frac{b}{6(1+a^2)x^2} + \frac{2ab^2}{3(1+a^2)^2 x} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{(1-3a^2)b^3 \log(x)}{3(1+a^2)^3} + \frac{((1-3a^2)b^3) \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, a+bx \right)}{3(1+a^2)} \\
 &= -\frac{b}{6(1+a^2)x^2} + \frac{2ab^2}{3(1+a^2)^2 x} + \frac{a(3-a^2)b^3 \tan^{-1}(a+bx)}{3(1+a^2)^3} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{(1-3a^2)b^3 \log(x)}{3(1+a^2)^3}
 \end{aligned}$$

**Mathematica [C]** time = 0.13592, size = 128, normalized size = 0.99

$$\frac{2(3a^2 - 1)b^3 x^3 \log(x) - (a - i)bx((a + i)(a^2 - 4abx + 1) + i(a - i)^2 b^2 x^2 \log(a + bx + i)) - 2(a^2 + 1)^3 \tan^{-1}(a + bx) + i}{6(a^2 + 1)^3 x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/x^4, x]

[Out] (-2\*(1 + a^2)^3\*ArcTan[a + b\*x] + 2\*(-1 + 3\*a^2)\*b^3\*x^3\*Log[x] + I\*(I + a)^3\*b^3\*x^3\*Log[I - a - b\*x] - (-I + a)\*b\*x\*((I + a)\*(1 + a^2 - 4\*a\*b\*x) + I

$*(-I + a)^2 * b^2 * x^2 * \text{Log}[I + a + b * x]) / (6 * (1 + a^2)^3 * x^3)$

**Maple [A]** time = 0.043, size = 162, normalized size = 1.3

$$\frac{\arctan(bx + a)}{3x^3} - \frac{b^3 \ln(1 + (bx + a)^2) a^2}{2(a^2 + 1)^3} + \frac{b^3 \ln(1 + (bx + a)^2)}{6(a^2 + 1)^3} - \frac{b^3 \arctan(bx + a) a^3}{3(a^2 + 1)^3} + \frac{b^3 \arctan(bx + a) a}{(a^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/x^4, x)

[Out]  $-1/3 * \arctan(b * x + a) / x^3 - 1/2 * b^3 / (a^2 + 1)^3 * \ln(1 + (b * x + a)^2) * a^2 + 1/6 * b^3 / (a^2 + 1)^3 * \ln(1 + (b * x + a)^2) - 1/3 * b^3 / (a^2 + 1)^3 * \arctan(b * x + a) * a^3 + b^3 / (a^2 + 1)^3 * \arctan(b * x + a) * a - 1/6 * b / (a^2 + 1) / x^2 + b^3 / (a^2 + 1)^3 * \ln(b * x) * a^2 - 1/3 * b^3 / (a^2 + 1)^3 * \ln(b * x) + 2/3 * a * b^2 / (a^2 + 1)^2 / x$

**Maxima [A]** time = 1.54328, size = 223, normalized size = 1.73

$$\frac{1}{6} \left( \frac{2(a^3 - 3a)b^2 \arctan\left(\frac{b^2x + ab}{b}\right)}{a^6 + 3a^4 + 3a^2 + 1} + \frac{(3a^2 - 1)b^2 \log(b^2x^2 + 2abx + a^2 + 1)}{a^6 + 3a^4 + 3a^2 + 1} - \frac{2(3a^2 - 1)b^2 \log(x)}{a^6 + 3a^4 + 3a^2 + 1} - \frac{4abx - a^2 - 1}{(a^4 + 2a^2 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^4, x, algorithm="maxima")

[Out]  $-1/6 * (2 * (a^3 - 3 * a) * b^2 * \arctan((b^2 * x + a * b) / b) / (a^6 + 3 * a^4 + 3 * a^2 + 1) + (3 * a^2 - 1) * b^2 * \log(b^2 * x^2 + 2 * a * b * x + a^2 + 1) / (a^6 + 3 * a^4 + 3 * a^2 + 1) - 2 * (3 * a^2 - 1) * b^2 * \log(x) / (a^6 + 3 * a^4 + 3 * a^2 + 1) - (4 * a * b * x - a^2 - 1) / ((a^4 + 2 * a^2 + 1) * x^2)) * b - 1/3 * \arctan(b * x + a) / x^3$

**Fricas [A]** time = 1.65264, size = 321, normalized size = 2.49

$$\frac{(3a^2 - 1)b^3x^3 \log(b^2x^2 + 2abx + a^2 + 1) - 2(3a^2 - 1)b^3x^3 \log(x) - 4(a^3 + a)b^2x^2 + (a^4 + 2a^2 + 1)bx + 2((a^3 - 3a)^2 - 1)}{6(a^6 + 3a^4 + 3a^2 + 1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^4, x, algorithm="fricas")

[Out]  $-1/6 * ((3 * a^2 - 1) * b^3 * x^3 * \log(b^2 * x^2 + 2 * a * b * x + a^2 + 1) - 2 * (3 * a^2 - 1) * b^3 * x^3 * \log(x) - 4 * (a^3 + a) * b^2 * x^2 + (a^4 + 2 * a^2 + 1) * b * x + 2 * ((a^3 - 3 * a)^2 - 1) * b^2 * x^2 + (a^4 + 2 * a^2 + 1) * b * x + 2 * ((a^3 - 3 * a)^2 - 1)) / (6 * (a^6 + 3 * a^4 + 3 * a^2 + 1) * x^3)$

**Sympy [B]** time = 26.9557, size = 1127, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/x\*\*4,x)

[Out] Piecewise(((3\*I\*b\*\*4\*x\*\*4\*atan(b\*x - I)/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) + 6\*b\*\*3\*x\*\*3\*atan(b\*x - I)/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) + 3\*I\*b\*\*3\*x\*\*3/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) + 3\*b\*\*2\*x\*\*2/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) - 24\*b\*x\*atan(b\*x - I)/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) + 2\*I\*b\*x/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) + 48\*I\*atan(b\*x - I)/(72\*b\*x\*\*4 - 144\*I\*x\*\*3) - 8/(72\*b\*x\*\*4 - 144\*I\*x\*\*3), Eq(a, -I)), (-3\*I\*b\*\*4\*x\*\*4\*atan(b\*x + I)/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) + 6\*b\*\*3\*x\*\*3\*atan(b\*x + I)/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) - 3\*I\*b\*\*3\*x\*\*3/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) + 3\*b\*\*2\*x\*\*2/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) - 24\*b\*x\*atan(b\*x + I)/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) - 2\*I\*b\*x/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) - 48\*I\*atan(b\*x + I)/(72\*b\*x\*\*4 + 144\*I\*x\*\*3) - 8/(72\*b\*x\*\*4 + 144\*I\*x\*\*3), Eq(a, I)), (-2\*a\*\*6\*atan(a + b\*x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - a\*\*4\*b\*x/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 6\*a\*\*4\*atan(a + b\*x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 2\*a\*\*3\*b\*\*3\*x\*\*3\*atan(a + b\*x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) + 4\*a\*\*3\*b\*\*2\*x\*\*2/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) + 6\*a\*\*2\*b\*\*3\*x\*\*3\*log(x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 3\*a\*\*2\*b\*\*3\*x\*\*3\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 2\*a\*\*2\*b\*\*3\*x\*\*3/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 2\*a\*\*2\*b\*x/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 6\*a\*\*2\*atan(a + b\*x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) + 6\*a\*b\*\*3\*x\*\*3\*atan(a + b\*x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) + 4\*a\*b\*\*2\*x\*\*2/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 2\*b\*\*3\*x\*\*3\*log(x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) + b\*\*3\*x\*\*3\*log(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 2\*b\*\*3\*x\*\*3/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - b\*x/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3) - 2\*atan(a + b\*x)/(6\*a\*\*6\*x\*\*3 + 18\*a\*\*4\*x\*\*3 + 18\*a\*\*2\*x\*\*3 + 6\*x\*\*3), True))

**Giac [A]** time = 1.1008, size = 239, normalized size = 1.85

$$-\frac{1}{6}b \left( \frac{(3a^2b^2 - b^2) \log(b^2x^2 + 2abx + a^2 + 1)}{a^6 + 3a^4 + 3a^2 + 1} - \frac{2(3a^2b^2 - b^2) \log(|x|)}{a^6 + 3a^4 + 3a^2 + 1} + \frac{2(a^3b^3 - 3ab^3) \arctan(bx + a)}{(a^6 + 3a^4 + 3a^2 + 1)b} + \frac{a^4 + 2a^2 - b^2}{(a^6 + 3a^4 + 3a^2 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/x^4,x, algorithm="giac")

[Out] -1/6\*b\*((3\*a^2\*b^2 - b^2)\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1)/(a^6 + 3\*a^4 + 3\*a^2 + 1) - 2\*(3\*a^2\*b^2 - b^2)\*log(abs(x))/(a^6 + 3\*a^4 + 3\*a^2 + 1) + 2\*(a^3\*b^3 - 3\*a\*b^3)\*arctan(b\*x + a)/((a^6 + 3\*a^4 + 3\*a^2 + 1)\*b) + (a^4 + 2\*a^2 - 4\*(a^3\*b + a\*b)\*x + 1)/((a^2 + 1)^3\*x^2) - 1/3\*arctan(b\*x + a)/x^3



$$3.52 \quad \int \frac{\tan^{-1}(a+bx)}{c+dx^3} dx$$

**Optimal.** Leaf size=863

$$\frac{i \log(ia + ibx + 1) \log\left(\frac{b(\sqrt[3]{dx} + \sqrt[3]{c})}{\sqrt[3]{d(i-a)+b\sqrt[3]{c}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i \log(-ia - ibx + 1) \log\left(\frac{b(\sqrt[3]{dx} + \sqrt[3]{c})}{b\sqrt[3]{c-(a+i)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1} \log(ia + ibx + 1) \log\left(\frac{b(\sqrt[3]{c} - \sqrt[3]{-1})}{b\sqrt[3]{c} - \sqrt[3]{-1}(i-a)}\right)}{6c^{2/3}\sqrt[3]{d}}$$

[Out]  $((-I/6)*\text{Log}[1 + I*a + I*b*x]*\text{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} + (I - a)*d^{1/3})])/(c^{2/3}*d^{1/3}) + ((I/6)*\text{Log}[1 - I*a - I*b*x]*\text{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} - (I + a)*d^{1/3})])/(c^{2/3}*d^{1/3}) + ((-1)^{1/6}*\text{Log}[1 + I*a + I*b*x]*\text{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} - (-1)^{1/3}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((-1)^{1/6}*\text{Log}[1 - I*a - I*b*x]*\text{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/3}*(I + a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) + ((-1)^{5/6}*\text{Log}[1 + I*a + I*b*x]*\text{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{2/3}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((-1)^{5/6}*\text{Log}[1 - I*a - I*b*x]*\text{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/6}*(1 - I*a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((I/6)*\text{PolyLog}[2, (d^{1/3}*(I - a - b*x))/(b*c^{1/3} + (I - a)*d^{1/3})])/(c^{2/3}*d^{1/3}) + ((-1)^{5/6}*\text{PolyLog}[2, -((( -1)^{1/6}*d^{1/3}*(I - a - b*x))/(I*b*c^{1/3} - (-1)^{1/6}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) + ((-1)^{1/6}*\text{PolyLog}[2, -((( -1)^{1/3}*d^{1/3}*(I - a - b*x))/(b*c^{1/3} - (-1)^{1/3}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) + ((I/6)*\text{PolyLog}[2, -(d^{1/3}*(I + a + b*x))/(b*c^{1/3} - (I + a)*d^{1/3})])/(c^{2/3}*d^{1/3}) - ((-1)^{1/6}*\text{PolyLog}[2, ((-1)^{1/3}*d^{1/3}*(I + a + b*x))/(b*c^{1/3} + (-1)^{1/3}*(I + a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((-1)^{5/6}*\text{PolyLog}[2, -((( -1)^{2/3}*d^{1/3}*(I + a + b*x))/(b*c^{1/3} - (-1)^{2/3}*(I + a)*d^{1/3})])/(6*c^{2/3}*d^{1/3})$

**Rubi [A]** time = 1.20694, antiderivative size = 863, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5051, 2409, 2394, 2393, 2391}

$$\frac{i \log(ia + ibx + 1) \log\left(\frac{b(\sqrt[3]{dx} + \sqrt[3]{c})}{\sqrt[3]{d(i-a)+b\sqrt[3]{c}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i \log(-ia - ibx + 1) \log\left(\frac{b(\sqrt[3]{dx} + \sqrt[3]{c})}{b\sqrt[3]{c-(a+i)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1} \log(ia + ibx + 1) \log\left(\frac{b(\sqrt[3]{c} - \sqrt[3]{-1})}{b\sqrt[3]{c} - \sqrt[3]{-1}(i-a)}\right)}{6c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/(c + d\*x^3), x]

[Out]  $((-I/6)*\text{Log}[1 + I*a + I*b*x]*\text{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} + (I - a)*d^{1/3})])/(c^{2/3}*d^{1/3}) + ((I/6)*\text{Log}[1 - I*a - I*b*x]*\text{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} - (I + a)*d^{1/3})])/(c^{2/3}*d^{1/3}) + ((-1)^{1/6}*\text{Log}[1 + I*a + I*b*x]*\text{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} - (-1)^{1/3}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((-1)^{1/6}*\text{Log}[1 - I*a - I*b*x]*\text{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/3}*(I + a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) + ((-1)^{5/6}*\text{Log}[1 + I*a + I*b*x]*\text{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{2/3}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((-1)^{5/6}*\text{Log}[1 - I*a - I*b*x]*\text{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/6}*(1 - I*a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((I/6)*\text{PolyLog}[2, (d^{1/3}*(I - a - b*x))/(b*c^{1/3} + (I - a)*d^{1/3})])/(c^{2/3}*d^{1/3}) + ((-1)^{5/6}*\text{PolyLog}[2, -((( -1)^{1/6}*d^{1/3}*(I - a - b*x))/(I*b*c^{1/3} - (-1)^{1/6}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) + ((-1)^{1/6}*\text{PolyLog}[2, -((( -1)^{1/3}*d^{1/3}*(I - a - b*x))/(b*c^{1/3} - (-1)^{1/3}*(I - a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) + ((I/6)*\text{PolyLog}[2, -(d^{1/3}*(I + a + b*x))/(b*c^{1/3} - (I + a)*d^{1/3})])/(c^{2/3}*d^{1/3}) - ((-1)^{1/6}*\text{PolyLog}[2, ((-1)^{1/3}*d^{1/3}*(I + a + b*x))/(b*c^{1/3} + (-1)^{1/3}*(I + a)*d^{1/3})])/(6*c^{2/3}*d^{1/3}) - ((-1)^{5/6}*\text{PolyLog}[2, -((( -1)^{2/3}*d^{1/3}*(I + a + b*x))/(b*c^{1/3} - (-1)^{2/3}*(I + a)*d^{1/3})])/(6*c^{2/3}*d^{1/3})$

$$\frac{a - b*x)}{(b*c^{(1/3)} - (-1)^{(1/3)}*(I - a)*d^{(1/3)})}]/(6*c^{(2/3)}*d^{(1/3)}) + ((I/6)*PolyLog[2, -((d^{(1/3)}*(I + a + b*x))/(b*c^{(1/3)} - (I + a)*d^{(1/3)}))]/(c^{(2/3)}*d^{(1/3)}) - ((-1)^{(1/6)}*PolyLog[2, ((-1)^{(1/3)}*d^{(1/3)}*(I + a + b*x))/(b*c^{(1/3)} + (-1)^{(1/3)}*(I + a)*d^{(1/3)})])/(6*c^{(2/3)}*d^{(1/3)}) - ((-1)^{(5/6)}*PolyLog[2, -(((-1)^{(2/3)}*d^{(1/3)}*(I + a + b*x))/(b*c^{(1/3)} - (-1)^{(2/3)}*(I + a)*d^{(1/3)})])/(6*c^{(2/3)}*d^{(1/3)})$$

### Rule 5051

```
Int[ArcTan[(a_) + (b_)*(x_)]/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

### Rule 2409

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_)*((f_) + (g_
)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x
)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

### Rule 2394

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))/((f_) + (g_)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_))/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{c+dx^3} dx &= \frac{1}{2}i \int \frac{\log(1-ia-ibx)}{c+dx^3} dx - \frac{1}{2}i \int \frac{\log(1+ia+ibx)}{c+dx^3} dx \\
&= \frac{1}{2}i \int \left( \frac{\log(1-ia-ibx)}{3c^{2/3}(-\sqrt[3]{c}-\sqrt[3]{dx})} - \frac{\log(1-ia-ibx)}{3c^{2/3}(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{dx})} - \frac{\log(1-ia-ibx)}{3c^{2/3}(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{dx})} \right) dx - \frac{1}{2}i \int \left( \frac{\log(1+ia+ibx)}{3c^{2/3}(-\sqrt[3]{c}-\sqrt[3]{dx})} - \frac{\log(1+ia+ibx)}{3c^{2/3}(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{dx})} - \frac{\log(1+ia+ibx)}{3c^{2/3}(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{dx})} \right) dx \\
&= -\frac{i \int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-\sqrt[3]{dx}} dx}{6c^{2/3}} - \frac{i \int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{dx}} dx}{6c^{2/3}} - \frac{i \int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{dx}} dx}{6c^{2/3}} + \frac{i \int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}-\sqrt[3]{dx}} dx}{6c^{2/3}} + \frac{i \int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{dx}} dx}{6c^{2/3}} + \frac{i \int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{dx}} dx}{6c^{2/3}} \\
&= -\frac{i \log(1+ia+ibx) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c+(i-a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i \log(1-ia-ibx) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c-(i+a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1} \log(1+ia) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c+(i-a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&= -\frac{i \log(1+ia+ibx) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c+(i-a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i \log(1-ia-ibx) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c-(i+a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1} \log(1+ia) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c-(i+a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&= -\frac{i \log(1+ia+ibx) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c+(i-a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i \log(1-ia-ibx) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c-(i+a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1} \log(1+ia) \log\left(\frac{b(\sqrt[3]{c}+\sqrt[3]{dx})}{b\sqrt[3]{c-(i+a)\sqrt[3]{d}}}\right)}{6c^{2/3}\sqrt[3]{d}}
\end{aligned}$$

**Mathematica [A]** time = 0.804645, size = 701, normalized size = 0.81

$$-i \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{d}(a+bx-i)}{-b\sqrt[3]{c+(a-i)\sqrt[3]{d}}}\right) + (-1)^{5/6} \operatorname{PolyLog}\left(2, \frac{\sqrt[6]{-1}\sqrt[3]{d}(a+bx-i)}{\sqrt[6]{-1}(a-i)\sqrt[3]{d}+ib\sqrt[3]{c}}\right) + \sqrt[6]{-1} \operatorname{PolyLog}\left(2, \frac{\sqrt[6]{-1}\sqrt[3]{d}(a+bx-i)}{b\sqrt[3]{c}+\sqrt[6]{-1}(a-i)\sqrt[3]{d}}\right) + i \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{d}(a+bx-i)}{-b\sqrt[3]{c+(a-i)\sqrt[3]{d}}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/(c + d\*x^3), x]

[Out]  $((-1)*\operatorname{Log}[1 + I*a + I*b*x]*\operatorname{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} - (-1 + a)*d^{1/3})] + I*\operatorname{Log}[(-1)*(I + a + b*x)]*\operatorname{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} - (I + a)*d^{1/3})] + (-1)^{1/6}*\operatorname{Log}[1 + I*a + I*b*x]*\operatorname{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/3}*(-I + a)*d^{1/3})] - (-1)^{1/6}*\operatorname{Log}[(-1)*(I + a + b*x)]*\operatorname{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/3}*(I + a)*d^{1/3})] - (-1)^{5/6}*\operatorname{Log}[(-1)*(I + a + b*x)]*\operatorname{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/6}*(1 - I*a)*d^{1/3})] + (-1)^{5/6}*\operatorname{Log}[1 + I*a + I*b*x]*\operatorname{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} - (-1)^{2/3}*(-I + a)*d^{1/3})] - I*\operatorname{PolyLog}[2, (d^{1/3}*(-I + a + b*x))/(-b*c^{1/3}) + (-I + a)*d^{1/3})] + (-1)^{5/6}*\operatorname{PolyLog}[2, ((-1)^{1/6}*d^{1/3}*(-I + a + b*x))/(I*b*c^{1/3} + (-1)^{1/6}*(-I + a)*d^{1/3})] + (-1)^{1/6}*\operatorname{PolyLog}[2, ((-1)^{1/3}*d^{1/3}*(-I + a + b*x))/(b*c^{1/3} + (-1)^{1/3}*(-I + a)*d^{1/3})] + I*\operatorname{PolyLog}[2, (d^{1/3}*(I + a + b*x))/(-b*c^{1/3}) + (I + a)*d^{1/3})] - (-1)^{1/6}*\operatorname{PolyLog}[2, ((-1)^{1/3}*d^{1/3}*(I + a + b*x))/(b*c^{1/3} + (-1)^{1/3}*(I + a)*d^{1/3})] - (-1)^{5/6}*\operatorname{PolyLog}[2, ((-1)^{2/3}*d^{1/3}*(I + a + b*x))/(-b*c^{1/3}) + (-1)^{2/3}*(I + a)*d^{1/3})]/(6*c^{2/3}*d^{1/3}))$

**Maple [C]** time = 0.636, size = 631, normalized size = 0.7

$$\frac{2b^2}{3} \sum_{R1=\operatorname{RootOf}\left(\left(3ia^2d+a^3d-cb^3-id-3ad\right)_Z^6+\left(3ia^2d+3a^3d-3cb^3+3id+3ad\right)_Z^4+\left(-3ia^2d+3a^3d-3cb^3-3id+3ad\right)_Z^2-3ia^2d+a^3d-cb^3+id-3ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(b*x+a)/(d*x^3+c),x)`

[Out] 
$$\frac{2}{3}b^2 \sum \left( \frac{1}{(a^3 d^2 \sqrt{R_1^4 + 3I a^2 d \sqrt{R_1^4 - b^3 c} \sqrt{R_1^4 - 3a d \sqrt{R_1^4 - I d \sqrt{R_1^4 + 2a^3 d \sqrt{R_1^2 + 2I a^2 d \sqrt{R_1^2 - 2b^3 c} \sqrt{R_1^2 + 2I d \sqrt{R_1^2 + a^3 d - I a^2 d - c b^3 + a d - I d}}}}}} \right) (I \arctan(bx+a) \ln\left(\frac{\sqrt{R_1 - (1 + I(bx+a))}}{(1 + (bx+a)^2)^{1/2}}\right) / \sqrt{R_1} + \operatorname{dilog}\left(\frac{\sqrt{R_1 - (1 + I(bx+a))}}{(1 + (bx+a)^2)^{1/2}} / \sqrt{R_1}\right), \sqrt{R_1} = \operatorname{RootOf}\left((3I a^2 d + a^3 d - c b^3 - I d - 3a d) \sqrt{Z^6} + (3I a^2 d + 3a^3 d - 3c b^3 + 3I d + 3a d) \sqrt{Z^4} + (-3I a^2 d + 3a^3 d - 3c b^3 - 3I d + 3a d) \sqrt{Z^2} - 3I a^2 d + a^3 d - c b^3 + I d - 3a d\right) \right) + \frac{2}{3}b^2 \sum \left( \frac{\sqrt{R_1^2}}{(a^3 d^2 \sqrt{R_1^4 + 3I a^2 d \sqrt{R_1^4 - b^3 c} \sqrt{R_1^4 - 3a d \sqrt{R_1^4 - I d \sqrt{R_1^4 + 2a^3 d \sqrt{R_1^2 + 2I a^2 d \sqrt{R_1^2 - 2b^3 c} \sqrt{R_1^2 + 2I d \sqrt{R_1^2 + a^3 d - I a^2 d - c b^3 + a d - I d}}}}}} \right) (I \arctan(bx+a) \ln\left(\frac{\sqrt{R_1 - (1 + I(bx+a))}}{(1 + (bx+a)^2)^{1/2}}\right) / \sqrt{R_1} + \operatorname{dilog}\left(\frac{\sqrt{R_1 - (1 + I(bx+a))}}{(1 + (bx+a)^2)^{1/2}} / \sqrt{R_1}\right), \sqrt{R_1} = \operatorname{RootOf}\left((3I a^2 d + a^3 d - c b^3 - I d - 3a d) \sqrt{Z^6} + (3I a^2 d + 3a^3 d - 3c b^3 + 3I d + 3a d) \sqrt{Z^4} + (-3I a^2 d + 3a^3 d - 3c b^3 - 3I d + 3a d) \sqrt{Z^2} - 3I a^2 d + a^3 d - c b^3 + I d - 3a d\right) \right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\arctan(bx+a)}{dx^3+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="fricas")`

[Out] `integral(arctan(b*x + a)/(d*x^3 + c), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(b*x+a)/(d*x**3+c),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(d*x^3 + c), x)
```

### 3.53 $\int \frac{\tan^{-1}(a+bx)}{c+dx^2} dx$

**Optimal.** Leaf size=543

$$-\frac{i\text{PolyLog}\left(2, -\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{i\text{PolyLog}\left(2, -\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

```
[Out] ((-I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
```

**Rubi [A]** time = 0.607336, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\text{PolyLog}\left(2, -\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{i\text{PolyLog}\left(2, -\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^2), x]
```

```
[Out] ((-I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d])
```

#### Rule 5051

```
Int[ArcTan[(a_) + (b_)*(x_)]/((c_) + (d_)*(x_)^(n_.)), x_Symbol] := Dist[I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

#### Rule 2409

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

$\wedge n]^p, (f + g*x^r)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, r\}, x] \&\& \text{IntegerQ}[p, 0] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \mid\mid (\text{IntegerQ}[r] \&\& \text{NeQ}[r, 1]))$

#### Rule 2394

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{n_.})*(b_.)]/((f_.) + (g_.)*(x_.))], x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])/g, x] - \text{Dist}[(b*e^n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0]$

#### Rule 2393

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))]*(b_.)]/((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c*(e*f - d*g), 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{n_.})]/(x_.), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(a + bx)}{c + dx^2} dx &= \frac{1}{2}i \int \frac{\log(1 - ia - ibx)}{c + dx^2} dx - \frac{1}{2}i \int \frac{\log(1 + ia + ibx)}{c + dx^2} dx \\ &= \frac{1}{2}i \int \left( \frac{\sqrt{-c} \log(1 - ia - ibx)}{2c(\sqrt{-c} - \sqrt{dx})} + \frac{\sqrt{-c} \log(1 - ia - ibx)}{2c(\sqrt{-c} + \sqrt{dx})} \right) dx - \frac{1}{2}i \int \left( \frac{\sqrt{-c} \log(1 + ia + ibx)}{2c(\sqrt{-c} - \sqrt{dx})} + \frac{\sqrt{-c} \log(1 + ia + ibx)}{2c(\sqrt{-c} + \sqrt{dx})} \right) dx \\ &= -\frac{i \int \frac{\log(1 - ia - ibx)}{\sqrt{-c} - \sqrt{dx}} dx}{4\sqrt{-c}} - \frac{i \int \frac{\log(1 - ia - ibx)}{\sqrt{-c} + \sqrt{dx}} dx}{4\sqrt{-c}} + \frac{i \int \frac{\log(1 + ia + ibx)}{\sqrt{-c} - \sqrt{dx}} dx}{4\sqrt{-c}} + \frac{i \int \frac{\log(1 + ia + ibx)}{\sqrt{-c} + \sqrt{dx}} dx}{4\sqrt{-c}} \\ &= -\frac{i \log(1 + ia + ibx) \log\left(\frac{b(\sqrt{-c} - \sqrt{dx})}{b\sqrt{-c} - (i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i \log(1 - ia - ibx) \log\left(\frac{b(\sqrt{-c} - \sqrt{dx})}{b\sqrt{-c} + (i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i \log(1 + ia + ibx) \log\left(\frac{b(\sqrt{-c} + \sqrt{dx})}{b\sqrt{-c} - (i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\ &= -\frac{i \log(1 + ia + ibx) \log\left(\frac{b(\sqrt{-c} - \sqrt{dx})}{b\sqrt{-c} - (i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i \log(1 - ia - ibx) \log\left(\frac{b(\sqrt{-c} - \sqrt{dx})}{b\sqrt{-c} + (i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i \log(1 + ia + ibx) \log\left(\frac{b(\sqrt{-c} + \sqrt{dx})}{b\sqrt{-c} - (i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\ &= -\frac{i \log(1 + ia + ibx) \log\left(\frac{b(\sqrt{-c} - \sqrt{dx})}{b\sqrt{-c} - (i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i \log(1 - ia - ibx) \log\left(\frac{b(\sqrt{-c} - \sqrt{dx})}{b\sqrt{-c} + (i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i \log(1 + ia + ibx) \log\left(\frac{b(\sqrt{-c} + \sqrt{dx})}{b\sqrt{-c} - (i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \end{aligned}$$

**Mathematica [A]** time = 0.329175, size = 409, normalized size = 0.75

$$i \left( -\text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx-i)}{-b\sqrt{-c+(a-i)\sqrt{d}}}\right) + \text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx-i)}{b\sqrt{-c+(a-i)\sqrt{d}}}\right) + \text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+i)}{-b\sqrt{-c+(a+i)\sqrt{d}}}\right) - \text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c+(a+i)\sqrt{d}}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/(c + d\*x^2), x]

[Out] ((-I/4)\*(Log[1 + I\*a + I\*b\*x]\*Log[(b\*(Sqrt[-c] - Sqrt[d]\*x))/(b\*Sqrt[-c] + (-I + a)\*Sqrt[d]]) - Log[(-I)\*(I + a + b\*x)]\*Log[(b\*(Sqrt[-c] - Sqrt[d]\*x))

$$\begin{aligned} & / (b\sqrt{-c} + (I + a)\sqrt{d}) - \text{Log}[1 + I*a + I*b*x] * \text{Log}[(b(\sqrt{-c} + \sqrt{d}*x)) / (b\sqrt{-c} - (-I + a)\sqrt{d})] + \text{Log}[(-I)(I + a + b*x)] * \text{Log}[(b(\sqrt{-c} + \sqrt{d}*x)) / (b\sqrt{-c} - (I + a)\sqrt{d})] - \text{PolyLog}[2, (\sqrt{d}*(-I + a + b*x)) / (-b\sqrt{-c}) + (-I + a)\sqrt{d}] + \text{PolyLog}[2, (\sqrt{d}*(-I + a + b*x)) / (b\sqrt{-c} + (-I + a)\sqrt{d})] + \text{PolyLog}[2, (\sqrt{d}*(I + a + b*x)) / (-b\sqrt{-c}) + (I + a)\sqrt{d}] - \text{PolyLog}[2, (\sqrt{d}*(I + a + b*x)) / (b\sqrt{-c} + (I + a)\sqrt{d})] / (\sqrt{-c}*\sqrt{d}) \end{aligned}$$

**Maple [B]** time = 0.713, size = 2192, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(d\*x^2+c), x)

[Out]  $\frac{1}{2}I/b(b^2cd)^{1/2}/c/(a^2d+cb^2+2(b^2cd)^{1/2}d)\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))\arctan(bx+a)-1/2I/b(b^2cd)^{1/2}/c/(a^2d+cb^2-2(b^2cd)^{1/2}d)\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))\arctan(bx+a)+Ib/(a^2d+cb^2-2(b^2cd)^{1/2}d)\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))\arctan(bx+a)+Ib/(a^2d+cb^2+2(b^2cd)^{1/2}d)\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))\arctan(bx+a)-1/2b/d(b^2cd)^{1/2}/(a^2d+cb^2-2(b^2cd)^{1/2}d)\arctan(bx+a)^2+b/(a^2d+cb^2-2(b^2cd)^{1/2}d)\arctan(bx+a)^2-1/2/b(b^2cd)^{1/2}/c/(a^2d+cb^2-2(b^2cd)^{1/2}d)\arctan(bx+a)^2-1/2/b(b^2cd)^{1/2}/c/(a^2d+cb^2-2(b^2cd)^{1/2}d)\arctan(bx+a)^2a^2-1/4b/d(b^2cd)^{1/2}/(a^2d+cb^2-2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))+1/2b/(a^2d+cb^2-2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))-1/4/b(b^2cd)^{1/2}/c/(a^2d+cb^2-2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))-1/4/b(b^2cd)^{1/2}/c/(a^2d+cb^2-2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))\arctan(bx+a)a^2+1/2Ib/d(b^2cd)^{1/2}/(a^2d+cb^2+2(b^2cd)^{1/2}d)\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))\arctan(bx+a)-1/2I/b(b^2cd)^{1/2}/c/(a^2d+cb^2-2(b^2cd)^{1/2}d)\ln(1-(2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2+2(b^2cd)^{1/2}d))\arctan(bx+a)+1/2b/d(b^2cd)^{1/2}/(a^2d+cb^2+2(b^2cd)^{1/2}d)\arctan(bx+a)^2+b/(a^2d+cb^2+2(b^2cd)^{1/2}d)\arctan(bx+a)^2+1/2/b(b^2cd)^{1/2}/c/(a^2d+cb^2+2(b^2cd)^{1/2}d)\arctan(bx+a)^2+1/2/b(b^2cd)^{1/2}/c/(a^2d+cb^2+2(b^2cd)^{1/2}d)\arctan(bx+a)^2a^2+1/4b/d(b^2cd)^{1/2}/(a^2d+cb^2+2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))+1/2b/(a^2d+cb^2+2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))+1/4/b(b^2cd)^{1/2}/c/(a^2d+cb^2+2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))+1/4/b(b^2cd)^{1/2}/c/(a^2d+cb^2+2(b^2cd)^{1/2}d)*\text{polylog}(2, (2Iad+a^2d+cb^2-d)(1+I(bx+a))^2/(1+(bx+a)^2)/(-a^2d-cb^2-2(b^2cd)^{1/2}d))$



$$-c*b^2-2*(b^2*c*d)^{(1/2)-d))*a^2$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(d\*x^2+c),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx+a)}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(d\*x^2+c),x, algorithm="fricas")

[Out] integral(arctan(b\*x + a)/(d\*x^2 + c), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(d\*x\*\*2+c),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{dx^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(d\*x^2+c),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(d\*x^2 + c), x)

### 3.54 $\int \frac{\tan^{-1}(a+bx)}{c+dx} dx$

**Optimal.** Leaf size=152

$$-\frac{i\text{PolyLog}\left(2, 1 - \frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{2d} + \frac{i\text{PolyLog}\left(2, 1 - \frac{2}{1-i(a+bx)}\right)}{2d} + \frac{\tan^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d} - \frac{\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}$$

[Out] -((ArcTan[a + b\*x]\*Log[2/(1 - I\*(a + b\*x))])/d) + (ArcTan[a + b\*x]\*Log[(2\*b\*(c + d\*x))/((b\*c + I\*d - a\*d)\*(1 - I\*(a + b\*x)))]/d) + ((I/2)\*PolyLog[2, 1 - 2/(1 - I\*(a + b\*x))])/d - ((I/2)\*PolyLog[2, 1 - (2\*b\*(c + d\*x))/((b\*c + I\*d - a\*d)\*(1 - I\*(a + b\*x)))]/d)

**Rubi [A]** time = 0.138446, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{i\text{PolyLog}\left(2, 1 - \frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{2d} + \frac{i\text{PolyLog}\left(2, 1 - \frac{2}{1-i(a+bx)}\right)}{2d} + \frac{\tan^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d} - \frac{\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/(c + d\*x), x]

[Out] -((ArcTan[a + b\*x]\*Log[2/(1 - I\*(a + b\*x))])/d) + (ArcTan[a + b\*x]\*Log[(2\*b\*(c + d\*x))/((b\*c + I\*d - a\*d)\*(1 - I\*(a + b\*x)))]/d) + ((I/2)\*PolyLog[2, 1 - 2/(1 - I\*(a + b\*x))])/d - ((I/2)\*PolyLog[2, 1 - (2\*b\*(c + d\*x))/((b\*c + I\*d - a\*d)\*(1 - I\*(a + b\*x)))]/d)

#### Rule 5047

Int[((a\_.) + ArcTan[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcTan[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[p, 0]

#### Rule 4856

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTan[c\*x])\*Log[2/(1 - I\*c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))]/(1 + c^2\*x^2), x], x] + Simp[((a + b\*ArcTan[c\*x])\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_.) + (e\_.)\*(x\_))]/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rubi steps

$$\int \frac{\tan^{-1}(a + bx)}{c + dx} dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{bc-ad}{b} + \frac{x}{b}} dx, x, a + bx\right)}{b}$$

$$= -\frac{\tan^{-1}(a + bx) \log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2} dx\right)}{d}$$

$$= -\frac{\tan^{-1}(a + bx) \log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{i\text{Li}_2\left(1 - \frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}$$

$$= -\frac{\tan^{-1}(a + bx) \log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{i\text{Li}_2\left(1 - \frac{2}{1-i(a+bx)}\right)}{2d}$$

**Mathematica [A]** time = 0.0195464, size = 231, normalized size = 1.52

$$\frac{i\text{PolyLog}\left(2, -\frac{id(1-i(a+bx))}{-ad+bc-id}\right)}{2d} - \frac{i\text{PolyLog}\left(2, \frac{id(1+i(a+bx))}{-ad+bc+id}\right)}{2d} + \frac{i \log(1 - i(a + bx)) \log\left(-\frac{i\left(\frac{bc-ad}{b} + \frac{d(a+bx)}{b}\right)}{-\frac{d}{b} - \frac{i(bc-ad)}{b}}\right)}{2d} - \frac{i \log(1 + i(a + bx)) \log\left(\frac{i\left(\frac{bc-ad}{b} + \frac{d(a+bx)}{b}\right)}{-\frac{d}{b} + \frac{i(bc-ad)}{b}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/(c + d\*x), x]

[Out] ((I/2)\*Log[1 - I\*(a + b\*x)]\*Log[(-I)\*((b\*c - a\*d)/b + (d\*(a + b\*x))/b)]/(- (d/b) - (I\*(b\*c - a\*d))/b))/d - ((I/2)\*Log[1 + I\*(a + b\*x)]\*Log[(I\*((b\*c - a\*d)/b + (d\*(a + b\*x))/b)]/(- (d/b) + (I\*(b\*c - a\*d))/b))/d + ((I/2)\*PolyLog[2, ((-I)\*d\*(1 - I\*(a + b\*x)))/(b\*c - I\*d - a\*d)]/d - ((I/2)\*PolyLog[2, (I\*d\*(1 + I\*(a + b\*x)))/(b\*c + I\*d - a\*d)]/d

**Maple [A]** time = 0.054, size = 198, normalized size = 1.3

$$\frac{\ln(d(bx + a) - ad + bc) \arctan(bx + a)}{d} + \frac{\frac{i}{2} \ln(d(bx + a) - ad + bc)}{d} \ln\left(\frac{id - d(bx + a)}{bc + id - ad}\right) - \frac{\frac{i}{2} \ln(d(bx + a) - ad + bc)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(d\*x+c), x)

[Out] ln(d\*(b\*x+a)-a\*d+b\*c)/d\*arctan(b\*x+a)+1/2\*I\*ln(d\*(b\*x+a)-a\*d+b\*c)/d\*ln((I\*d-d\*(b\*x+a))/(b\*c+I\*d-a\*d))-1/2\*I\*ln(d\*(b\*x+a)-a\*d+b\*c)/d\*ln((I\*d+d\*(b\*x+a))/(I\*d+a\*d-b\*c))+1/2\*I/d\*dilog((I\*d-d\*(b\*x+a))/(b\*c+I\*d-a\*d))-1/2\*I/d\*dilog((I\*d+d\*(b\*x+a))/(I\*d+a\*d-b\*c))

---

**Maxima [B]** time = 1.97429, size = 383, normalized size = 2.52

$$\frac{\arctan(bx + a) \log(dx + c)}{d} - \frac{\arctan\left(\frac{b^2x+ab}{b}\right) \log(dx + c)}{d} - \frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2abcd+(a^2+1)d^2}, \frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2abcd+(a^2+1)d^2}\right) \log(b^2x^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out] arctan(b\*x + a)\*log(d\*x + c)/d - arctan((b^2\*x + a\*b)/b)\*log(d\*x + c)/d - 1/2\*(arctan2((b\*d^2\*x + b\*c\*d)/(b^2\*c^2 - 2\*a\*b\*c\*d + (a^2 + 1)\*d^2), (b^2\*c^2 - a\*b\*c\*d + (b^2\*c\*d - a\*b\*d^2)\*x)/(b^2\*c^2 - 2\*a\*b\*c\*d + (a^2 + 1)\*d^2))\*log(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1) - arctan(b\*x + a)\*log((b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)/(b^2\*c^2 - 2\*a\*b\*c\*d + (a^2 + 1)\*d^2)) + I\*dilog((I\*b\*d\*x + (I\*a + 1)\*d)/(-I\*b\*c + (I\*a + 1)\*d)) - I\*dilog((I\*b\*d\*x + (I\*a - 1)\*d)/(-I\*b\*c + (I\*a - 1)\*d)))/d

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out] integral(arctan(b\*x + a)/(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atan}(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(d\*x+c),x)

[Out] Integral(atan(a + b\*x)/(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(d\*x + c), x)

$$3.55 \quad \int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}} dx$$

**Optimal.** Leaf size=244

$$\frac{idPolyLog\left(2, \frac{c(-a-bx+i)}{-ac+bd+ic}\right)}{2c^2} - \frac{idPolyLog\left(2, \frac{c(a+bx+i)}{-bd+(a+i)c}\right)}{2c^2} + \frac{id \log(ia + ibx + 1) \log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2} - \frac{id \log(-ia - ibx + 1) \log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}$$

[Out]  $-\frac{((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])}{(2*b*c)} - \frac{((1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])}{(2*b*c)} - \frac{((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d + c*x))/((I + a)*c - b*d))])}{c^2} + \frac{((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d + c*x))/((I - a)*c + b*d)])}{c^2} + \frac{((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*c - a*c + b*d)])}{c^2} - \frac{((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d)])}{c^2}$

**Rubi [A]** time = 0.239131, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{idPolyLog\left(2, \frac{c(-a-bx+i)}{-ac+bd+ic}\right)}{2c^2} - \frac{idPolyLog\left(2, \frac{c(a+bx+i)}{-bd+(a+i)c}\right)}{2c^2} + \frac{id \log(ia + ibx + 1) \log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2} - \frac{id \log(-ia - ibx + 1) \log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/(c + d/x), x]

[Out]  $-\frac{((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])}{(2*b*c)} - \frac{((1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])}{(2*b*c)} - \frac{((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d + c*x))/((I + a)*c - b*d))])}{c^2} + \frac{((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d + c*x))/((I - a)*c + b*d)])}{c^2} + \frac{((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*c - a*c + b*d)])}{c^2} - \frac{((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d)])}{c^2}$

#### Rule 5051

Int[ArcTan[(a\_) + (b\_.)\*(x\_)]/((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Dist[I/2, Int[Log[1 - I\*a - I\*b\*x]/(c + d\*x^n), x], x] - Dist[I/2, Int[Log[1 + I\*a + I\*b\*x]/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]

#### Rule 2409

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IntegerQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
]; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}} dx &= \frac{1}{2}i \int \frac{\log(1-ia-ibx)}{c+\frac{d}{x}} dx - \frac{1}{2}i \int \frac{\log(1+ia+ibx)}{c+\frac{d}{x}} dx \\ &= \frac{1}{2}i \int \left( \frac{\log(1-ia-ibx)}{c} - \frac{d \log(1-ia-ibx)}{c(d+cx)} \right) dx - \frac{1}{2}i \int \left( \frac{\log(1+ia+ibx)}{c} - \frac{d \log(1+ia+ibx)}{c(d+cx)} \right) dx \\ &= \frac{i \int \log(1-ia-ibx) dx}{2c} - \frac{i \int \log(1+ia+ibx) dx}{2c} - \frac{(id) \int \frac{\log(1-ia-ibx)}{d+cx} dx}{2c} + \frac{(id) \int \frac{\log(1+ia+ibx)}{d+cx} dx}{2c} \\ &= -\frac{id \log(1-ia-ibx) \log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} + \frac{id \log(1+ia+ibx) \log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} - \frac{\text{Subst}(\int \log(x) dx, x, \dots)}{2bc} \\ &= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} - \frac{id \log(1-ia-ibx) \log\left(-\frac{b}{i}\right)}{2c^2} \\ &= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} - \frac{id \log(1-ia-ibx) \log\left(-\frac{b}{i}\right)}{2c^2} \end{aligned}$$

**Mathematica [B]** time = 11.0746, size = 771, normalized size = 3.16

$$ibd(bd-ac)\text{PolyLog}\left(2, \exp\left(2i\left(\tan^{-1}(a+bx) - \tan^{-1}\left(a - \frac{bd}{c}\right)\right)\right)\right) + ibd(ac-bd)\text{PolyLog}\left(2, -e^{2i \tan^{-1}(a+bx)}\right) + bcd\sqrt{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x), x]
```

```
[Out] (-2*a^2*c^2*ArcTan[a + b*x] + 2*a*b*c*d*ArcTan[a + b*x] + I*a*b*c*d*Pi*ArcTan[a + b*x] - I*b^2*d^2*Pi*ArcTan[a + b*x] - 2*a*b*c^2*x*ArcTan[a + b*x] + 2*b^2*c*d*x*ArcTan[a + b*x] + (2*I)*a*b*c*d*ArcTan[a - (b*d)/c]*ArcTan[a + b*x] - (2*I)*b^2*d^2*ArcTan[a - (b*d)/c]*ArcTan[a + b*x] - b*c*d*ArcTan[a + b*x]^2 + I*a*b*c*d*ArcTan[a + b*x]^2 - I*b^2*d^2*ArcTan[a + b*x]^2 + (b*c*
```

$$d\sqrt{1+a^2-(2abd)/c+(b^2d^2)/c^2}\operatorname{ArcTan}[a+bx]^2/E^{(I\operatorname{ArcTan}[a-(b*d)/c]+a*b*c*d*\pi*\log[1+E^{((-2*I)\operatorname{ArcTan}[a+bx])}] - b^2*d^2*\pi*\log[1+E^{((-2*I)\operatorname{ArcTan}[a+bx])}] - 2*a*b*c*d*\operatorname{ArcTan}[a+bx]*\log[1+E^{(2*I)\operatorname{ArcTan}[a+bx])}] + 2*b^2*d^2*\operatorname{ArcTan}[a+bx]*\log[1+E^{(2*I)\operatorname{ArcTan}[a+bx])}] - 2*a*b*c*d*\operatorname{ArcTan}[a-(b*d)/c]*\log[1-E^{(2*I)*(-\operatorname{ArcTan}[a-(b*d)/c]+\operatorname{ArcTan}[a+bx])}] + 2*b^2*d^2*\operatorname{ArcTan}[a-(b*d)/c]*\log[1-E^{(2*I)*(-\operatorname{ArcTan}[a-(b*d)/c]+\operatorname{ArcTan}[a+bx])}] + 2*a*b*c*d*\operatorname{ArcTan}[a+bx]*\log[1-E^{(2*I)*(-\operatorname{ArcTan}[a-(b*d)/c]+\operatorname{ArcTan}[a+bx])}] - 2*b^2*d^2*\operatorname{ArcTan}[a+bx]*\log[1-E^{(2*I)*(-\operatorname{ArcTan}[a-(b*d)/c]+\operatorname{ArcTan}[a+bx])}] - 2*a*c^2*\log[1/\sqrt{1+(a+bx)^2}] + 2*b*c*d*\log[1/\sqrt{1+(a+bx)^2}] - a*b*c*d*\pi*\log[1/\sqrt{1+(a+bx)^2}] + b^2*d^2*\pi*\log[1/\sqrt{1+(a+bx)^2}] + 2*a*b*c*d*\operatorname{ArcTan}[a-(b*d)/c]*\log[-\sin[\operatorname{ArcTan}[a-(b*d)/c]-\operatorname{ArcTan}[a+bx]]] - 2*b^2*d^2*\operatorname{ArcTan}[a-(b*d)/c]*\log[-\sin[\operatorname{ArcTan}[a-(b*d)/c]-\operatorname{ArcTan}[a+bx]]] + I*b*d*(a*c-b*d)*\operatorname{PolyLog}[2,-E^{(2*I)\operatorname{ArcTan}[a+bx]}] + I*b*d*(-a*c+b*d)*\operatorname{PolyLog}[2,E^{(2*I)*(-\operatorname{ArcTan}[a-(b*d)/c]+\operatorname{ArcTan}[a+bx])}]]/(b*c^2*(-2*a*c+2*b*d))$$

**Maple [A]** time = 0.06, size = 317, normalized size = 1.3

$$\frac{x \arctan(bx+a)}{c} + \frac{\arctan(bx+a)a}{bc} - \frac{\arctan(bx+a)d \ln(c(bx+a)-ac+bd)}{c^2} - \frac{\ln(a^2c^2-2abcd+b^2d^2+2(c(bx+a)-ac+bd)^2)}{2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(c+1/x\*d),x)

[Out]  $\arctan(b*x+a)/c*x+1/b*\arctan(b*x+a)/c*a-\arctan(b*x+a)*d/c^2*\ln(c*(b*x+a)-a*c+b*d)-1/2/b/c*\ln(a^2*c^2-2*a*b*c*d+b^2*d^2+2*(c*(b*x+a)-a*c+b*d)*a*c-2*(c*(b*x+a)-a*c+b*d)*b*d+(c*(b*x+a)-a*c+b*d)^2+c^2)-1/2*I/c^2*d*\ln(c*(b*x+a)-a*c+b*d)*\ln((I*c-c*(b*x+a))/(I*c-a*c+b*d))+1/2*I/c^2*d*\ln(c*(b*x+a)-a*c+b*d)*\ln((I*c+c*(b*x+a))/(I*c+a*c-b*d))-1/2*I/c^2*d*\operatorname{dilog}((I*c-c*(b*x+a))/(I*c-a*c+b*d))+1/2*I/c^2*d*\operatorname{dilog}((I*c+c*(b*x+a))/(I*c+a*c-b*d))$

**Maxima [A]** time = 2.01758, size = 383, normalized size = 1.57

$$\frac{bd \arctan(bx+a) \log\left(-\frac{b^2c^2x^2+2b^2cdx+b^2d^2}{2abcd-b^2d^2-(a^2+1)c^2}\right) + i bd \operatorname{Li}_2\left(-\frac{ibcx+(ia-1)c}{(-ia+1)c+ibd}\right) - i bd \operatorname{Li}_2\left(-\frac{ibcx+(ia+1)c}{(-ia-1)c+ibd}\right) - 2(bcx+ac) \arctan(bx+a)}{2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x),x, algorithm="maxima")

[Out]  $-1/2*(b*d*\arctan(b*x+a)*\log(-(b^2*c^2*x^2+2*b^2*c*d*x+b^2*d^2)/(2*a*b*c*d-b^2*d^2-(a^2+1)*c^2))+I*b*d*\operatorname{dilog}(-(I*b*c*x+(I*a-1)*c)/((-I*a+1)*c+I*b*d))-I*b*d*\operatorname{dilog}(-(I*b*c*x+(I*a+1)*c)/((-I*a-1)*c+I*b*d))-2*(b*c*x+a*c)*\arctan(b*x+a)-(b*d*\arctan^2(-(b*c^2*x+b*c*d)/(2*a*b*c*d-b^2*d^2-(a^2+1)*c^2),(a*b*c*d-b^2*d^2+(a*b*c^2-b^2*c*d)*x)/(2*a*b*c*d-b^2*d^2-(a^2+1)*c^2))-c*\log(b^2*x^2+2*a*b*x+a^2+1))/(b*c^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \arctan(bx + a)}{cx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x),x, algorithm="fricas")

[Out] integral(x\*arctan(b\*x + a)/(c\*x + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(c+d/x),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(c + d/x), x)



$$3.56 \quad \int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}} dx$$

**Optimal.** Leaf size=668

$$\frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(-a-bx+i)}}{a(-\sqrt{-c})-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c}(ia+ibx+1)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}$$

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-((b
*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3/2
) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*S
qrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I*a
- I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))]
)/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt[-
c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*
PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])
/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1 +
I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt
[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) - ((
I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] +
b*Sqrt[d])])/(-c)^(3/2)
```

**Rubi [A]** time = 0.854872, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(-a-bx+i)}}{a(-\sqrt{-c})-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c}(ia+ibx+1)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+i)}{a\sqrt{-c}-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x^2), x]
```

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-((b
*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3/2
) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*S
qrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I*a
- I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))]
)/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt[-
c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*
PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])
/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1 +
I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt
[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) - ((
I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] +
b*Sqrt[d])])/(-c)^(3/2)
```

**Rule 5051**

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2409

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]]^p, (f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IntegerQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}} dx &= \frac{1}{2}i \int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^2}} dx - \frac{1}{2}i \int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^2}} dx \\
&= \frac{1}{2}i \int \left( \frac{\log(1-ia-ibx)}{c} - \frac{d \log(1-ia-ibx)}{c(d+cx^2)} \right) dx - \frac{1}{2}i \int \left( \frac{\log(1+ia+ibx)}{c} - \frac{d \log(1+ia+ibx)}{c(d+cx^2)} \right) dx \\
&= \frac{i \int \log(1-ia-ibx) dx}{2c} - \frac{i \int \log(1+ia+ibx) dx}{2c} - \frac{(id) \int \frac{\log(1-ia-ibx)}{d+cx^2} dx}{2c} + \frac{(id) \int \frac{\log(1+ia+ibx)}{d+cx^2} dx}{2c} \\
&= -\frac{\text{Subst}(\int \log(x) dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int \log(x) dx, x, 1+ia+ibx)}{2bc} - \frac{(id) \int \left( \frac{\log(1-ia-ibx)}{2\sqrt{d}(\sqrt{d}-\sqrt{-c})} \right) dx}{4c} \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} - \frac{(i\sqrt{d}) \int \frac{\log(1-ia-ibx)}{\sqrt{d}-\sqrt{-c}} dx}{4c} \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d} \log(1+ia+ibx) \log(\dots)}{4(-c)^3} \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d} \log(1+ia+ibx) \log(\dots)}{4(-c)^3} \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d} \log(1+ia+ibx) \log(\dots)}{4(-c)^3}
\end{aligned}$$

**Mathematica [B]** time = 21.489, size = 1536, normalized size = 2.3

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/(c + d/x^2), x]

[Out] ((a + b\*x)\*ArcTan[a + b\*x] + Log[1/Sqrt[1 + (a + b\*x)^2]])/(b\*c) - (Sqrt[d] \* (-2\*Sqrt[c]\*ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d]))\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]] - 2\*a^2\*Sqrt[c]\*ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]] + 2\*Sqrt[c]\*ArcTan[(I + a)\*Sqrt[c]]/(b\*Sqrt[d])\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]] + 2\*a^2\*Sqrt[c]\*ArcTan[(I + a)\*Sqrt[c]]/(b\*Sqrt[d])\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]] - 2\*b\*Sqrt[d]\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]^2 + (b\*Sqrt[d]\*Sqrt[(-I + a)^2\*c + b^2\*d]/(b^2\*d))\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]^2)/E^(I\*ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])) - (I\*a\*b\*Sqrt[d]\*Sqrt[(-I + a)^2\*c + b^2\*d]/(b^2\*d))\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]^2)/E^(I\*ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + (b\*Sqrt[d]\*Sqrt[((I + a)^2\*c + b^2\*d)/(b^2\*d))\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]^2)/E^(I\*ArcTan[(I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + (I\*a\*b\*Sqrt[d]\*Sqrt[((I + a)^2\*c + b^2\*d)/(b^2\*d))\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]^2)/E^(I\*ArcTan[(I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + 4\*(1 + a^2)\*Sqrt[c]\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]\*ArcTan[a + b\*x] + (2\*I)\*Sqrt[c]\*ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])\*Log[1 - E^((-2\*I)\*(ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + ArcTan[(Sqrt[c]\*x)/Sqrt[d]])] + (2\*I)\*a^2\*Sqrt[c]\*ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])\*Log[1 - E^((-2\*I)\*(ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + ArcTan[(Sqrt[c]\*x)/Sqrt[d]])] + (2\*I)\*Sqrt[c]\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]\*Log[1 - E^((-2\*I)\*(ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + ArcTan[(Sqrt[c]\*x)/Sqrt[d]])] + (2\*I)\*a^2\*Sqrt[c]\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]\*Log[1 - E^((-2\*I)\*(ArcTan[(-I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + ArcTan[(Sqrt[c]\*x)/Sqrt[d]])] - (2\*I)\*Sqrt[c]\*ArcTan[(I + a)\*Sqrt[c]]/(b\*Sqrt[d])\*Log[1 - E^((-2\*I)\*(ArcTan[(I + a)\*Sqrt[c]]/(b\*Sqrt[d])) + ArcTan[(Sqrt[c]\*x)/Sqrt[d]])] + ArcTan[(Sqrt[c]\*x)/Sqrt[d]]



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(c+d/x\*\*2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x^2), x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(c + d/x^2), x)

$$3.57 \quad \int \frac{\tan^{-1}(a+bx)}{c + \frac{d}{x^3}} dx$$

**Optimal.** Leaf size=933

result too large to display

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)]/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-((b
*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3))])]/c^(4/3) + ((I/6)*d
^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((b*
(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3))
])/((6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) -
(-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/
3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)
*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/3)) + ((-1)
^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x)
)/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/3)) - ((-1)^(1/6)*d
^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c^(
1/3) - b*d^(1/3))])/((6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1/6)
*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/((6*c^
(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))
/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2, (
(-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))
])/((6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a
+ b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/3))
```

**Rubi [A]** time = 1.36743, antiderivative size = 933, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5051, 2409, 2389, 2295, 2394, 2393, 2391}

$$-\frac{(ia + ibx + 1) \log(ia + ibx + 1)}{2bc} + \frac{i\sqrt[3]{d} \log\left(\frac{b(\sqrt[3]{cx + \sqrt[3]{d}})}{\sqrt[3]{c(i-a)+b\sqrt[3]{d}}}\right) \log(ia + ibx + 1)}{6c^{4/3}} - \frac{\sqrt[6]{-1}\sqrt[3]{d} \log\left(-\frac{b(\sqrt[3]{d} - \sqrt[3]{-1}\sqrt[3]{cx})}{\sqrt[3]{-1(i-a)}\sqrt[3]{c-b}\sqrt[3]{d}}\right) \log(ia + ibx)}{6c^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/(c + d/x^3), x]

```
[Out] -((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c) - ((1 - I*a - I*b*x)*Log[
(-I)*(I + a + b*x)]/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-((b
*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3))])]/c^(4/3) + ((I/6)*d
^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((b*
(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3))
])/((6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) -
(-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/
3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)
*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/3)) + ((-1)
^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x)
)/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/3)) - ((-1)^(1/6)*d
^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c^(
1/3) - b*d^(1/3))])/((6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1/6)
*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/((6*c^
(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/3)
+ b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))
/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2, (
(-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))
])/((6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a
+ b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/((6*c^(4/3))
```

$$\begin{aligned} & \left( \frac{4}{3} \right) + \left( \frac{I}{6} d^{1/3} \text{PolyLog}[2, (c^{1/3}(I - a - b*x)) / ((I - a)c^{1/3} + b*d^{1/3})] \right) / c^{4/3} - \left( \frac{I}{6} d^{1/3} \text{PolyLog}[2, (c^{1/3}(I + a + b*x)) / ((I + a)c^{1/3} - b*d^{1/3})] \right) / c^{4/3} \\ & + \left( (-1)^{5/6} d^{1/3} \text{PolyLog}[2, (-1)^{2/3} c^{1/3} (I + a + b*x) / ((-1)^{2/3} (I + a) c^{1/3} - b*d^{1/3})] \right) / (6*c^{4/3}) \\ & + \left( (-1)^{1/6} d^{1/3} \text{PolyLog}[2, (-1)^{1/3} c^{1/3} (I + a + b*x) / ((-1)^{1/3} (I + a) c^{1/3} + b*d^{1/3})] \right) / (6*c^{4/3}) \end{aligned}$$
**Rule 5051**

```
Int[ArcTan[(a_) + (b_)*(x_)]/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

**Rule 2409**

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_)*((f_) + (g_
)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

**Rule 2389**

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

**Rule 2295**

```
Int[Log[(c_)*(x_)^(n_)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

**Rule 2394**

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))/((f_) + (g_)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

**Rule 2393**

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_))/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

**Rule 2391**

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

**Rubi steps**

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}} dx &= \frac{1}{2}i \int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^3}} dx - \frac{1}{2}i \int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^3}} dx \\
&= \frac{1}{2}i \int \left( \frac{\log(1-ia-ibx)}{c} - \frac{d \log(1-ia-ibx)}{c(d+cx^3)} \right) dx - \frac{1}{2}i \int \left( \frac{\log(1+ia+ibx)}{c} - \frac{d \log(1+ia+ibx)}{c(d+cx^3)} \right) dx \\
&= \frac{i \int \log(1-ia-ibx) dx}{2c} - \frac{i \int \log(1+ia+ibx) dx}{2c} - \frac{(id) \int \frac{\log(1-ia-ibx)}{d+cx^3} dx}{2c} + \frac{(id) \int \frac{\log(1+ia+ibx)}{d+cx^3} dx}{2c} \\
&= \frac{\text{Subst}(\int \log(x) dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int \log(x) dx, x, 1+ia+ibx)}{2bc} - \frac{(id) \int \left( -\frac{\log(1-ia-ibx)}{3d^{2/3}(-\sqrt[3]{d}-\sqrt[3]{cx})} \right) dx}{6c} + \dots \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} + \frac{(i\sqrt[3]{d}) \int \frac{\log(1-ia-ibx)}{-\sqrt[3]{d}-\sqrt[3]{cx}} dx}{6c} + \dots \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d} \log(1-ia-ibx) \log\left(-\frac{\sqrt[3]{d}}{-\sqrt[3]{d}-\sqrt[3]{cx}}\right)}{6c^{4/3}} + \dots \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d} \log(1-ia-ibx) \log\left(-\frac{\sqrt[3]{d}}{-\sqrt[3]{d}-\sqrt[3]{cx}}\right)}{6c^{4/3}} + \dots \\
&= -\frac{(1+ia+ibx) \log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx) \log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d} \log(1-ia-ibx) \log\left(-\frac{\sqrt[3]{d}}{-\sqrt[3]{d}-\sqrt[3]{cx}}\right)}{6c^{4/3}} + \dots
\end{aligned}$$

**Mathematica [C]** time = 7.07409, size = 933, normalized size = 1.

$$6 \left( (a+bx) \tan^{-1}(a+bx) + \log\left(\frac{1}{\sqrt{(a+bx)^2+1}}\right) \right) - b^3 d \text{RootSum} \left[ c \#1^3 a^3 + 3c \#1^2 a^3 + ca^3 + 3c \#1 a^3 + 3ic \#1^3 a^2 + 3ic \#1^2 a^2 - \dots \right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/(c + d/x^3), x]

[Out] (6\*((a + b\*x)\*ArcTan[a + b\*x] + Log[1/Sqrt[1 + (a + b\*x)^2]]) - b^3\*d\*RootSum[I\*c - 3\*a\*c - (3\*I)\*a^2\*c + a^3\*c - b^3\*d - (3\*I)\*c\*#1 + 3\*a\*c\*#1 - (3\*I)\*a^2\*c\*#1 + 3\*a^3\*c\*#1 - 3\*b^3\*d\*#1 + (3\*I)\*c\*#1^2 + 3\*a\*c\*#1^2 + (3\*I)\*a^2\*c\*#1^2 + 3\*a^3\*c\*#1^2 - 3\*b^3\*d\*#1^2 - I\*c\*#1^3 - 3\*a\*c\*#1^3 + (3\*I)\*a^2\*c\*#1^3 + a^3\*c\*#1^3 - b^3\*d\*#1^3 & , (-Pi\*ArcTan[a + b\*x]) - 2\*ArcTan[a + b\*x]^2 + (2\*I)\*ArcTan[a + b\*x]\*ArcTanh[(-1 + #1)/(1 + #1)] + I\*Pi\*Log[1 + E^((-2\*I)\*ArcTan[a + b\*x])] + (2\*I)\*ArcTan[a + b\*x]\*Log[1 - E^((2\*I)\*ArcTan[a + b\*x] - 2\*ArcTanh[(-1 + #1)/(1 + #1)])] - 2\*ArcTanh[(-1 + #1)/(1 + #1)]\*Log[1 - E^((2\*I)\*ArcTan[a + b\*x] - 2\*ArcTanh[(-1 + #1)/(1 + #1)])] - I\*Pi\*Log[1/Sqrt[1 + (a + b\*x)^2]] + 2\*ArcTanh[(-1 + #1)/(1 + #1)]\*Log[Sin[ArcTan[a + b\*x] + I\*ArcTanh[(-1 + #1)/(1 + #1)]]] + PolyLog[2, E^((2\*I)\*ArcTan[a + b\*x] - 2\*ArcTanh[(-1 + #1)/(1 + #1)])] - 2\*ArcTan[a + b\*x]^2\*#1 + Pi\*ArcTan[a + b\*x]\*#1^2 - (2\*I)\*ArcTan[a + b\*x]\*ArcTanh[(-1 + #1)/(1 + #1)]\*#1^2 - I\*Pi\*Log[1 + E^((-2\*I)\*ArcTan[a + b\*x])]\*#1^2 - (2\*I)\*ArcTan[a + b\*x]\*Log[1 - E^((2\*I)\*ArcTan[a + b\*x] - 2\*ArcTanh[(-1 + #1)/(1 + #1)])]\*#1^2 + 2\*ArcTanh[(-1 + #1)/(1 + #1)]\*Log[1 - E^((2\*I)\*ArcTan[a + b\*x] - 2\*ArcTanh[(-1 + #1)/(1 + #1)])]\*#1^2 + I\*Pi\*Log[1/Sqrt[1 + (a + b\*x)^2]]\*#1^2 - 2\*ArcTanh[(-1 + #1)/(1 + #1)]\*Log[Sin[ArcTan[a + b\*x] + I\*ArcTanh[(-1 + #1)/(1 + #1)]]]



```

]#1^2 - PolyLog[2, E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)
]])#1^2 + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*Sqrt[#1/(1 + #1)
]^2 + 4*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1*Sqrt[#1/(1 + #1)
]^2 + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1^2*Sqrt[#1/(1 + #1)
]^2))/(-a*c) - (2*I)*a^2*c + a^3*c - b^3*d + 2*a*c*#1 + 2*a^3*c*#1 - 2*b^3
*d*#1 - a*c*#1^2 + (2*I)*a^2*c*#1^2 + a^3*c*#1^2 - b^3*d*#1^2) & ])/(6*b*c)

```

**Maple [C]** time = 0.658, size = 682, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^3), x)
```

```
[Out] arctan(b*x+a)/c*x+1/b*arctan(b*x+a)/c*a-1/2/b/c*ln(1+(b*x+a)^2)-2/3*b^2/c*d
*sum(1/(a^3*c*_R1^4+3*I*_R1^4*a^2*c-b^3*d*_R1^4-3*a*c*_R1^4-I*c*_R1^4+2*a^3
*c*_R1^2+2*I*_R1^2*a^2*c-2*b^3*d*_R1^2+2*_R1^2*a*c+2*I*c*_R1^2+a^3*c-I*a^2*
c-b^3*d+a*c-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a)))/(1+(b*x+a)^2)^(1/2)
))/_R1)+dilog((_R1-(1+I*(b*x+a)))/(1+(b*x+a)^2)^(1/2))/_R1)), _R1=RootOf((3*I*
a^2*c+a^3*c-b^3*d-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*c-3*b^3*d+3*I*c+3*a*c)*_
Z^4+(-3*I*a^2*c+3*a^3*c-3*b^3*d-3*I*c+3*a*c)*_Z^2-3*I*a^2*c+a^3*c-b^3*d+I*c
-3*a*c))-2/3*b^2/c*d*sum(_R1^2/(a^3*c*_R1^4+3*I*_R1^4*a^2*c-b^3*d*_R1^4-3*a
*c*_R1^4-I*c*_R1^4+2*a^3*c*_R1^2+2*I*_R1^2*a^2*c-2*b^3*d*_R1^2+2*_R1^2*a*c+
2*I*c*_R1^2+a^3*c-I*a^2*c-b^3*d+a*c-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x
+a)))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a)))/(1+(b*x+a)^2)^(1/2)
))/_R1)), _R1=RootOf((3*I*a^2*c+a^3*c-b^3*d-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*
c-3*b^3*d+3*I*c+3*a*c)*_Z^4+(-3*I*a^2*c+3*a^3*c-3*b^3*d-3*I*c+3*a*c)*_Z^2-3
*I*a^2*c+a^3*c-b^3*d+I*c-3*a*c))

```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \arctan(bx + a)}{cx^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3), x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(b*x + a)/(c*x^3 + d), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(c+d/x\*\*3), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (bx + a)}{c + \frac{d}{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x^3), x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(c + d/x^3), x)

$$3.58 \quad \int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}} dx$$

**Optimal.** Leaf size=673

$$\frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a-id}}}\right)}{d^2} + \frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc+\sqrt{-a-id}}}\right)}{d^2} - \frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a+id}}}\right)}{d^2} - \frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc+\sqrt{-a+id}}}\right)}{d^2}$$

[Out] ((2\*I)\*Sqrt[I + a]\*ArcTan[(Sqrt[b]\*Sqrt[x])/Sqrt[I + a]]/(Sqrt[b]\*d) - ((2\*I)\*Sqrt[I - a]\*ArcTanh[(Sqrt[b]\*Sqrt[x])/Sqrt[I - a]]/(Sqrt[b]\*d) + (I\*c\*Log[(d\*(Sqrt[-I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-I - a]\*d)]\*Log[c + d\*Sqrt[x]])/d^2 - (I\*c\*Log[(d\*(Sqrt[I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[I - a]\*d)]\*Log[c + d\*Sqrt[x]])/d^2 + (I\*c\*Log[-((d\*(Sqrt[-I - a] + Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-I - a]\*d))]\*Log[c + d\*Sqrt[x]])/d^2 - (I\*c\*Log[-((d\*(Sqrt[I - a] + Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[I - a]\*d))]\*Log[c + d\*Sqrt[x]])/d^2 + (I\*Sqrt[x]\*Log[1 - I\*a - I\*b\*x])/d - (I\*c\*Log[c + d\*Sqrt[x]]\*Log[1 - I\*a - I\*b\*x])/d^2 - (I\*Sqrt[x]\*Log[1 + I\*a + I\*b\*x])/d + (I\*c\*Log[c + d\*Sqrt[x]]\*Log[1 + I\*a + I\*b\*x])/d^2 + (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-I - a]\*d)]/d^2 + (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-I - a]\*d)]/d^2 - (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[I - a]\*d)]/d^2 - (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[I - a]\*d)]/d^2

**Rubi [A]** time = 0.898424, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {5051, 2408, 2466, 2448, 321, 205, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a-id}}}\right)}{d^2} + \frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc+\sqrt{-a-id}}}\right)}{d^2} - \frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a+id}}}\right)}{d^2} - \frac{icPolyLog\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc+\sqrt{-a+id}}}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/(c + d\*Sqrt[x]), x]

[Out] ((2\*I)\*Sqrt[I + a]\*ArcTan[(Sqrt[b]\*Sqrt[x])/Sqrt[I + a]]/(Sqrt[b]\*d) - ((2\*I)\*Sqrt[I - a]\*ArcTanh[(Sqrt[b]\*Sqrt[x])/Sqrt[I - a]]/(Sqrt[b]\*d) + (I\*c\*Log[(d\*(Sqrt[-I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-I - a]\*d)]\*Log[c + d\*Sqrt[x]])/d^2 - (I\*c\*Log[(d\*(Sqrt[I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[I - a]\*d)]\*Log[c + d\*Sqrt[x]])/d^2 + (I\*c\*Log[-((d\*(Sqrt[-I - a] + Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-I - a]\*d))]\*Log[c + d\*Sqrt[x]])/d^2 - (I\*c\*Log[-((d\*(Sqrt[I - a] + Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[I - a]\*d))]\*Log[c + d\*Sqrt[x]])/d^2 + (I\*Sqrt[x]\*Log[1 - I\*a - I\*b\*x])/d - (I\*c\*Log[c + d\*Sqrt[x]]\*Log[1 - I\*a - I\*b\*x])/d^2 - (I\*Sqrt[x]\*Log[1 + I\*a + I\*b\*x])/d + (I\*c\*Log[c + d\*Sqrt[x]]\*Log[1 + I\*a + I\*b\*x])/d^2 + (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-I - a]\*d)]/d^2 + (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-I - a]\*d)]/d^2 - (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[I - a]\*d)]/d^2 - (I\*c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[I - a]\*d)]/d^2

**Rule 5051**

Int[ArcTan[(a\_) + (b\_.)\*(x\_)]/((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Dist[I/2, Int[Log[1 - I\*a - I\*b\*x]/(c + d\*x^n), x], x] - Dist[I/2, Int[Log[1 + I

\*a + I\*b\*x]/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]

#### Rule 2408

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := With[{k = Denominator[r]}, Dist[k, Subst[Int[x^(k - 1)\*(f + g\*x^(k\*r))^q\*(a + b\*Log[c\*(d + e\*x^k)^n])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IntegerQ[p, 0]

#### Rule 2466

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p])^q, x^m\*(f + g\*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]

#### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2462

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[f + g\*x]\*(a + b\*Log[c\*(d + e\*x^n)^p])/g, x] - Dist[(b\*e\*n\*p)/g, Int[(x^(n - 1)\*Log[f + g\*x])/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x

)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^n)]/(x\_.), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{-1}(a + bx)}{c + d\sqrt{x}} dx &= \frac{1}{2}i \int \frac{\log(1 - ia - ibx)}{c + d\sqrt{x}} dx - \frac{1}{2}i \int \frac{\log(1 + ia + ibx)}{c + d\sqrt{x}} dx \\
 &= i \operatorname{Subst} \left( \int \frac{x \log(1 - ia - ibx^2)}{c + dx} dx, x, \sqrt{x} \right) - i \operatorname{Subst} \left( \int \frac{x \log(1 + ia + ibx^2)}{c + dx} dx, x, \sqrt{x} \right) \\
 &= i \operatorname{Subst} \left( \int \left( \frac{\log(1 - ia - ibx^2)}{d} - \frac{c \log(1 - ia - ibx^2)}{d(c + dx)} \right) dx, x, \sqrt{x} \right) - i \operatorname{Subst} \left( \int \left( \frac{\log(1 + ia + ibx^2)}{d} - \frac{c \log(1 + ia + ibx^2)}{d(c + dx)} \right) dx, x, \sqrt{x} \right) \\
 &= \frac{i \operatorname{Subst} \left( \int \log(1 - ia - ibx^2) dx, x, \sqrt{x} \right)}{d} - \frac{i \operatorname{Subst} \left( \int \log(1 + ia + ibx^2) dx, x, \sqrt{x} \right)}{d} - \frac{(ic) \operatorname{Subst} \left( \int \frac{\log(1 - ia - ibx^2)}{c + dx} dx, x, \sqrt{x} \right)}{d} + \frac{(ic) \operatorname{Subst} \left( \int \frac{\log(1 + ia + ibx^2)}{c + dx} dx, x, \sqrt{x} \right)}{d} \\
 &= \frac{i\sqrt{x} \log(1 - ia - ibx)}{d} - \frac{ic \log(c + d\sqrt{x}) \log(1 - ia - ibx)}{d^2} - \frac{i\sqrt{x} \log(1 + ia + ibx)}{d} + \frac{ic \log(c + d\sqrt{x}) \log(1 + ia + ibx)}{d^2} \\
 &= \frac{i\sqrt{x} \log(1 - ia - ibx)}{d} - \frac{ic \log(c + d\sqrt{x}) \log(1 - ia - ibx)}{d^2} - \frac{i\sqrt{x} \log(1 + ia + ibx)}{d} + \frac{ic \log(c + d\sqrt{x}) \log(1 + ia + ibx)}{d^2} \\
 &= \frac{2i\sqrt{i+a} \tan^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}} \right)}{\sqrt{bd}} - \frac{2i\sqrt{i-a} \tanh^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}} \right)}{\sqrt{bd}} + \frac{i\sqrt{x} \log(1 - ia - ibx)}{d} - \frac{ic \log(c + d\sqrt{x}) \log(1 - ia - ibx)}{d^2} \\
 &= \frac{2i\sqrt{i+a} \tan^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}} \right)}{\sqrt{bd}} - \frac{2i\sqrt{i-a} \tanh^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}} \right)}{\sqrt{bd}} + \frac{ic \log \left( \frac{d(\sqrt{-i-a} - \sqrt{b}\sqrt{x})}{\sqrt{bc} + \sqrt{-i-ad}} \right) \log(c + d\sqrt{x})}{d^2} - \frac{ic \log \left( \frac{d(\sqrt{-i-a} - \sqrt{b}\sqrt{x})}{\sqrt{bc} + \sqrt{-i-ad}} \right) \log(c + d\sqrt{x})}{d^2} \\
 &= \frac{2i\sqrt{i+a} \tan^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}} \right)}{\sqrt{bd}} - \frac{2i\sqrt{i-a} \tanh^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}} \right)}{\sqrt{bd}} + \frac{ic \log \left( \frac{d(\sqrt{-i-a} - \sqrt{b}\sqrt{x})}{\sqrt{bc} + \sqrt{-i-ad}} \right) \log(c + d\sqrt{x})}{d^2} - \frac{ic \log \left( \frac{d(\sqrt{-i-a} - \sqrt{b}\sqrt{x})}{\sqrt{bc} + \sqrt{-i-ad}} \right) \log(c + d\sqrt{x})}{d^2} \\
 &= \frac{2i\sqrt{i+a} \tan^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}} \right)}{\sqrt{bd}} - \frac{2i\sqrt{i-a} \tanh^{-1} \left( \frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}} \right)}{\sqrt{bd}} + \frac{ic \log \left( \frac{d(\sqrt{-i-a} - \sqrt{b}\sqrt{x})}{\sqrt{bc} + \sqrt{-i-ad}} \right) \log(c + d\sqrt{x})}{d^2} - \frac{ic \log \left( \frac{d(\sqrt{-i-a} - \sqrt{b}\sqrt{x})}{\sqrt{bc} + \sqrt{-i-ad}} \right) \log(c + d\sqrt{x})}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.517766, size = 604, normalized size = 0.9

$$i \left( c \operatorname{PolyLog} \left( 2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a-id}}} \right) + c \operatorname{PolyLog} \left( 2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc+\sqrt{-a-id}}} \right) - c \operatorname{PolyLog} \left( 2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a+id}}} \right) - c \operatorname{PolyLog} \left( 2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc+\sqrt{-a+id}}} \right) + c \log \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/(c + d\*Sqrt[x]), x]

[Out] (I\*((2\*Sqrt[I + a]\*d\*ArcTan[(Sqrt[b]\*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2\*Sqrt[I - a]\*d\*ArcTanh[(Sqrt[b]\*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c\*Log[(d\*(Sqrt[-I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-I - a]\*d)]\*Log[c + d\*Sqrt[x]] - c\*Log[(d\*(Sqrt[I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[I - a]\*d)]\*Log[c + d\*Sqrt[x]] + c\*Log[(d\*(Sqrt[-I - a] + Sqrt[b]\*Sqrt[x]))/(-(Sqrt[b]\*c) + Sqrt[-I - a]\*d)]\*Log[c + d\*Sqrt[x]] - c\*Log[(d\*(Sqrt[I - a] + Sqrt[b]\*Sqrt[x]))/(-(Sqrt[b]\*c) + Sqrt[I - a]\*d)]\*Log[c + d\*Sqrt[x]] - d\*Sqrt[x]\*Log[1 + I\*a + I\*b\*x] + c\*Log[c + d\*Sqrt[x]]\*Log[1 + I\*a + I\*b\*x] + d\*Sqrt[x]\*Log[(-I)\*(I + a + b\*x)] - c\*Log[c + d\*Sqrt[x]]\*Log[(-I)\*(I + a + b\*x)] + c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-I - a]\*d)] + c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-I - a]\*d)] - c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[I - a]\*d)] - c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[I - a]\*d)))/d^2

**Maple [C]** time = 0.338, size = 344, normalized size = 0.5

$$2 \frac{\arctan(bx+a)\sqrt{x}}{d} - 2 \frac{\arctan(bx+a)c \ln(c+d\sqrt{x})}{d^2} + c \sum_{_R1=\text{RootOf}(b^2_Z^4-4cb^2_Z^3+(2abd^2+6b^2c^2)_Z^2+(-4bcad^2-4c^3b^2)_Z+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(c+d\*x^(1/2)), x)

[Out] 2\*arctan(b\*x+a)/d\*x^(1/2)-2\*arctan(b\*x+a)\*c/d^2\*ln(c+d\*x^(1/2))+c\*sum(1/(\_R1^2\*b-2\*\_R1\*b\*c+a\*d^2+b\*c^2)\*(ln(c+d\*x^(1/2))\*ln((-d\*x^(1/2)+\_R1-c)/\_R1)+dilog((-d\*x^(1/2)+\_R1-c)/\_R1)), \_R1=RootOf(b^2\*\_Z^4-4\*c\*b^2\*\_Z^3+(2\*a\*b\*d^2+6\*b^2\*c^2)\*\_Z^2+(-4\*a\*b\*c\*d^2-4\*b^2\*c^3)\*\_Z+a^2\*d^4+2\*a\*b\*c^2\*d^2+b^2\*c^4+d^4))-sum((\_R^2-2\*\_R\*c+c^2)/(\_R^3\*b-3\*\_R^2\*b\*c+\_R\*a\*d^2+3\*\_R\*b\*c^2-a\*c\*d^2-b\*c^3)\*ln(d\*x^(1/2)-\_R+c), \_R=RootOf(b^2\*\_Z^4-4\*c\*b^2\*\_Z^3+(2\*a\*b\*d^2+6\*b^2\*c^2)\*\_Z^2+(-4\*a\*b\*c\*d^2-4\*b^2\*c^3)\*\_Z+a^2\*d^4+2\*a\*b\*c^2\*d^2+b^2\*c^4+d^4))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{d\sqrt{x}+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d\*x^(1/2)), x, algorithm="maxima")

[Out] integrate(arctan(b\*x + a)/(d\*sqrt(x) + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d\sqrt{x}\arctan(bx+a)-c\arctan(bx+a)}{d^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d\*x^(1/2)),x, algorithm="fricas")

[Out] integral((d\*sqrt(x)\*arctan(b\*x + a) - c\*arctan(b\*x + a))/(d^2\*x - c^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(c+d\*x\*\*(1/2)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx+a)}{d\sqrt{x}+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d\*x^(1/2)),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(d\*sqrt(x) + c), x)

$$3.59 \quad \int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx$$

**Optimal.** Leaf size=770

$$-\frac{id^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x+d})}{-\sqrt{bd+\sqrt{-a-ic}}}\right)}{c^3} + \frac{id^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x+d})}{-\sqrt{bd+\sqrt{-a+ic}}}\right)}{c^3} - \frac{id^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x+d})}{\sqrt{bd+\sqrt{-a-ic}}}\right)}{c^3} + \frac{id^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x+d})}{\sqrt{bd+\sqrt{-a+ic}}}\right)}{c^3}$$

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2)
- (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
+ (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
- (I*d^2*Log[(c*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
+ (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
- (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])/c^2
+ (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3
+ (I*d*Sqrt[x]*Log[1 + I*a + I*b*x])/c^2
- ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
- (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3
- ((1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])/(2*b*c)
- (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3
+ (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3
- (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3
+ (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```

**Rubi [A]** time = 0.982378, antiderivative size = 770, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 16, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$ , Rules used = {5051, 2408, 2476, 2448, 321, 205, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$-\frac{id^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x+d})}{-\sqrt{bd+\sqrt{-a-ic}}}\right)}{c^3} + \frac{id^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x+d})}{-\sqrt{bd+\sqrt{-a+ic}}}\right)}{c^3} - \frac{id^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x+d})}{\sqrt{bd+\sqrt{-a-ic}}}\right)}{c^3} + \frac{id^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x+d})}{\sqrt{bd+\sqrt{-a+ic}}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/Sqrt[x]), x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2)
- (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
+ (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
- (I*d^2*Log[(c*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
+ (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3
- (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])/c^2
+ (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3
+ (I*d*Sqrt[x]*Log[1 + I*a + I*b*x])/c^2
- ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
- (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3
- ((1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])/(2*b*c)
- (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3
+ (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3
- (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3
+ (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```



Rule 5051

```
Int[ArcTan[(a_) + (b_)*(x_)]/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rule 2408

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_)*((f_) + (g_
.)*(x_)^(r_))^(q_), x_Symbol] := With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2476

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)]*(b_))^(q_)*(x_)^(m
_)*((f_) + (g_)*(x_)^(s_))^(r_), x_Symbol] := Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)], x_Symbol] := Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2454

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)]*(b_))^(q_)*(x_)^(m
_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_)*(x_)^(n_)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p])/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]*(b_.))^(p_.)*((h_.)*(x_)
)^(m_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx &= \frac{1}{2}i \int \frac{\log(1-ia-ibx)}{c+\frac{d}{\sqrt{x}}} dx - \frac{1}{2}i \int \frac{\log(1+ia+ibx)}{c+\frac{d}{\sqrt{x}}} dx \\
&= i \operatorname{Subst} \left( \int \frac{x \log(1-ia-ibx^2)}{c+\frac{d}{x}} dx, x, \sqrt{x} \right) - i \operatorname{Subst} \left( \int \frac{x \log(1+ia+ibx^2)}{c+\frac{d}{x}} dx, x, \sqrt{x} \right) \\
&= i \operatorname{Subst} \left( \int \left( -\frac{d \log(1-ia-ibx^2)}{c^2} + \frac{x \log(1-ia-ibx^2)}{c} + \frac{d^2 \log(1-ia-ibx^2)}{c^2(d+cx)} \right) dx, x, \sqrt{x} \right) \\
&= \frac{i \operatorname{Subst} \left( \int x \log(1-ia-ibx^2) dx, x, \sqrt{x} \right)}{c} - \frac{i \operatorname{Subst} \left( \int x \log(1+ia+ibx^2) dx, x, \sqrt{x} \right)}{c} \quad (id) \\
&= -\frac{id\sqrt{x} \log(1-ia-ibx)}{c^2} + \frac{id^2 \log(d+c\sqrt{x}) \log(1-ia-ibx)}{c^3} + \frac{id\sqrt{x} \log(1+ia+ibx)}{c^2} - \frac{id^2 \log(d+c\sqrt{x}) \log(1+ia+ibx)}{c^3} \\
&= -\frac{id\sqrt{x} \log(1-ia-ibx)}{c^2} + \frac{id^2 \log(d+c\sqrt{x}) \log(1-ia-ibx)}{c^3} + \frac{id\sqrt{x} \log(1+ia+ibx)}{c^2} - \frac{id^2 \log(d+c\sqrt{x}) \log(1+ia+ibx)}{c^3} \\
&= -\frac{2i\sqrt{i+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{bc^2}} + \frac{2i\sqrt{i-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{bc^2}} - \frac{id\sqrt{x} \log(1-ia-ibx)}{c^2} + \frac{id^2 \log(d+c\sqrt{x}) \log(1-ia-ibx)}{c^3} \\
&= -\frac{2i\sqrt{i+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{bc^2}} + \frac{2i\sqrt{i-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{bc^2}} - \frac{id^2 \log\left(\frac{c(\sqrt{-i-a}-\sqrt{b}\sqrt{x})}{\sqrt{-i-ac}+\sqrt{bd}}\right) \log(d+c\sqrt{x})}{c^3} \\
&= -\frac{2i\sqrt{i+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{bc^2}} + \frac{2i\sqrt{i-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{bc^2}} - \frac{id^2 \log\left(\frac{c(\sqrt{-i-a}-\sqrt{b}\sqrt{x})}{\sqrt{-i-ac}+\sqrt{bd}}\right) \log(d+c\sqrt{x})}{c^3} \\
&= -\frac{2i\sqrt{i+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{bc^2}} + \frac{2i\sqrt{i-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{bc^2}} - \frac{id^2 \log\left(\frac{c(\sqrt{-i-a}-\sqrt{b}\sqrt{x})}{\sqrt{-i-ac}+\sqrt{bd}}\right) \log(d+c\sqrt{x})}{c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.718889, size = 666, normalized size = 0.86

$$i \left( -2d^2 \left( \operatorname{PolyLog} \left( 2, \frac{\sqrt{b}(c\sqrt{x+d})}{\sqrt{bd}-\sqrt{-a-ic}} \right) + \operatorname{PolyLog} \left( 2, \frac{\sqrt{b}(c\sqrt{x+d})}{\sqrt{bd}+\sqrt{-a-ic}} \right) + \log(c\sqrt{x+d}) \left( \log \left( \frac{c(-\sqrt{b}\sqrt{x}+\sqrt{-a-i})}{\sqrt{bd}+\sqrt{-a-ic}} \right) + \log \left( \frac{c(\sqrt{b}\sqrt{x}+\sqrt{-a-i})}{-\sqrt{bd}+\sqrt{-a-ic}} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a + b\*x]/(c + d/Sqrt[x]),x]

[Out] ((I/2)\*(4\*c\*d\*(Sqrt[x] - (Sqrt[I + a]\*ArcTan[(Sqrt[b]\*Sqrt[x])/Sqrt[I + a]])/Sqrt[b]) - 4\*c\*d\*(Sqrt[x] - (Sqrt[I - a]\*ArcTanh[(Sqrt[b]\*Sqrt[x])/Sqrt[I - a]])/Sqrt[b]) + 2\*c\*d\*Sqrt[x]\*Log[1 + I\*a + I\*b\*x] - (c^2\*(-I + a + b\*x)\*Log[1 + I\*a + I\*b\*x])/b - 2\*d^2\*Log[d + c\*Sqrt[x]]\*Log[1 + I\*a + I\*b\*x] - 2\*c\*d\*Sqrt[x]\*Log[(-I)\*(I + a + b\*x)] + (c^2\*(I + a + b\*x)\*Log[(-I)\*(I + a + b\*x)])/b + 2\*d^2\*Log[d + c\*Sqrt[x]]\*Log[(-I)\*(I + a + b\*x)] - 2\*d^2\*((Log[(c\*(Sqrt[-I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[-I - a]\*c + Sqrt[b]\*d)] + Log[(c\*(Sqrt[-I - a] + Sqrt[b]\*Sqrt[x]))/(Sqrt[-I - a]\*c - Sqrt[b]\*d)])\*Log[d + c\*Sqrt[x]] + PolyLog[2, (Sqrt[b]\*(d + c\*Sqrt[x]))/(-Sqrt[-I - a]\*c) + Sqrt[b]\*d] + PolyLog[2, (Sqrt[b]\*(d + c\*Sqrt[x]))/(Sqrt[-I - a]\*c + Sqrt[b]\*d)]) + 2\*d^2\*((Log[(c\*(Sqrt[I - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[I - a]\*c + Sqrt[b]\*d)] + Log[(c\*(Sqrt[I - a] + Sqrt[b]\*Sqrt[x]))/(Sqrt[I - a]\*c - Sqrt[b]\*d)])\*Log[d + c\*Sqrt[x]] + PolyLog[2, (Sqrt[b]\*(d + c\*Sqrt[x]))/(-Sqrt[I - a]\*c) + Sqrt[b]\*d] + PolyLog[2, (Sqrt[b]\*(d + c\*Sqrt[x]))/(Sqrt[I - a]\*c + Sqrt[b]\*d)])

$b]*d]] + \text{Log}[(c*(\text{Sqrt}[I - a] + \text{Sqrt}[b]*\text{Sqrt}[x]))/(\text{Sqrt}[I - a]*c - \text{Sqrt}[b]*d)] + \text{Log}[d + c*\text{Sqrt}[x]] + \text{PolyLog}[2, (\text{Sqrt}[b]*(d + c*\text{Sqrt}[x]))/(-(\text{Sqrt}[I - a]*c) + \text{Sqrt}[b]*d)] + \text{PolyLog}[2, (\text{Sqrt}[b]*(d + c*\text{Sqrt}[x]))/(\text{Sqrt}[I - a]*c + \text{Sqrt}[b]*d)])))/c^3$

**Maple [C]** time = 0.318, size = 377, normalized size = 0.5

$$\frac{x \arctan (bx + a)}{c} - 2 \frac{\arctan (bx + a) d \sqrt{x}}{c^2} + 2 \frac{\arctan (bx + a) d^2 \ln (d + c \sqrt{x})}{c^3} - \frac{d^2}{c} \quad \text{\_R1=RootOf}(b^2\_Z^4 - 4b^2d\_Z^3 + (2c^2ab + 6b^2d^2 - c^2)\_Z^2 + (2cd - c^2)\_Z + a^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(c+d/x^(1/2)),x)

[Out] arctan(b\*x+a)/c\*x-2\*arctan(b\*x+a)/c^2\*d\*x^(1/2)+2\*arctan(b\*x+a)/c^3\*d^2\*ln(d+c\*x^(1/2))-1/c\*d^2\*sum(1/(\_R1^2\*b-2\*\_R1\*b\*d+a\*c^2+b\*d^2)\*(ln(d+c\*x^(1/2))\*ln((-c\*x^(1/2)+\_R1-d)/\_R1)+dilog((-c\*x^(1/2)+\_R1-d)/\_R1)),\_R1=RootOf(b^2\*\_Z^4-4\*b^2\*d\*\_Z^3+(2\*a\*b\*c^2+6\*b^2\*d^2)\*\_Z^2+(-4\*a\*b\*c^2\*d-4\*b^2\*d^3)\*\_Z+a^2\*c^4+2\*a\*b\*c^2\*d^2+b^2\*d^4+c^4))-1/2/c\*sum((\_R^3-5\*\_R^2\*d+7\*\_R\*d^2-3\*d^3)/(\_R^3\*b-3\*\_R^2\*b\*d+\_R\*a\*c^2+3\*\_R\*b\*d^2-a\*c^2\*d-b\*d^3)\*ln(c\*x^(1/2)-\_R+d),\_R=RootOf(b^2\*\_Z^4-4\*b^2\*d\*\_Z^3+(2\*a\*b\*c^2+6\*b^2\*d^2)\*\_Z^2+(-4\*a\*b\*c^2\*d-4\*b^2\*d^3)\*\_Z+a^2\*c^4+2\*a\*b\*c^2\*d^2+b^2\*d^4+c^4))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan (bx + a)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")

[Out] integrate(arctan(b\*x + a)/(c + d/sqrt(x)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx \arctan (bx + a) - d \sqrt{x} \arctan (bx + a)}{c^2x - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")

[Out] integral((c\*x\*arctan(b\*x + a) - d\*sqrt(x)\*arctan(b\*x + a))/(c^2\*x - d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(c+d/x\*\*(1/2)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(c+d/x^(1/2)),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(c + d/sqrt(x)), x)

### 3.60 $\int \frac{\tan^{-1}(a+bx)}{1+x^2} dx$

**Optimal.** Leaf size=274

$$-\frac{1}{4}\text{PolyLog}\left(2, -\frac{-a-bx+i}{a-i(1-b)}\right) + \frac{1}{4}\text{PolyLog}\left(2, -\frac{-a-bx+i}{a-i(b+1)}\right) - \frac{1}{4}\text{PolyLog}\left(2, \frac{a+bx+i}{a-ib+i}\right) + \frac{1}{4}\text{PolyLog}\left(2, \frac{a+bx+i}{a+i(b+1)}\right)$$

[Out] (Log[(b\*(I - x))/(a + I\*(1 + b))]\*Log[1 - I\*a - I\*b\*x])/4 - (Log[-((b\*(I + x))/(a + I\*(1 - b)))]\*Log[1 - I\*a - I\*b\*x])/4 - (Log[(b\*(I - x))/(a - I\*(1 - b))]\*Log[1 + I\*a + I\*b\*x])/4 + (Log[-((b\*(I + x))/(a - I\*(1 + b)))]\*Log[1 + I\*a + I\*b\*x])/4 - PolyLog[2, -((I - a - b\*x)/(a - I\*(1 - b)))]/4 + PolyLog[2, -((I - a - b\*x)/(a - I\*(1 + b)))]/4 - PolyLog[2, (I + a + b\*x)/(I + a - I\*b)]/4 + PolyLog[2, (I + a + b\*x)/(a + I\*(1 + b))]/4

**Rubi [A]** time = 0.270686, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{1}{4}\text{PolyLog}\left(2, -\frac{-a-bx+i}{a-i(1-b)}\right) + \frac{1}{4}\text{PolyLog}\left(2, -\frac{-a-bx+i}{a-i(b+1)}\right) - \frac{1}{4}\text{PolyLog}\left(2, \frac{a+bx+i}{a-ib+i}\right) + \frac{1}{4}\text{PolyLog}\left(2, \frac{a+bx+i}{a+i(b+1)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b\*x]/(1 + x^2), x]

[Out] (Log[(b\*(I - x))/(a + I\*(1 + b))]\*Log[1 - I\*a - I\*b\*x])/4 - (Log[-((b\*(I + x))/(a + I\*(1 - b)))]\*Log[1 - I\*a - I\*b\*x])/4 - (Log[(b\*(I - x))/(a - I\*(1 - b))]\*Log[1 + I\*a + I\*b\*x])/4 + (Log[-((b\*(I + x))/(a - I\*(1 + b)))]\*Log[1 + I\*a + I\*b\*x])/4 - PolyLog[2, -((I - a - b\*x)/(a - I\*(1 - b)))]/4 + PolyLog[2, -((I - a - b\*x)/(a - I\*(1 + b)))]/4 - PolyLog[2, (I + a + b\*x)/(I + a - I\*b)]/4 + PolyLog[2, (I + a + b\*x)/(a + I\*(1 + b))]/4

#### Rule 5051

Int[ArcTan[(a\_) + (b\_)\*(x\_)]/((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Dist[I/2, Int[Log[1 - I\*a - I\*b\*x]/(c + d\*x^n), x], x] - Dist[I/2, Int[Log[1 + I\*a + I\*b\*x]/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]

#### Rule 2409

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_)^(r\_))^(q\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IntegerQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))

#### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_)))/((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))]\*(b\_)))/((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x]



$$\begin{aligned} & )/b-a/b)^{2+1}/(-I*b-I-a))*\arctan((b*x+a)/b-a/b)+1/2/(I*b+I+a)*\ln(1-(I+a-I*b) \\ & )*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^{2+1}/(-I*b-I-a))*\arctan((b*x+a)/ \\ & b-a/b)-1/2*I/(I*b+I+a)*\ln(1-(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a \\ & /b)^{2+1}/(-I*b-I-a))*\arctan((b*x+a)/b-a/b)*a-1/2*I*b/(I*b+I+a)*\arctan((b*x+ \\ & a)/b-a/b)^2-1/4*I*b/(I*b+I+a)*\text{polylog}(2,(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/( \\ & ((b*x+a)/b-a/b)^{2+1}/(-I*b-I-a))-1/2*I/(I*b+I+a)*\arctan((b*x+a)/b-a/b)^2-1/ \\ & 2/(I*b+I+a)*\arctan((b*x+a)/b-a/b)^2*a-1/4*I/(I*b+I+a)*\text{polylog}(2,(I+a-I*b)*( \\ & 1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^{2+1}/(-I*b-I-a))-1/4/(I*b+I+a)*\text{poly} \\ & \log(2,(I+a-I*b)*(1+I*((b*x+a)/b-a/b))^2/(((b*x+a)/b-a/b)^{2+1}/(-I*b-I-a))*a \end{aligned}$$

**Maxima [A]** time = 1.8351, size = 443, normalized size = 1.62

$$\frac{1}{8}b \left( \frac{8 \arctan(x) \arctan\left(\frac{b^2x+ab}{b}\right)}{b} - \frac{4 \arctan(x) \arctan\left(\frac{ab+(b^2+b)x}{a^2+b^2+2b+1}, \frac{abx+a^2+b+1}{a^2+b^2+2b+1}\right) - 4 \arctan(x) \arctan\left(\frac{ab+(b^2-b)x}{a^2+b^2-2b+1}, \frac{abx+a^2+b+1}{a^2+b^2-2b+1}\right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(x^2+1),x, algorithm="maxima")

[Out]  $\frac{1}{8}b*(8*\arctan(x)*\arctan((b^2*x + a*b)/b)/b - (4*\arctan(x)*\arctan2((a*b + (b^2 + b)*x)/(a^2 + b^2 + 2*b + 1), (a*b*x + a^2 + b + 1)/(a^2 + b^2 + 2*b + 1)) - 4*\arctan(x)*\arctan2((a*b + (b^2 - b)*x)/(a^2 + b^2 - 2*b + 1), (a*b*x + a^2 - b + 1)/(a^2 + b^2 - 2*b + 1)) + \log(x^2 + 1)*\log((b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^2 + b^2 + 2*b + 1)) - \log(x^2 + 1)*\log((b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^2 + b^2 - 2*b + 1)) + 2*\text{dilog}(-I*b*x - b)/(I*a + b + 1)) - 2*\text{dilog}(-I*b*x - b)/(I*a + b - 1)) + 2*\text{dilog}((I*b*x + b)/(-I*a + b + 1)) - 2*\text{dilog}((I*b*x + b)/(-I*a + b - 1)))/b + \arctan(b*x + a)*\arctan(x) - \arctan(x)*\arctan((b^2*x + a*b)/b)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx+a)}{x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(x^2+1),x, algorithm="fricas")

[Out] integral(arctan(b\*x + a)/(x^2 + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atan}(a + bx)}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(x\*\*2+1),x)

[Out] Integral(atan(a + b\*x)/(x\*\*2 + 1), x)



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(x^2+1),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/(x^2 + 1), x)

### 3.61 $\int \frac{\tan^{-1}(d+ex)}{a+bx^2} dx$

**Optimal.** Leaf size=543

$$-\frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(-d-ex+i)}}{-\sqrt{-ae+\sqrt{b}(-d+i)}}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(-d-ex+i)}}{\sqrt{-ae+\sqrt{b}(-d+i)}}\right)}{4\sqrt{-a}\sqrt{b}} - \frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(d+ex+i)}}{-\sqrt{-ae+\sqrt{b}(d+i)}}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(d+ex+i)}}{\sqrt{-ae+\sqrt{b}(d+i)}}\right)}{4\sqrt{-a}\sqrt{b}}$$

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b])
```

**Rubi [A]** time = 0.600126, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5051, 2409, 2394, 2393, 2391}

$$-\frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(-d-ex+i)}}{-\sqrt{-ae+\sqrt{b}(-d+i)}}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(-d-ex+i)}}{\sqrt{-ae+\sqrt{b}(-d+i)}}\right)}{4\sqrt{-a}\sqrt{b}} - \frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(d+ex+i)}}{-\sqrt{-ae+\sqrt{b}(d+i)}}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\text{PolyLog}\left(2, \frac{\sqrt{b(d+ex+i)}}{\sqrt{-ae+\sqrt{b}(d+i)}}\right)}{4\sqrt{-a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x^2), x]
```

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b])
```

#### Rule 5051

```
Int[ArcTan[(a_) + (b_)*(x_)]/((c_) + (d_)*(x_)^(n_.)), x_Symbol] :> Dist[I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

#### Rule 2409

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

$\wedge n]^p, (f + g*x^r)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, r\}, x] \&\& \text{IntegerQ}[p, 0] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \mid\mid (\text{IntegerQ}[r] \&\& \text{NeQ}[r, 1]))$

#### Rule 2394

$\text{Int}[(a + \text{Log}[(c + (d + (e * (x)^n)] * b)) / ((f + (g * (x)))], x\_Symbol] :> \text{Simp}[(\text{Log}[(e * (f + g * x)) / (e * f - d * g)] * (a + b * \text{Log}[c * (d + e * x)^n]) / g, x] - \text{Dist}[(b * e * n) / g, \text{Int}[\text{Log}[(e * (f + g * x)) / (e * f - d * g)] / (d + e * x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e * f - d * g, 0]$

#### Rule 2393

$\text{Int}[(a + \text{Log}[(c + (d + (e * (x))) * b]) / ((f + (g * (x)))], x\_Symbol] :> \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b * \text{Log}[1 + (c * e * x) / g]) / x, x], x, f + g * x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e * f - d * g, 0] \&\& \text{EqQ}[g + c * (e * f - d * g), 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c + (d + (e * (x)^n))] / (x)], x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, -(c * e * x^n)] / n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c * d, 1]$

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^{-1}(d+ex)}{a+bx^2} dx &= \frac{1}{2}i \int \frac{\log(1-id-iox)}{a+bx^2} dx - \frac{1}{2}i \int \frac{\log(1+id+iox)}{a+bx^2} dx \\ &= \frac{1}{2}i \int \left( \frac{\sqrt{-a} \log(1-id-iox)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \log(1-id-iox)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx - \frac{1}{2}i \int \left( \frac{\sqrt{-a} \log(1+id+iox)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \log(1+id+iox)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx \\ &= -\frac{i \int \frac{\log(1-id-iox)}{\sqrt{-a}-\sqrt{bx}} dx}{4\sqrt{-a}} - \frac{i \int \frac{\log(1-id-iox)}{\sqrt{-a}+\sqrt{bx}} dx}{4\sqrt{-a}} + \frac{i \int \frac{\log(1+id+iox)}{\sqrt{-a}-\sqrt{bx}} dx}{4\sqrt{-a}} + \frac{i \int \frac{\log(1+id+iox)}{\sqrt{-a}+\sqrt{bx}} dx}{4\sqrt{-a}} \\ &= \frac{i \log\left(\frac{e(\sqrt{-a}-\sqrt{bx})}{\sqrt{b}(i+d)+\sqrt{-ae}}\right) \log(1-id-iox)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log\left(-\frac{e(\sqrt{-a}+\sqrt{bx})}{\sqrt{b}(i+d)-\sqrt{-ae}}\right) \log(1-id-iox)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log\left(-\frac{e(\sqrt{-a}-\sqrt{bx})}{\sqrt{b}(i-d)-\sqrt{-ae}}\right) \log(1+id+iox)}{4\sqrt{-a}\sqrt{b}} \\ &= \frac{i \log\left(\frac{e(\sqrt{-a}-\sqrt{bx})}{\sqrt{b}(i+d)+\sqrt{-ae}}\right) \log(1-id-iox)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log\left(-\frac{e(\sqrt{-a}+\sqrt{bx})}{\sqrt{b}(i+d)-\sqrt{-ae}}\right) \log(1-id-iox)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log\left(-\frac{e(\sqrt{-a}-\sqrt{bx})}{\sqrt{b}(i-d)-\sqrt{-ae}}\right) \log(1+id+iox)}{4\sqrt{-a}\sqrt{b}} \\ &= \frac{i \log\left(\frac{e(\sqrt{-a}-\sqrt{bx})}{\sqrt{b}(i+d)+\sqrt{-ae}}\right) \log(1-id-iox)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log\left(-\frac{e(\sqrt{-a}+\sqrt{bx})}{\sqrt{b}(i+d)-\sqrt{-ae}}\right) \log(1-id-iox)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log\left(-\frac{e(\sqrt{-a}-\sqrt{bx})}{\sqrt{b}(i-d)-\sqrt{-ae}}\right) \log(1+id+iox)}{4\sqrt{-a}\sqrt{b}} \end{aligned}$$

**Mathematica [A]** time = 0.354393, size = 409, normalized size = 0.75

$$i \left( \text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex-i)}{-\sqrt{-ae}+\sqrt{b}(d-i)}\right) - \text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex-i)}{\sqrt{-ae}+\sqrt{b}(d-i)}\right) - \text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex+i)}{-\sqrt{-ae}+\sqrt{b}(d+i)}\right) + \text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex+i)}{\sqrt{-ae}+\sqrt{b}(d+i)}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[d + e\*x]/(a + b\*x^2), x]

[Out] ((I/4)\*(-(Log[(e\*(Sqrt[-a] - Sqrt[b]\*x))/(Sqrt[b]\*(-I + d) + Sqrt[-a]\*e)]\*Log[1 + I\*d + I\*e\*x]) + Log[(e\*(Sqrt[-a] + Sqrt[b]\*x)/(-Sqrt[b]\*(-I + d))



$$-b*d^2-2*(a*b*e^2)^{(1/2)-b))*d^2$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(e\*x+d)/(b\*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ex+d)}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(e\*x+d)/(b\*x^2+a),x, algorithm="fricas")

[Out] integral(arctan(e\*x + d)/(b\*x^2 + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(e\*x+d)/(b\*x\*\*2+a),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ex+d)}{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(e\*x+d)/(b\*x^2+a),x, algorithm="giac")

[Out] integrate(arctan(e\*x + d)/(b\*x^2 + a), x)

### 3.62 $\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2} dx$

**Optimal.** Leaf size=367

$$\frac{i\text{PolyLog}\left(2, 1 + \frac{2(-e(b-\sqrt{b^2-4ac})-2c(d+ex)+2cd)}{(1-i(d+ex))(-e\sqrt{b^2-4ac}+be-2cd+2ic)}\right)}{2\sqrt{b^2-4ac}} + \frac{i\text{PolyLog}\left(2, 1 + \frac{2(-e(\sqrt{b^2-4ac}+b)-2c(d+ex)+2cd)}{(1-i(d+ex))(e(\sqrt{b^2-4ac}+b)+2c(-d+i))}\right)}{2\sqrt{b^2-4ac}} + \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}$$

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c] - (ArcTan[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c] + ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c])
```

**Rubi [A]** time = 0.672732, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {618, 206, 6728, 5047, 4856, 2402, 2315, 2447}

$$\frac{i\text{PolyLog}\left(2, 1 + \frac{2(-e(b-\sqrt{b^2-4ac})-2c(d+ex)+2cd)}{(1-i(d+ex))(-e\sqrt{b^2-4ac}+be-2cd+2ic)}\right)}{2\sqrt{b^2-4ac}} + \frac{i\text{PolyLog}\left(2, 1 + \frac{2(-e(\sqrt{b^2-4ac}+b)-2c(d+ex)+2cd)}{(1-i(d+ex))(e(\sqrt{b^2-4ac}+b)+2c(-d+i))}\right)}{2\sqrt{b^2-4ac}} + \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x + c*x^2), x]
```

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c] - (ArcTan[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c] + ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))]/Sqrt[b^2 - 4*a*c])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

#### Rule 5047

```
Int[((a_) + ArcTan[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(m
_), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

#### Rule 4856

```
Int[((a_) + ArcTan[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

#### Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

#### Rule 2315

```
Int[Log[(c_)*(x_)/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

#### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2} dx &= \int \left( \frac{2c \tan^{-1}(d+ex)}{\sqrt{b^2-4ac}(b-\sqrt{b^2-4ac}+2cx)} - \frac{2c \tan^{-1}(d+ex)}{\sqrt{b^2-4ac}(b+\sqrt{b^2-4ac}+2cx)} \right) dx \\
&= \frac{(2c) \int \frac{\tan^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{\tan^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} \\
&= \frac{(2c) \text{Subst} \left( \int \frac{\tan^{-1}(x)}{\frac{-2cd+(b-\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac}} - \frac{(2c) \text{Subst} \left( \int \frac{\tan^{-1}(x)}{\frac{-2cd+(b+\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac}} \\
&= \frac{\tan^{-1}(d+ex) \log \left( \frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2ic-2cd+be-\sqrt{b^2-4ac})(1-i(d+ex))} \right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex) \log \left( \frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(i-d)+(b+\sqrt{b^2-4ac})e)(1-i(d+ex))} \right)}{\sqrt{b^2-4ac}} \\
&= \frac{\tan^{-1}(d+ex) \log \left( \frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2ic-2cd+be-\sqrt{b^2-4ac})(1-i(d+ex))} \right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex) \log \left( \frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(i-d)+(b+\sqrt{b^2-4ac})e)(1-i(d+ex))} \right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

**Mathematica [A]** time = 0.412683, size = 443, normalized size = 1.21

$$i \left( -\text{PolyLog} \left( 2, \frac{2c(d+ex-i)}{e(\sqrt{b^2-4ac}-b)+2c(d-i)} \right) + \text{PolyLog} \left( 2, \frac{2c(d+ex-i)}{-e(\sqrt{b^2-4ac}+b)+2c(d-i)} \right) + \text{PolyLog} \left( 2, \frac{2c(d+ex+i)}{e(\sqrt{b^2-4ac}-b)+2c(d+i)} \right) - \text{PolyLog} \left( 2, \frac{2c(d+ex+i)}{-e(\sqrt{b^2-4ac}+b)+2c(d+i)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[d + e\*x]/(a + b\*x + c\*x^2), x]

[Out] ((I/2)\*(Log[(e\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x))/(2\*c\*(I + d) + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]\*Log[1 - I\*(d + e\*x)] - Log[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(-2\*c\*(I + d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*Log[1 - I\*(d + e\*x)] - Log[(e\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x))/(2\*c\*(-I + d) + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]\*Log[1 + I\*(d + e\*x)] + Log[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(-2\*c\*(-I + d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*Log[1 + I\*(d + e\*x)] - PolyLog[2, (2\*c\*(-I + d + e\*x))/(2\*c\*(-I + d) + (-b + Sqrt[b^2 - 4\*a\*c])\*e)] + PolyLog[2, (2\*c\*(-I + d + e\*x))/(2\*c\*(-I + d) - (b + Sqrt[b^2 - 4\*a\*c])\*e)] + PolyLog[2, (2\*c\*(I + d + e\*x))/(2\*c\*(I + d) + (-b + Sqrt[b^2 - 4\*a\*c])\*e)] - PolyLog[2, (2\*c\*(I + d + e\*x))/(2\*c\*(I + d) - (b + Sqrt[b^2 - 4\*a\*c])\*e)]))/Sqrt[b^2 - 4\*a\*c]

**Maple [B]** time = 1.036, size = 4743, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.





```

*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c
-b^2))^(1/2)-c))*arctan(e*x+d)*b^2-1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b
^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c
+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^
2*(4*a*c-b^2))^(1/2)-c))*b^2-1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a
*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*b^2+1/8*e*(e^2*
(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)
+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)
^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*b^2+1/4*e*(e^2*(4*a*c
-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ar
ctan(e*x+d)^2*b^2+1/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d
^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2*d^2-1/e*(e^2*(4*a*c-b^2))^(1
/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+
d)^2*d^2+1/2/e*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^
2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+c*d^2-c)*(1+I
*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*d
^2+I*e/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*d*c+a
*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*
(4*a*c-b^2))^(1/2)-c))*arctan(e*x+d)+1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2
-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e
*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b
^2))^(1/2)-c))-1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(e^2*(4*a
*c-b^2))^(1/2)+c)*arctan(e*x+d)^2-1/8*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*
e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*d*c+a*e^2-b*e*d+
c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2)
)^(1/2)-c))+1/4*e*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-
b^2))^(1/2)+c)*arctan(e*x+d)^2

```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(ex+d)}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(e*x + d)/(c*x^2 + b*x + a), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(c*x**2+b*x+a), x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ex + d)}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a), x, algorithm="giac")
```

```
[Out] integrate(arctan(e*x + d)/(c*x^2 + b*x + a), x)
```

$$3.63 \quad \int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} dx$$

**Optimal.** Leaf size=132

$$\frac{i\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{2i \tan^{-1}(a+bx) \tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

[Out]  $((-2*I)*\text{ArcTan}[a + b*x]*\text{ArcTan}[\text{Sqrt}[1 + I*(a + b*x)]/\text{Sqrt}[1 - I*(a + b*x]])/b + (I*\text{PolyLog}[2, ((-I)*\text{Sqrt}[1 + I*(a + b*x)])/\text{Sqrt}[1 - I*(a + b*x)])]/b - (I*\text{PolyLog}[2, (I*\text{Sqrt}[1 + I*(a + b*x)])/\text{Sqrt}[1 - I*(a + b*x)])]/b$

**Rubi [A]** time = 0.0962101, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5055, 4886}

$$\frac{i\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{2i \tan^{-1}(a+bx) \tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcTan}[a + b*x]/\text{Sqrt}[1 + a^2 + 2*a*b*x + b^2*x^2], x]$

[Out]  $((-2*I)*\text{ArcTan}[a + b*x]*\text{ArcTan}[\text{Sqrt}[1 + I*(a + b*x)]/\text{Sqrt}[1 - I*(a + b*x]])/b + (I*\text{PolyLog}[2, ((-I)*\text{Sqrt}[1 + I*(a + b*x)])/\text{Sqrt}[1 - I*(a + b*x)])]/b - (I*\text{PolyLog}[2, (I*\text{Sqrt}[1 + I*(a + b*x)])/\text{Sqrt}[1 - I*(a + b*x)])]/b$

#### Rule 5055

$\text{Int}[(a_.) + \text{ArcTan}[(c_.) + (d_.)*(x_.)]*(b_.)]^{(p_.)} * ((A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)^{(q_.)}, x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(C/d^2 + (C*x^2)/d^2)^q * (a + b*\text{ArcTan}[x])^p, x], x, c + d*x], x] /;$   $\text{FreeQ}\{a, b, c, d, A, B, C, p, q\}, x$  &&  $\text{EqQ}[B*(1 + c^2) - 2*A*c*d, 0]$  &&  $\text{EqQ}[2*c*C - B*d, 0]$

#### Rule 4886

$\text{Int}[(a_.) + \text{ArcTan}[(c_.)*(x_.)]*(b_.)]/\text{Sqrt}[(d_.) + (e_.)*(x_.)^2], x\_Symbol] \rightarrow \text{Simp}[(-2*I*(a + b*\text{ArcTan}[c*x])*\text{ArcTan}[\text{Sqrt}[1 + I*c*x]/\text{Sqrt}[1 - I*c*x]])/(c*\text{Sqrt}[d]), x] + (\text{Simp}[(I*b*\text{PolyLog}[2, -((I*\text{Sqrt}[1 + I*c*x])/ \text{Sqrt}[1 - I*c*x])])]/(c*\text{Sqrt}[d]), x) - \text{Simp}[(I*b*\text{PolyLog}[2, (I*\text{Sqrt}[1 + I*c*x])/ \text{Sqrt}[1 - I*c*x])]/(c*\text{Sqrt}[d]), x) /;$   $\text{FreeQ}\{a, b, c, d, e\}, x$  &&  $\text{EqQ}[e, c^2*d]$  &&  $\text{GtQ}[d, 0]$

#### Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}} dx, x, a+bx\right)}{b} = -\frac{2i \tan^{-1}(a+bx) \tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\text{Li}_2\left(-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\text{Li}_2\left(\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

**Mathematica [A]** time = 0.101348, size = 97, normalized size = 0.73

$$\frac{i\text{PolyLog}\left(2, -ie^{i\tan^{-1}(a+bx)}\right) - i\text{PolyLog}\left(2, ie^{i\tan^{-1}(a+bx)}\right) + \tan^{-1}(a+bx)\left(\log\left(1 - ie^{i\tan^{-1}(a+bx)}\right) - \log\left(1 + ie^{i\tan^{-1}(a+bx)}\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/Sqrt[1 + a^2 + 2\*a\*b\*x + b^2\*x^2], x]

[Out] (ArcTan[a + b\*x]\*(Log[1 - I\*E^(I\*ArcTan[a + b\*x])]) - Log[1 + I\*E^(I\*ArcTan[a + b\*x])]) + I\*PolyLog[2, (-I)\*E^(I\*ArcTan[a + b\*x])] - I\*PolyLog[2, I\*E^(I\*ArcTan[a + b\*x])])/b

**Maple [A]** time = 0.397, size = 143, normalized size = 1.1

$$-\frac{\arctan(bx+a)}{b} \ln\left(1 + i(1 + i(bx+a)) \frac{1}{\sqrt{1+(bx+a)^2}}\right) + \frac{\arctan(bx+a)}{b} \ln\left(1 - i(1 + i(bx+a)) \frac{1}{\sqrt{1+(bx+a)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/(b^2\*x^2+2\*a\*b\*x+a^2+1)^(1/2), x)

[Out] -1/b\*arctan(b\*x+a)\*ln(1+I\*(1+I\*(b\*x+a))/(1+(b\*x+a)^2)^(1/2))+1/b\*arctan(b\*x+a)\*ln(1-I\*(1+I\*(b\*x+a))/(1+(b\*x+a)^2)^(1/2))+I/b\*dilog(1+I\*(1+I\*(b\*x+a))/(1+(b\*x+a)^2)^(1/2))-I/b\*dilog(1-I\*(1+I\*(b\*x+a))/(1+(b\*x+a)^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(b^2\*x^2+2\*a\*b\*x+a^2+1)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx+a)}{\sqrt{b^2x^2+2abx+a^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(b^2\*x^2+2\*a\*b\*x+a^2+1)^(1/2), x, algorithm="fricas")

[Out] integral(arctan(b\*x + a)/sqrt(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}(a + bx)}{\sqrt{a^2 + 2abx + b^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/(b\*\*2\*x\*\*2+2\*a\*b\*x+a\*\*2+1)\*\*(1/2),x)

[Out] Integral(atan(a + b\*x)/sqrt(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{\sqrt{b^2x^2 + 2abx + a^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/(b^2\*x^2+2\*a\*b\*x+a^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/sqrt(b^2\*x^2 + 2\*a\*b\*x + a^2 + 1), x)

$$3.64 \quad \int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} dx$$

**Optimal.** Leaf size=216

$$\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} - \frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} - \frac{2i\sqrt{(a+bx)^2+1}\tan^{-1}(a+bx)}{b\sqrt{c(a+bx)^2}}$$

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
+ b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt[c + c*(a + b*x)^2])
```

**Rubi [A]** time = 0.162288, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {5055, 4890, 4886}

$$\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} - \frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} - \frac{2i\sqrt{(a+bx)^2+1}\tan^{-1}(a+bx)}{b\sqrt{c(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2], x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
+ b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt[c + c*(a + b*x)^2])
```

#### Rule 5055

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_.) + (
C_.)*(x_)^2)^(q_.), x_Symbol] := Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

#### Rule 4890

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/Sqrt[(d_.) + (e_.)*(x_)^2], x_S
ymbol] := Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
IGtQ[p, 0] && !GtQ[d, 0]
```

#### Rule 4886

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -((I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c
*x])])]/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 -
I*c*x]])]/(c*Sqrt[d]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{c+cx^2}} dx, x, a+bx\right)}{b}$$

$$= \frac{\sqrt{1+(a+bx)^2} \text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}} dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}}$$

$$= -\frac{2i\sqrt{1+(a+bx)^2} \tan^{-1}(a+bx) \tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2} \text{Li}_2\left(-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

**Mathematica [A]** time = 0.0636544, size = 125, normalized size = 0.58

$$\frac{\sqrt{(a+bx)^2+1} \left( i \text{PolyLog}\left(2, -ie^{i \tan^{-1}(a+bx)}\right) - i \text{PolyLog}\left(2, ie^{i \tan^{-1}(a+bx)}\right) + \tan^{-1}(a+bx) \left( \log\left(1 - ie^{i \tan^{-1}(a+bx)}\right) - \log\left(1 + ie^{i \tan^{-1}(a+bx)}\right) \right) \right)}{b\sqrt{c((a+bx)^2+1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/Sqrt[(1 + a^2)\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2], x]

[Out] (Sqrt[1 + (a + b\*x)^2]\*(ArcTan[a + b\*x]\*(Log[1 - I\*E^(I\*ArcTan[a + b\*x])] - Log[1 + I\*E^(I\*ArcTan[a + b\*x])]) + I\*PolyLog[2, (-I)\*E^(I\*ArcTan[a + b\*x]]) - I\*PolyLog[2, I\*E^(I\*ArcTan[a + b\*x])]))/(b\*Sqrt[c\*(1 + (a + b\*x)^2)])

**Maple [A]** time = 0.439, size = 176, normalized size = 0.8

$$\frac{i}{bc} \left( i \arctan(bx+a) \ln \left( 1 + i(1+i(bx+a)) \frac{1}{\sqrt{1+(bx+a)^2}} \right) - i \arctan(bx+a) \ln \left( 1 - i(1+i(bx+a)) \frac{1}{\sqrt{1+(bx+a)^2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+b^2\*c\*x^2)^(1/2), x)

[Out] I\*(I\*arctan(b\*x+a)\*ln(1+I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))-I\*arctan(b\*x+a)\*ln(1-I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))+dilog(1+I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))-dilog(1-I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))\*((c\*(-I+a+b\*x))\*(I+a+b\*x))^(1/2)/(b^2\*x^2+2\*a\*b\*x+a^2+1)^(1/2)/b/c

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/2), x, algorithm="maxima")



[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\arctan(bx + a)}{\sqrt{b^2cx^2 + 2abcx + (a^2 + 1)c}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/2),x, algorithm="fricas")

[Out] integral(arctan(b\*x + a)/sqrt(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atan}(a + bx)}{\sqrt{c(a^2 + 2abx + b^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(b\*x+a)/((a\*\*2+1)\*c+2\*a\*b\*c\*x+c\*x\*\*2\*b\*\*2)\*\*(1/2),x)

[Out] Integral(atan(a + b\*x)/sqrt(c\*(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2 + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{\sqrt{b^2cx^2 + 2abcx + (a^2 + 1)c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/2),x, algorithm="giac")

[Out] integrate(arctan(b\*x + a)/sqrt(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c), x)

$$3.65 \quad \int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{\tan^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}, x\right)$$

[Out] Unintegrable[ArcTan[a + b\*x]/(1 + (a + b\*x)^2)^(1/3), x]

**Rubi [A]** time = 0.0379871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[a + b\*x]/(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3), x]

[Out] Defer[Subst][Defer[Int][ArcTan[x]/(1 + x^2)^(1/3), x], x, a + b\*x]/b

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{1+x^2}} dx, x, a+bx\right)}{b}$$

**Mathematica [A]** time = 0.363002, size = 163, normalized size = 7.41

$$\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1, \frac{4}{3}, \frac{4}{3}\right\}, \left\{\frac{11}{6}, \frac{7}{3}\right\}, \frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1} + 6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\tan^{-1}(a+bx)\text{Hypergeometric2F1}\left(1, \frac{4}{3}, \frac{11}{6}, (1+(a+bx)^2)^{-1}\right)}{(a+bx)^2+1}\right)}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{a^2+2abx+b^2x^2+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3), x]

[Out] (6\*Gamma[11/6]\*Gamma[7/3]\*(15 + 10\*(a + b\*x)\*ArcTan[a + b\*x] + (4\*(a + b\*x)\*ArcTan[a + b\*x]\*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b\*x)^2)^(-1)])/(1 + (a + b\*x)^2)) + (5\*2^(1/3)\*Sqrt[Pi]\*Gamma[5/3]\*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b\*x)^2)^(-1)]/(1 + (a + b\*x)^2))/(20\*b\*(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3)\*Gamma[11/6]\*Gamma[7/3])

**Maple [A]** time = 1.044, size = 0, normalized size = 0.

$$\int \arctan(bx+a) \frac{1}{\sqrt[3]{b^2x^2+2xab+a^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)`

[Out] `int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima")`

[Out] `integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas")`

[Out] `integral(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atan}(a + bx)}{\sqrt[3]{a^2 + 2abx + b^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)`

[Out] `Integral(atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

$$3.66 \quad \int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$$

**Optimal.** Leaf size=24

$$\text{Unintegrable}\left(\frac{\tan^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}, x\right)$$

[Out] Unintegrable[ArcTan[a + b\*x]/(c + c\*(a + b\*x)^2)^(1/3), x]

**Rubi [A]** time = 0.0505649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[a + b\*x]/((1 + a^2)\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2)^(1/3), x]

[Out] Defer[Subst][Defer[Int][ArcTan[x]/(c + c\*x^2)^(1/3), x], x, a + b\*x]/b

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{c+cx^2}} dx, x, a+bx\right)}{b}$$

**Mathematica [A]** time = 0.0728054, size = 165, normalized size = 6.88

$$\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1, \frac{4}{3}, \frac{4}{3}\right\}, \left\{\frac{11}{6}, \frac{7}{3}\right\}, \frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1} + 6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\tan^{-1}(a+bx)\text{HypergeometricPFQ}\left(\left\{1, \frac{4}{3}, \frac{4}{3}\right\}, \left\{\frac{11}{6}, \frac{7}{3}\right\}, \frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}\right)}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{c(a^2+2abx+b^2x^2+1)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTan[a + b\*x]/((1 + a^2)\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2)^(1/3), x]

[Out] (6\*Gamma[11/6]\*Gamma[7/3]\*(15 + 10\*(a + b\*x)\*ArcTan[a + b\*x] + (4\*(a + b\*x)\*ArcTan[a + b\*x]\*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b\*x)^2)^(-1)]))/(1 + (a + b\*x)^2) + (5\*2^(1/3)\*Sqrt[Pi]\*Gamma[5/3]\*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b\*x)^2)^(-1)]/(1 + (a + b\*x)^2))/(20\*b\*(c\*(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3)\*Gamma[11/6]\*Gamma[7/3])

**Maple [A]** time = 0.996, size = 0, normalized size = 0.

$$\int \arctan(bx+a) \frac{1}{\sqrt[3]{(a^2+1)c+2abcx+b^2cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)`

[Out] `int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{(b^2cx^2 + 2abcx + (a^2 + 1)c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm="maxima")`

[Out] `integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\arctan(bx + a)}{(b^2cx^2 + 2abcx + (a^2 + 1)c)^{\frac{1}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm="fricas")`

[Out] `integral(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atan}(a + bx)}{\sqrt[3]{c(a^2 + 2abx + b^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+c*x**2*b**2)**(1/3),x)`

[Out] `Integral(atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(bx + a)}{(b^2cx^2 + 2abcx + (a^2 + 1)c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+c*x^2*b^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

$$3.67 \quad \int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} dx$$

**Optimal.** Leaf size=187

$$\frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{1+(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} + \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{1+(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{\sqrt{(a+bx)^2+1}}{2b} + \frac{i \tan^{-1}\left(\frac{\sqrt{1+(a+bx)}}{\sqrt{1-i(a+bx)}}\right) \tan^{-1}(a+bx)}{b} + \frac{(a+bx)}{b}$$

[Out]  $-\operatorname{Sqrt}[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*\operatorname{Sqrt}[1 + (a + b*x)^2]*\operatorname{ArcTan}[a + b*x])/(2*b) + (I*\operatorname{ArcTan}[a + b*x]*\operatorname{ArcTan}[\operatorname{Sqrt}[1 + I*(a + b*x)]/\operatorname{Sqrt}[1 - I*(a + b*x)])]/b - ((I/2)*\operatorname{PolyLog}[2, ((-I)*\operatorname{Sqrt}[1 + I*(a + b*x)])/\operatorname{Sqrt}[1 - I*(a + b*x)])]/b + ((I/2)*\operatorname{PolyLog}[2, (I*\operatorname{Sqrt}[1 + I*(a + b*x)])/\operatorname{Sqrt}[1 - I*(a + b*x)])]/b$

**Rubi [A]** time = 0.215784, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {5057, 4952, 261, 4886}

$$\frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{1+(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} + \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{1+(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{\sqrt{(a+bx)^2+1}}{2b} + \frac{i \tan^{-1}\left(\frac{\sqrt{1+(a+bx)}}{\sqrt{1-i(a+bx)}}\right) \tan^{-1}(a+bx)}{b} + \frac{(a+bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*x)^2*\operatorname{ArcTan}[a + b*x]/\operatorname{Sqrt}[1 + a^2 + 2*a*b*x + b^2*x^2], x]$

[Out]  $-\operatorname{Sqrt}[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*\operatorname{Sqrt}[1 + (a + b*x)^2]*\operatorname{ArcTan}[a + b*x])/(2*b) + (I*\operatorname{ArcTan}[a + b*x]*\operatorname{ArcTan}[\operatorname{Sqrt}[1 + I*(a + b*x)]/\operatorname{Sqrt}[1 - I*(a + b*x)])]/b - ((I/2)*\operatorname{PolyLog}[2, ((-I)*\operatorname{Sqrt}[1 + I*(a + b*x)])/\operatorname{Sqrt}[1 - I*(a + b*x)])]/b + ((I/2)*\operatorname{PolyLog}[2, (I*\operatorname{Sqrt}[1 + I*(a + b*x)])/\operatorname{Sqrt}[1 - I*(a + b*x)])]/b$

#### Rule 5057

$\operatorname{Int}[(a + \operatorname{ArcTan}[c + (d*x)]*(b))^p*(e + (f*x))^m, x] \rightarrow \operatorname{Dist}[1/d, \operatorname{Subst}[\operatorname{Int}[(d*e - c*f)/d + (f*x)/d]^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*\operatorname{ArcTan}[x])^p, x], x, c + d*x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, A, B, C, m, p, q\}, x$  &&  $\operatorname{EqQ}[B*(1 + c^2) - 2*A*c*d, 0]$  &&  $\operatorname{EqQ}[2*c*C - B*d, 0]$

#### Rule 4952

$\operatorname{Int}[(a + \operatorname{ArcTan}[c*x])^p*(e + (f*x))^m/\operatorname{Sqrt}[d + (e*x)^2], x] \rightarrow \operatorname{Simp}[(f*(f*x))^{m-1}*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcTan}[c*x])^p]/(c^2*d*m), x] + (-\operatorname{Dist}[(b*f*p)/(c*m), \operatorname{Int}[(f*x)^{m-1}*(a + b*\operatorname{ArcTan}[c*x])^{p-1}]/\operatorname{Sqrt}[d + e*x^2], x], x] - \operatorname{Dist}[(f^2*(m-1))/(c^2*m), \operatorname{Int}[(f*x)^{m-2}*(a + b*\operatorname{ArcTan}[c*x])^p]/\operatorname{Sqrt}[d + e*x^2], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x$  &&  $\operatorname{EqQ}[e, c^2*d]$  &&  $\operatorname{GtQ}[p, 0]$  &&  $\operatorname{GtQ}[m, 1]$

#### Rule 261

$\operatorname{Int}[(x)^m*(a + (b*x)^n)^p, x] \rightarrow \operatorname{Simp}[(a + b*x^n)^{p+1}/(b*n*(p+1)), x] /;$   $\operatorname{FreeQ}\{a, b, m, n, p\}, x$  &&  $\operatorname{EqQ}[m, n-1]$  &&  $\operatorname{NeQ}[p, -1]$

#### Rule 4886



```
Int[(a_.) + ArcTan[(c_.)*(x_)]*(b_.)]/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -(I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c*x]])
/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

### Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} dx = \frac{\text{Subst}\left(\int \frac{x^2 \tan^{-1}(x)}{\sqrt{1+x^2}} dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2} \tan^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}} dx, x, a+bx\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, a+bx\right)}{2b}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2} \tan^{-1}(a+bx)}{2b} + \frac{i \tan^{-1}(a+bx) \tan^{-1}\left(\frac{\sqrt{1+(a+bx)^2}}{a+bx}\right)}{b}$$

**Mathematica [A]** time = 0.657008, size = 145, normalized size = 0.78

$$\frac{-i \text{PolyLog}\left(2, -ie^{i \tan^{-1}(a+bx)}\right) + i \text{PolyLog}\left(2, ie^{i \tan^{-1}(a+bx)}\right) - \sqrt{(a+bx)^2+1} + (a+bx)\sqrt{(a+bx)^2+1} \tan^{-1}(a+bx)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2], x]
```

```
[Out] (-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x] -
ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])] + ArcTan[a + b*x]*Log[1 +
I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I*Po
lyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b)
```

**Maple [A]** time = 0.619, size = 187, normalized size = 1.

$$\frac{\arctan(bx+a)xb + \arctan(bx+a)a - 1}{2b} \sqrt{b^2x^2 + 2xab + a^2 + 1} + \frac{\arctan(bx+a)}{2b} \ln\left(1 + i(1 + i(bx+a)) \frac{1}{\sqrt{1+(bx+a)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2), x)
```

```
[Out] 1/2*(arctan(b*x+a)*x*b+arctan(b*x+a)*a-1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b+1
/2/b*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a)))/(1+(b*x+a)^2)^(1/2))-1/2/b*arctan(b
*x+a)*ln(1-I*(1+I*(b*x+a)))/(1+(b*x+a)^2)^(1/2))-1/2*I/b*dilog(1+I*(1+I*(b*x
+a)))/(1+(b*x+a)^2)^(1/2))+1/2*I/b*dilog(1-I*(1+I*(b*x+a)))/(1+(b*x+a)^2)^(1/
2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^2 + 2abx + a^2) \arctan(bx + a)}{\sqrt{b^2x^2 + 2abx + a^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^2 \operatorname{atan}(a + bx)}{\sqrt{a^2 + 2abx + b^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2 \arctan(bx + a)}{\sqrt{b^2x^2 + 2abx + a^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

$$3.68 \quad \int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}} dx$$

**Optimal.** Leaf size=281

$$\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} + \frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} - \frac{\sqrt{c(a+bx)^2+c}}{2bc} + \frac{i\sqrt{(a+bx)^2+1}}{2b\sqrt{c(a+bx)^2+c}}$$

```
[Out] -Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcTan[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt[c + c*(a + b*x)^2])
```

**Rubi [A]** time = 0.331005, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {5057, 4952, 261, 4890, 4886}

$$\frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} + \frac{i\sqrt{(a+bx)^2+1}\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} - \frac{\sqrt{c(a+bx)^2+c}}{2bc} + \frac{i\sqrt{(a+bx)^2+1}}{2b\sqrt{c(a+bx)^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2], x]
```

```
[Out] -Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcTan[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)])]/(b*Sqrt[c + c*(a + b*x)^2])
```

#### Rule 5057

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.)*(A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)^(q_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(C/d^2 + (C*x^2)/d^2)^q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

#### Rule 4952

```
Int[(((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/Sqrt[(d_.) + (e_.)*(x_.)^2], x_Symbol] := Simp[(f*(f*x)^(m-1)*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x])^p)/(c^2*d*m), x] + (-Dist[(b*f*p)/(c*m), Int[((f*x)^(m-1)*(a + b*ArcTan[c*x])^(p-1))/Sqrt[d + e*x^2], x], x] - Dist[(f^2*(m-1))/(c^2*m), Int[((f*x)^(m-2)*(a + b*ArcTan[c*x])^p)/Sqrt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

#### Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

#### Rule 4890

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_)^(p_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && !GtQ[d, 0]
```

#### Rule 4886

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(-2*I*(a + b*ArcTan[c*x])*ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]])/(c*Sqrt[d]), x] + (Simp[(I*b*PolyLog[2, -(I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c*x]])/(c*Sqrt[d]), x] - Simp[(I*b*PolyLog[2, (I*Sqrt[1 + I*c*x])/Sqrt[1 - I*c*x]])/(c*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

#### Rubi steps

$$\begin{aligned} \int \frac{(a + bx)^2 \tan^{-1}(a + bx)}{\sqrt{(1 + a^2)c + 2abcx + b^2cx^2}} dx &= \frac{\text{Subst}\left(\int \frac{x^2 \tan^{-1}(x)}{\sqrt{c + cx^2}} dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx)\sqrt{c + c(a + bx)^2} \tan^{-1}(a + bx)}{2bc} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c + cx^2}} dx, x, a + bx\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c + cx^2}} dx, x, a + bx\right)}{2b} \\ &= -\frac{\sqrt{c + c(a + bx)^2}}{2bc} + \frac{(a + bx)\sqrt{c + c(a + bx)^2} \tan^{-1}(a + bx)}{2bc} - \frac{\sqrt{1 + (a + bx)^2} \text{Subst}\left(\int \frac{1}{\sqrt{c + cx^2}} dx, x, a + bx\right)}{2b\sqrt{c}} \\ &= -\frac{\sqrt{c + c(a + bx)^2}}{2bc} + \frac{(a + bx)\sqrt{c + c(a + bx)^2} \tan^{-1}(a + bx)}{2bc} + \frac{i\sqrt{1 + (a + bx)^2} \tan^{-1}\left(\frac{a + bx}{i}\right)}{b\sqrt{c}} \end{aligned}$$

**Mathematica [A]** time = 0.133035, size = 189, normalized size = 0.67

$$\frac{\sqrt{a^2 + 2abx + b^2x^2 + 1} \left( -i \text{PolyLog}\left(2, -ie^{i \tan^{-1}(a+bx)}\right) + i \text{PolyLog}\left(2, ie^{i \tan^{-1}(a+bx)}\right) - \sqrt{(a + bx)^2 + 1} + (a + bx)\sqrt{(a + bx)^2 + 1} \right)}{2b\sqrt{c}(a^2 + 2abx + b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2], x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x] - ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])]) + ArcTan[a + b*x]*Log[1 + I*E^(I*ArcTan[a + b*x])]) - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I*PolyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b*Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)])
```

**Maple [A]** time = 1.125, size = 222, normalized size = 0.8

$$\frac{\arctan(bx+a)xb + \arctan(bx+a)a - 1}{2bc} \sqrt{c(-i+a+bx)(i+a+bx)} - \frac{i}{bc} \left( i \arctan(bx+a) \ln \left( 1 + i(1+i(bx+a)) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+b^2\*c\*x^2)^(1/2),x)

[Out] 1/2\*(arctan(b\*x+a)\*x\*b+arctan(b\*x+a)\*a-1)\*(c\*(-I+a+b\*x)\*(I+a+b\*x))^(1/2)/b/c-1/2\*I\*(I\*arctan(b\*x+a)\*ln(1+I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))-I\*arctan(b\*x+a)\*ln(1-I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))+dilog(1+I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))-dilog(1-I\*(1+I\*(b\*x+a)))/(1+(b\*x+a)^2)^(1/2))\*(c\*(-I+a+b\*x)\*(I+a+b\*x))^(1/2)/(b^2\*x^2+2\*a\*b\*x+a^2+1)^(1/2)/b/c

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/2),x,algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(b^2x^2 + 2abx + a^2) \arctan(bx + a)}{\sqrt{b^2cx^2 + 2abcx + (a^2 + 1)c}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/2),x,algorithm="fricas")

[Out] integral((b^2\*x^2 + 2\*a\*b\*x + a^2)\*arctan(b\*x + a)/sqrt(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*atan(b\*x+a)/((a\*\*2+1)\*c+2\*a\*b\*c\*x+c\*x\*\*2\*b\*\*2)\*\*(1/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2 \arctan(bx + a)}{\sqrt{b^2cx^2 + 2abcx + (a^2 + 1)c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/2),x,  
algorithm="giac")

[Out] integrate((b\*x + a)^2\*arctan(b\*x + a)/sqrt(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c), x)

$$3.69 \quad \int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$$

**Optimal.** Leaf size=29

$$\text{Unintegrable}\left(\frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}, x\right)$$

[Out] Unintegrable[((a + b\*x)^2\*ArcTan[a + b\*x])/(1 + (a + b\*x)^2)^(1/3), x]

**Rubi [A]** time = 0.131172, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((a + b\*x)^2\*ArcTan[a + b\*x])/(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3), x]

[Out] Defer[Subst][Defer[Int][(x^2\*ArcTan[x])/(1 + x^2)^(1/3), x], x, a + b\*x]/b

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx = \frac{\text{Subst}\left(\int \frac{x^2 \tan^{-1}(x)}{\sqrt[3]{1+x^2}} dx, x, a+bx\right)}{b}$$

**Mathematica [A]** time = 1.43275, size = 181, normalized size = 6.24

$$3((a+bx)^2+1)^{2/3} \left( \frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left(\left\{1, \frac{4}{3}, \frac{4}{3}\right\}, \left\{\frac{11}{6}, \frac{7}{3}\right\}, \frac{1}{(a+bx)^2+1}\right)}{((a+bx)^2+1)^2} + \Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right) \left( \frac{24(a+bx)\tan^{-1}(a+bx)}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*x)^2\*ArcTan[a + b\*x])/(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3), x]

[Out] (-3\*(1 + (a + b\*x)^2)^(2/3)\*((5\*2^(1/3)\*Sqrt[Pi]\*Gamma[5/3]\*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b\*x)^2)^(-1)]/(1 + (a + b\*x)^2)^2 + Gamma[11/6]\*Gamma[7/3]\*(15 + 90/(1 + (a + b\*x)^2) + (24\*(a + b\*x)\*ArcTan[a + b\*x]\*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b\*x)^2)^(-1)]/(1 + (a + b\*x)^2)^2 + 5\*ArcTan[a + b\*x]\*(-4\*(a + b\*x) + 6\*Sin[2\*ArcTan[a + b\*x]])))/(140\*b\*Gamma[11/6]\*Gamma[7/3])

**Maple [A]** time = 1.329, size = 0, normalized size = 0.

$$\int (bx+a)^2 \arctan(bx+a) \frac{1}{\sqrt[3]{b^2x^2+2xab+a^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)`

[Out] `int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2 \arctan(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(b^2x^2 + 2abx + a^2) \arctan(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas")`

[Out] `integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^2 \operatorname{atan}(a + bx)}{\sqrt[3]{a^2 + 2abx + b^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)`

[Out] `Integral((a + b*x)**2*atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2 \arctan(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

$$3.70 \quad \int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable} \left( \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}, x \right)$$

[Out] Unintegrable[((a + b\*x)^2\*ArcTan[a + b\*x])/(c + c\*(a + b\*x)^2)^(1/3), x]

**Rubi [A]** time = 0.17908, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((a + b\*x)^2\*ArcTan[a + b\*x])/((1 + a^2)\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2)^(1/3), x]

[Out] Defer[Subst][Defer[Int] [(x^2\*ArcTan[x])/(c + c\*x^2)^(1/3), x], x, a + b\*x]/b

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}} dx = \frac{\text{Subst} \left( \int \frac{x^2 \tan^{-1}(x)}{\sqrt[3]{c+cx^2}} dx, x, a+bx \right)}{b}$$

**Mathematica [A]** time = 0.312256, size = 225, normalized size = 7.26

$$\frac{3\sqrt[3]{a^2+2abx+b^2x^2+1}((a+bx)^2+1)^{2/3} \left( \frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right)\text{HypergeometricPFQ}\left[\left\{1, \frac{4}{3}, \frac{4}{3}\right\}, \left\{\frac{11}{6}, \frac{7}{3}\right\}, \frac{1}{(a+bx)^2+1}\right]}{((a+bx)^2+1)^2} + \Gamma\left(\frac{11}{6}\right) \right)}{140b\Gamma\left(\frac{11}{6}\right) \Gamma\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*x)^2\*ArcTan[a + b\*x])/((1 + a^2)\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2)^(1/3), x]

[Out] (-3\*(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3)\*(1 + (a + b\*x)^2)^(2/3)\*((5\*2^(1/3))\*Sqrt[Pi]\*Gamma[5/3]\*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b\*x)^2)^(-1)]/(1 + (a + b\*x)^2)^2 + Gamma[11/6]\*Gamma[7/3]\*(15 + 90/(1 + (a + b\*x)^2) + (24\*(a + b\*x)\*ArcTan[a + b\*x]\*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b\*x)^2)^(-1)]/(1 + (a + b\*x)^2)^2 + 5\*ArcTan[a + b\*x]\*(-4\*(a + b\*x) + 6\*Sin[2\*ArcTan[a + b\*x]])))/(140\*b\*(c\*(1 + a^2 + 2\*a\*b\*x + b^2\*x^2)^(1/3)\*Gamma[11/6]\*Gamma[7/3])

---

**Maple [A]** time = 1.3, size = 0, normalized size = 0.

$$\int (bx + a)^2 \arctan(bx + a) \frac{1}{\sqrt[3]{(a^2 + 1)c + 2abcx + b^2cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+b^2\*c\*x^2)^(1/3),x)

[Out] int((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+b^2\*c\*x^2)^(1/3),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2 \arctan(bx + a)}{(b^2cx^2 + 2abcx + (a^2 + 1)c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/3),x,  
algorithm="maxima")

[Out] integrate((b\*x + a)^2\*arctan(b\*x + a)/(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c)^(1/3), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^2 + 2abx + a^2) \arctan(bx + a)}{(b^2cx^2 + 2abcx + (a^2 + 1)c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/3),x,  
algorithm="fricas")

[Out] integral((b^2\*x^2 + 2\*a\*b\*x + a^2)\*arctan(b\*x + a)/(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c)^(1/3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*atan(b\*x+a)/((a\*\*2+1)\*c+2\*a\*b\*c\*x+c\*x\*\*2\*b\*\*2)\*\*(1/3),  
x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^2 \arctan(bx + a)}{(b^2cx^2 + 2abcx + (a^2 + 1)c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arctan(b\*x+a)/((a^2+1)\*c+2\*a\*b\*c\*x+c\*x^2\*b^2)^(1/3), x,  
algorithm="giac")

[Out] integrate((b\*x + a)^2\*arctan(b\*x + a)/(b^2\*c\*x^2 + 2\*a\*b\*c\*x + (a^2 + 1)\*c)  
^(1/3), x)

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```



```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))]
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```